ELEMENTARY SCHOOL MATHEMATICS

SECOND EDITION

BOOK

4

BOOK

4

ELEMENTARY SCHOOL MATHEMATICS

SECOND EDITION

Authors

ROBERT E. EICHOLZ

PHARES G. O'DAFFER

Consulting authors

CHARLES F. BRUMFIEL

MERRILL E. SHANKS

ADDISON-WESLEY PUBLISHING COMPANY
Menlo Park, California • Reading, Massachusetts • London • Don Mills, Ontario

Book 4

Table of Contents

1 PLACE VALUE 1
- Sets, numbers, and numerals
- Grouping by tens
- Reading numerals, 2- to 7-digit
- Inequalities
- Introduction of billions and trillions
- Word problems
- Chapter review

2 ADDITION AND SUBTRACTION . 22
- Sets
- Addition and subtraction concepts
- Equations and solutions
- Inverse relation
- Number line
- Basic principles
- Facts through 18
- Column addition, 2- and 3-digit
- Word problems
- Use of the basic principles
- Reasoning
- Addition with regrouping
- Subtraction with regrouping
- Dollar and decimal-point notation for money
- Cumulative review
- Chapter review

GEOMETRY UNIT 1 64
- Simple closed curves
- Circles and points
- Circles and pairs of points
- 3 points and a circle

3 MULTIPLICATION AND DIVISION 72
- Number in a set
- Number of equivalent sets
- Multiplication and division concept
- Repeated addition and repeated subtraction
- Number-line games
- Basic principles
- Facts through 81
- Multiplication table
- Inverse relation
- Product sets
- Word problems
- Cumulative review
- Chapter review

4 MEASUREMENT 112
- Length
  - arbitrary units
  - inches and centimeters
  - inches, feet, yards, miles
- Area
  - counting squares
  - approximation
- Volume
  - counting cubes
  - liquid
- Perimeter
  - of polygons
  - comparison with area
- Word problems
- Chapter review

Copyright © *1968 by* ADDISON-WESLEY PUBLISHING COMPANY, INC.
Philippines Copyright 1968

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of the publisher. Printed in the United States of America. Published simultaneously in Canada.

HIJK7543210

5 SPECIAL PRODUCTS AND QUOTIENTS 134
- Products that are multiples of 10, 100, and 1000
- Special missing factors
- Quotients
- Use of the basic principles
- Reasoning
- A summary of the basic principles
- Chapter review

GEOMETRY UNIT 2 148
- Tangents to a circle
- Central and inscribed angles
- Inscribed circles
- Circumscribed circles

6 ESTIMATION 156
- Estimating sums
- Estimating differences
- Estimating products
- Estimating missing factors
- Estimating quotients
- Special attention to estimates leading to development of the division algorithm
- Estimation in word problems
- Cumulative review
- Chapter review

7 MULTIPLYING 168
- Use of the multiplication-addition principle
- Estimation
- Inequalities
- Products involving factors with two and three digits
- Word problems
- Volume and area exercises
- Cumulative review
- Chapter review

GEOMETRY UNIT 3 192
- Edges and faces of cubes
- Patterns for cubes
- Triangular pyramids
- Cylinders and cones

8 DIVIDING 200
- Estimation
- Inverse relation
- Quotients and remainders
- Repeated subtraction
- Inequalities
- Reasoning
- Long-division process (2-digit divisors)
- Averages
- Word problems
- Cumulative review
- Chapter review

9 NUMBER THEORY 232
- Even and odd numbers
- Multiples and factors
- Common factors and greatest common factors
- Prime numbers
- Function game
- Cumulative review
- Chapter review

COORDINATE GEOMETRY UNIT 4 . 244
- Using coordinates
- Graphing number pairs
- Graphing functions
- Negative numbers and graphing

Illustrations by Dick Cole

10 FRACTIONS 252
Fractions and number pairs
Fractions and sets
Fractions and parts of an object
Equivalent fractions
Sets of equivalent fractions
Fractions and measurement
A check (definition) for equivalent fractions
Lower, higher, and lowest terms
Improper fractions
Word problems
Cumulative review
Chapter review

11 RATIONAL NUMBERS 290
Fractions and numbers
On the number line
Names for rational numbers
Equality of rational numbers
Inequalities for rational numbers
Rational numbers greater than one
Addition of rational numbers (intuitive)
Whole numbers and rational numbers
Mixed numerals
Use in linear measurement
Word problems
Cumulative review
Chapter review

SUPPLEMENTARY EXERCISES . . 320

TABLES OF MEASURES . . . 337

BOOKS TO EXPLORE . . . 338

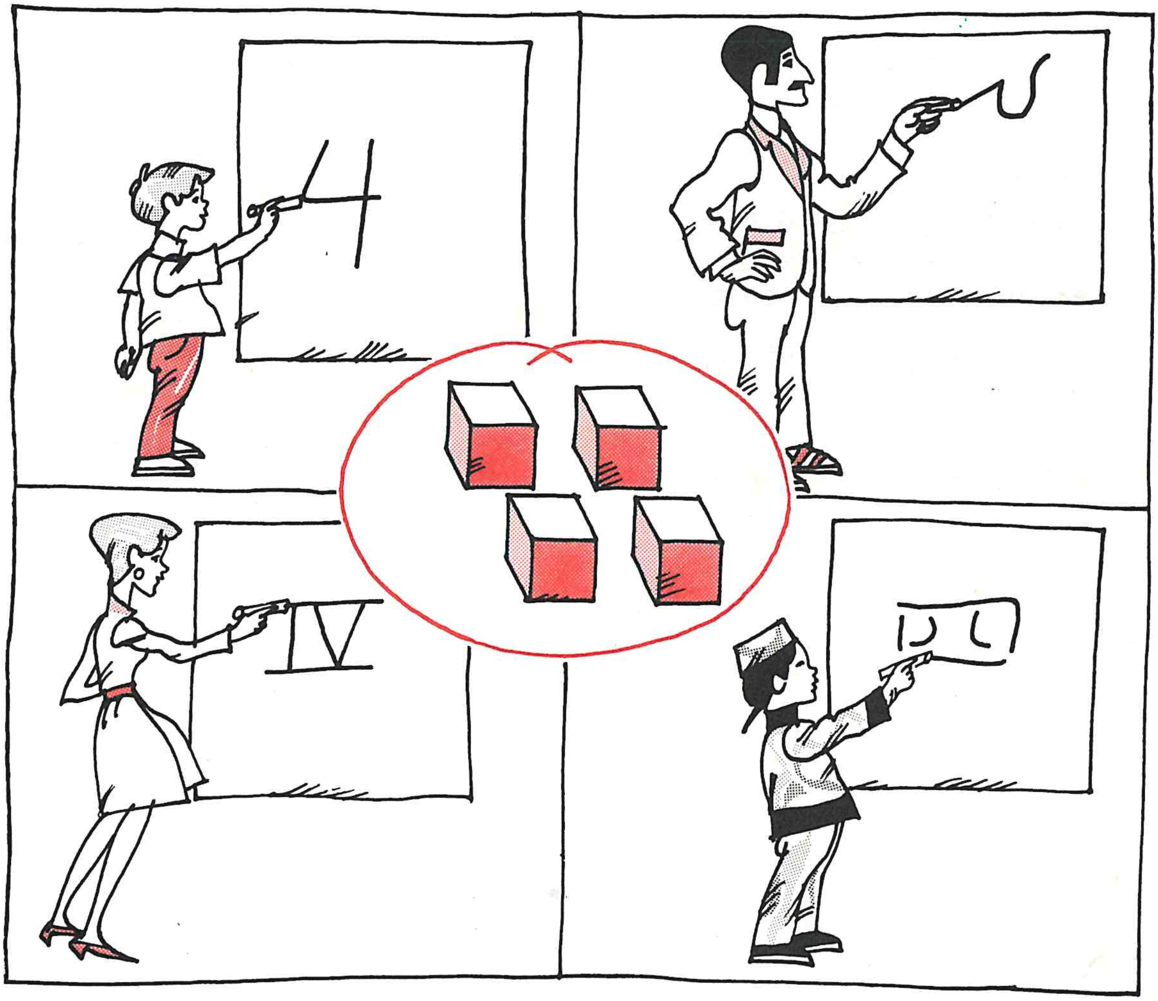
GLOSSARY 341

INDEX 345

CHAPTER 1 Place Value

Sets, numbers, and numerals

We look at a set and think of the number for that set. We can then write a numeral for that number. Many different numerals represent the same number. For example, there are many different ways to write a numeral for the number of cubes in the set ring below.



▶ Each person is **thinking** of the same **number.**

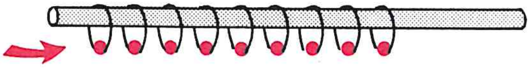
▶ Each person is **writing** a different **numeral** for the number.

Grouping by tens—2-digit numerals

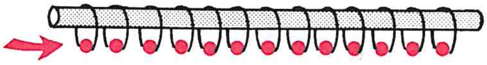
You can write the numeral for any whole number by using one or more of these **digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.**

DISCUSSION EXERCISES

1. Use one of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 to tell how many beads are in this picture.



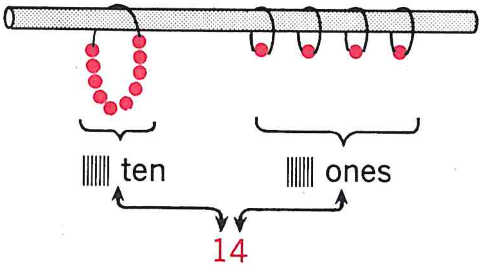
2. Use two of the digits to tell how many beads are in this picture.



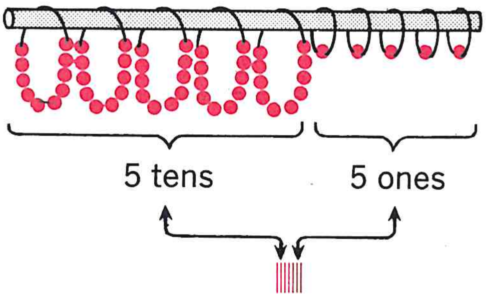
3. We agree to group by tens. The digit on the left tells how many tens. The digit on the right tells how many single beads.

[A] How many tens are shown?

[B] How many single beads are shown?

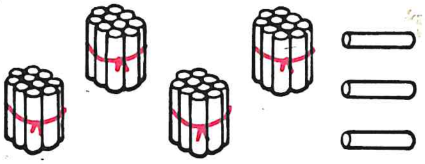


4. The two digits in this numeral are alike. As in exercise 3, each digit's place in the numeral gives important information about tens and ones. Find the numeral for the number of this set.



5. Find the numeral for the number of each set below.

A

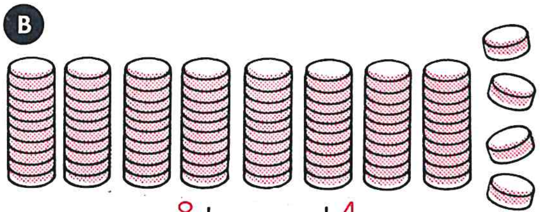


4 tens and 3

40 + 3

We write |||||.

B



8 tens and 4

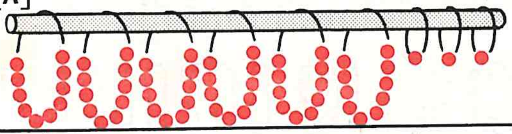
80 + 4

We write |||||.

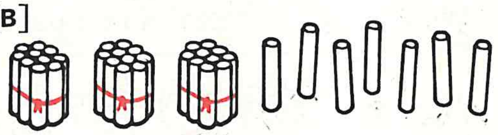
EXERCISES

1. Give the number for each set. Try not to count higher than 10.

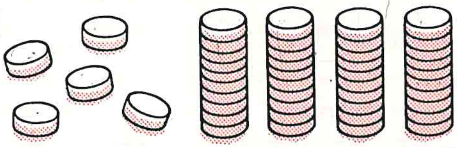
[A]



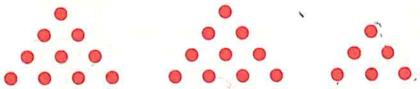
[B]



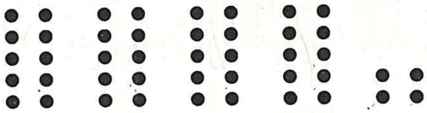
[C]



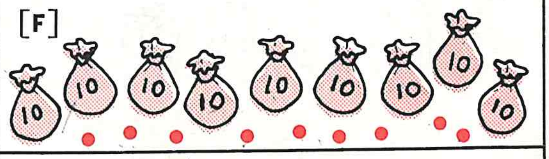
[D]



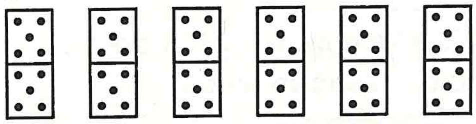
[E]



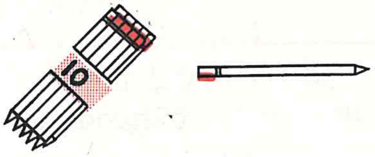
[F]



[G]



[H]



2. Write the 2-digit numeral for each exercise.

[A] 5 tens and 9
[B] 4 tens and 6
[C] 6 tens and 4
[D] 3 tens and 0
[E] 90 + 2
[F] 20 + 9
[G] 80 + 8
[H] 90 + 9
[I] 8 tens and 10
[J] 60 + 8
[K] 8 tens and 13
[L] 70 + 10

3. Give the correct digit for each ▨.

[A] 54 means ▨ tens and ▨ ones.
[B] 45 means ▨ tens and ▨ ones.
[C] 39 means ▨ ones and ▨ tens.
[D] 93 means ▨ ones and ▨ tens.

4. Solve the equations.

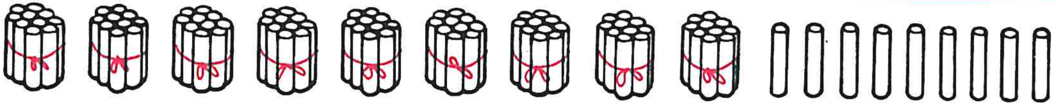
[A] $32 = \boxed{n} + 2$
[B] $57 = 50 + \boxed{n}$
[C] $78 = \boxed{n} + 8$
[D] $99 = 90 + \boxed{n}$

5. [A] Give the largest 2-digit number that has 7 as one of its digits.
[B] Give the smallest 2-digit number that has 3 as one of its digits.
[C] Give the smallest 2-digit number with both digits alike.
[D] Give the largest 2-digit number that has 0 as one of its digits.
[E] Give the largest 2-digit number with both digits alike.

3-digit numerals

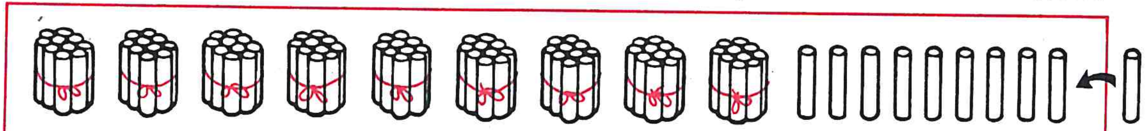
DISCUSSION EXERCISES

1. What 2-digit numeral do we use to tell how many sticks are shown below?



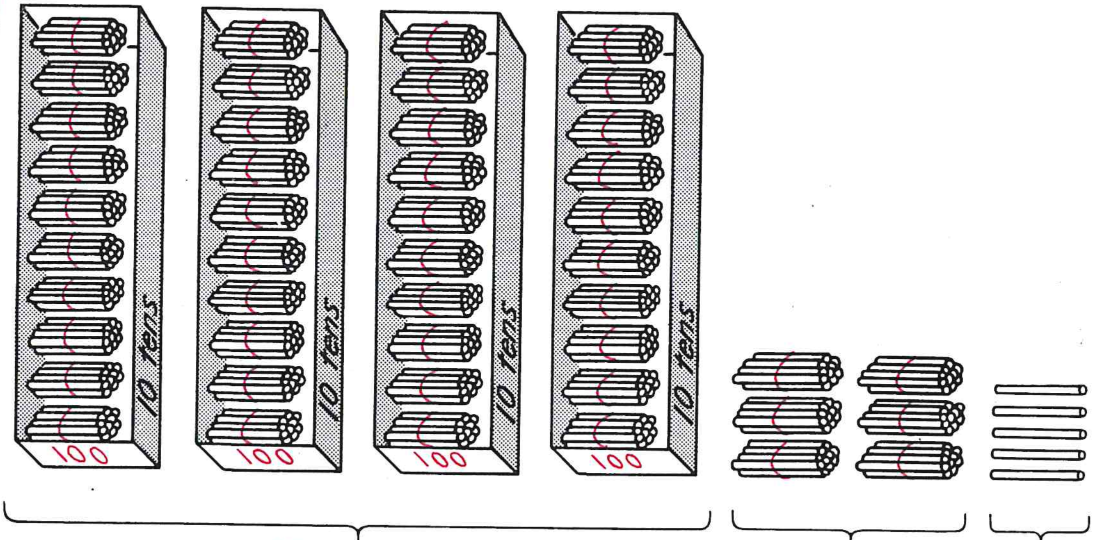
9 tens and 9. We write ▥.

2. What 3-digit numeral do we use to tell how many sticks are shown below?



10 tens and 0. We write ▥.
We say, "one hundred."

3. In a 3-digit numeral, the digit on the left tells how many hundreds. Give the number of hundreds, tens, and ones shown below.



▥ hundreds ▥ tens ▥ ones

There are 4 6 5 sticks.

The 4 is in the hundreds place. We say, "four hundred sixty-five" for 465.

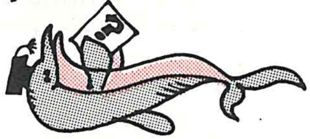
EXERCISES

1. Write the numeral for each exercise.

[A] three hundreds, five tens, and two
[B] eight hundreds, three tens, and seven
[C] nine hundreds, eight ones, and three tens
[D] six tens, four ones, and eight hundreds
[E] five ones, four tens, and three hundreds
[F] one hundred, nine tens, and four
[G] three hundred ninety-four
[H] seven hundred sixty-eight
[I] four hundred forty
[J] four hundred four

Think

I'm one more ten than 90,
10 tens in all you see.
My first letter is an H,
My last one is a D.



WHO AM I?

2. Give the missing digits.

[A] 563 means ▨ hundreds, ▨ tens, and ▨ ones.
[B] 702 means ▨ hundreds, ▨ tens, and ▨ ones.
[C] 300 means ▨ hundreds, ▨ tens, and ▨ ones.
[D] 395 means ▨ tens, ▨ hundreds, and ▨ ones.
[E] 593 means ▨ ones, ▨ hundreds, and ▨ tens.
[F] 673 means ▨ hundreds, ▨ ones, and ▨ tens.

3. Solve the equations.

[A] $324 = 300 + 20 + n$
[B] $617 = 600 + n + 7$
[C] $836 = n + 30 + 6$
[D] $409 = 400 + n + 9$
[E] $555 = n + 50 + 5$
[F] $760 = 700 + 60 + n$
[G] $999 = 900 + n + 9$
[H] $737 = n + 30 + 7$

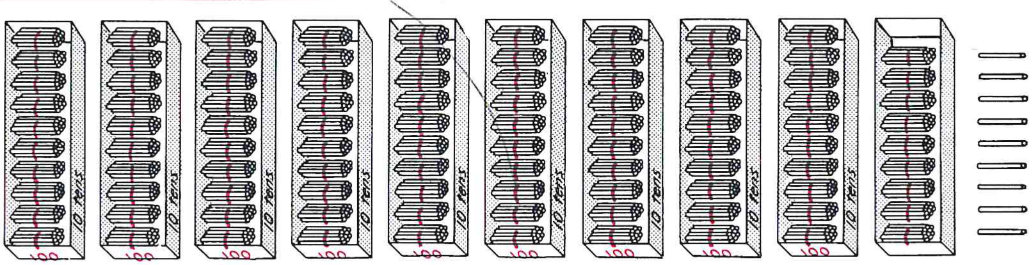
★ **4.** Solve the equations.

[A] $99 + 1 = n$
$100 + 99 = n$
[B] $199 = 100 + n$
$199 + 1 = n$
[C] $299 = 200 + n$
$299 + 1 = n$
[D] $399 = 300 + n$
$399 + 1 = n$
[E] $599 = n + 99$
$599 + 1 = n$
[F] $899 = 800 + n$
$899 + 1 = n$

4-digit numerals

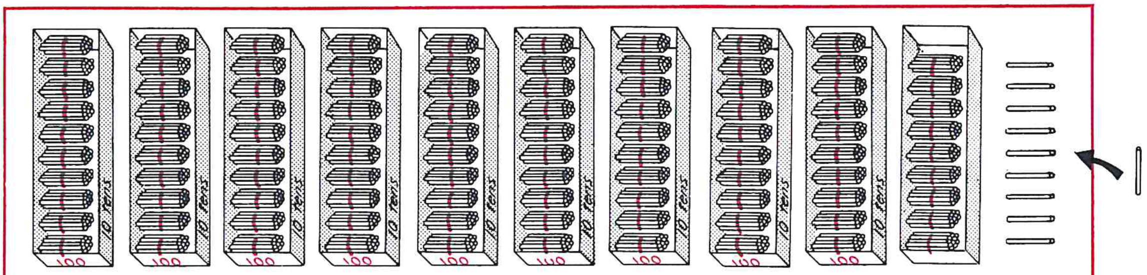
DISCUSSION EXERCISES

1. What 3-digit numeral do we use to tell how many sticks are shown below?



9 hundreds, 9 tens, and 9. We write ▥.

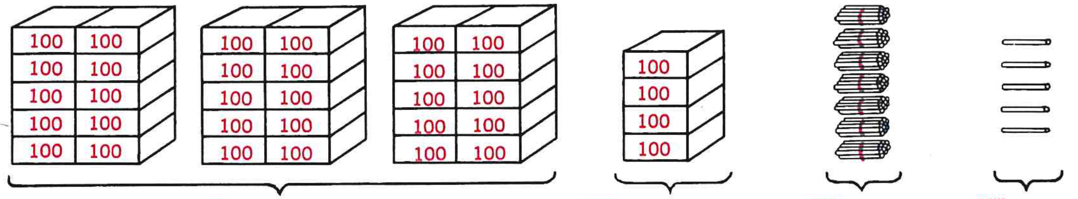
2. What 4-digit numeral do we use to tell how many sticks are shown below?



10 hundreds, 0 tens, and 0. We write ▥.

We say, "one thousand."

3. In a 4-digit numeral, the digit on the left tells how many thousands. Give the number of thousands, hundreds, tens, and ones shown below.



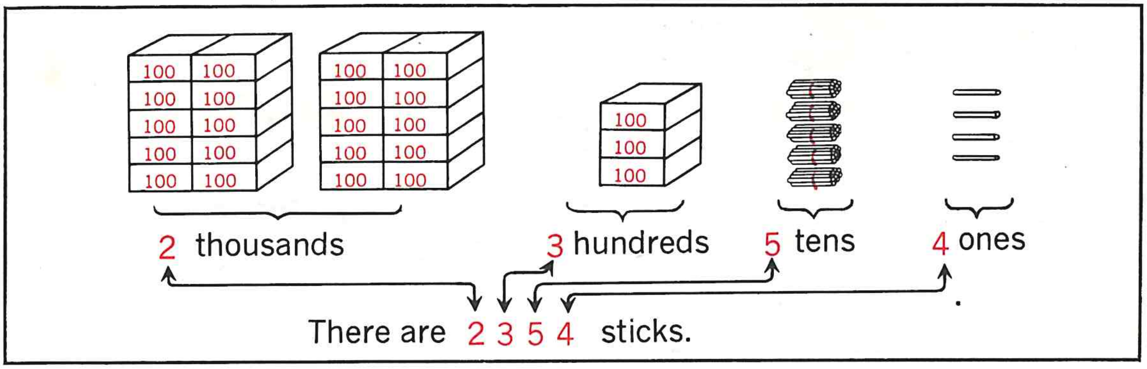
▥ thousands ▥ hundreds ▥ tens ▥ ones

There are 3 4 7 5 sticks.

The 3 is in the thousands place. We say,
"three thousand four hundred seventy-five" for 3475.

EXERCISES

1. Write the numeral for the number of sticks indicated.



There are 2 3 5 4 sticks.

2. Give the missing digits in the order indicated.

[A] 3476 means ▨ thousands, ▨ hundreds, ▨ tens, and ▨ ones.
[B] 6743 means ▨ thousands, ▨ hundreds, ▨ tens, and ▨ ones.
[C] 4007 means ▨ thousands, ▨ hundreds, ▨ tens, and ▨ ones.
[D] 4070 means ▨ thousands, ▨ hundreds, ▨ tens, and ▨ ones.
[E] 9803 means ▨ tens, ▨ thousands, ▨ hundreds, and ▨ ones.

3. Write the correct numeral for each exercise.

[A] six thousands, five hundreds, four tens, and three ones
[B] three thousand seven hundred sixty-two
[C] nine thousands, zero hundreds, four tens, and five
[D] nine thousand thirty-five
[E] four thousands, six hundreds, zero tens, and seven ones
[F] four thousand five hundred seven
[G] nine thousands, zero hundreds, zero tens, and three ones
[H] nine thousand two
[I] nine thousand nine hundred ninety-nine

4. Solve the equations.

[A] $2856 = 2000 + 800 + 50 + \boxed{n}$
[B] $3495 = 3000 + 400 + \boxed{n} + 5$
[C] $6284 = 6000 + \boxed{n} + 80 + 4$
[D] $9765 = \boxed{n} + 700 + 60 + 5$
[E] $4271 = 4000 + 200 + \boxed{n} + 1$
[F] $2465 = 2000 + \boxed{n} + 60 + 5$

Think

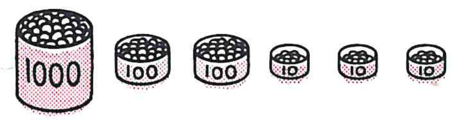
I'm a special number,
With 3 digits and no more.
Increase me by just one,
And the digit count is 4.

WHO AM I?

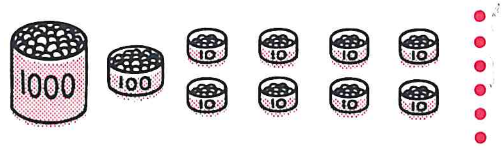
Comparison—place value

The numerals on the cans tell how many marbles are inside. In each exercise tell which picture (**A** or **B**) indicates the greater number of marbles.

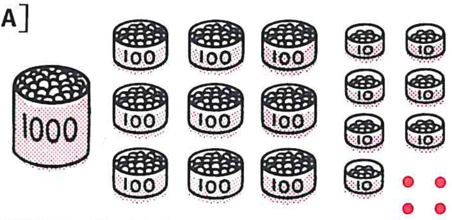
1. [**A**]



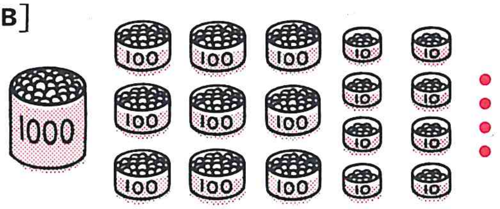
[**B**]



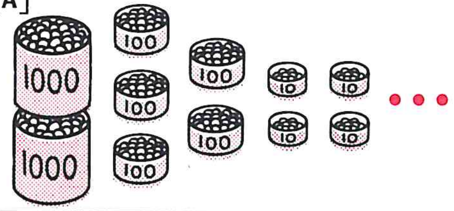
2. [**A**]



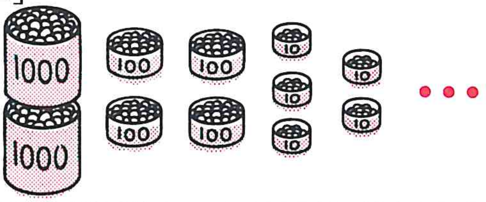
[**B**]



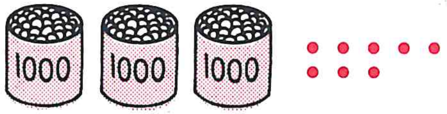
3. [**A**]



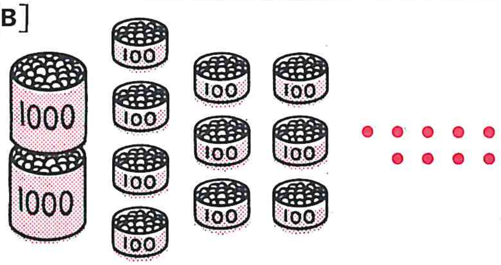
[**B**]



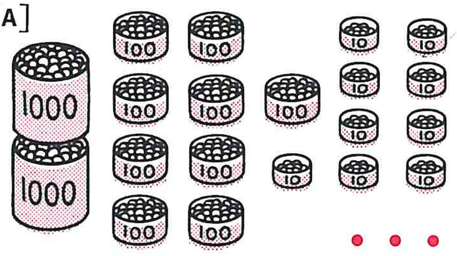
4. [**A**]



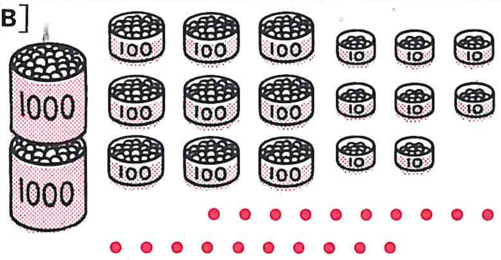
[**B**]



5. [**A**]



[**B**]



6. For each part above, give the number that tells how many marbles.

Place-value exercises

1. Solve the equations.

[A] $683 = 600 + \boxed{n} + 3$
[B] $896 = \boxed{n} + 90 + 6$
[C] $5436 = 5000 + 400 + 30 + \boxed{n}$
[D] $3579 = 3000 + 500 + \boxed{n} + 9$
[E] $6738 = 6000 + \boxed{n} + 30 + 8$
[F] $2645 = \boxed{n} + 600 + 40 + 5$
[G] $8604 = 8000 + 600 + \boxed{n} + 4$
[H] $3699 = 3000 + \boxed{n} + 90 + 9$

2. In each exercise, write the sign (<, >) that should go in each ●. Then write the words (greater than, less than) that should go in each blank.

[A] 730 ● 73
730 is __?__ 73.

[B] 703 ● 730
703 is __?__ 730.

[C] 7300 ● 7030
7300 is __?__ 7030.

[D] 593 ● 583
593 is 10 __?__ 583.

[E] 5635 ● 5365
5635 is __?__ 5365.

[F] 5423 ● 5523
5423 is 100 __?__ 5523.

[G] 6742 ● 5742
5742 is 1000 __?__ 6742.

[H] 3762 ● 3672
3762 is __?__ 3672.

[I] 5461 ● 5164
5164 is __?__ 5461.

[J] 8203 ● 8230
8203 is __?__ 8230.

3. Give the number that is 100 more than

[A] 7.
[B] 27.
[C] 327.
[D] 5327.
[E] 2007.
[F] 2700.
[G] 2341.
[H] 900.
[I] 8900.
[J] 9900.

4. Give the number that is 1000 less than

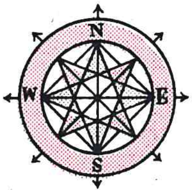
[A] 2000.
[B] 5234.
[C] 5003.
[D] 1900.
[E] 1004.
[F] 1000.
[G] 10,000.
[H] 5900.

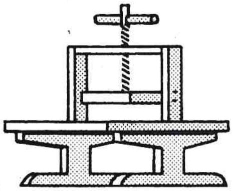
★**5.** Give the number that is 110 more than

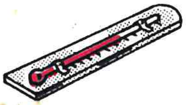
[A] 2000.
[B] 2500.
[C] 3740.
[D] 3462.
[E] 5445.
[F] 2920.
[G] 3490.
[H] 8990.

★**6.** [A] Give the largest 3-digit number that has the digits 2 and 5.
[B] Give the smallest 3-digit number that has only one 0 digit.
[C] Give the smallest 4-digit number that has the digits 5 and 3.
[D] Give the largest 4-digit number that has the digits 4, 7, 8, and 5.
[E] Give the largest 4-digit number with no two digits alike.

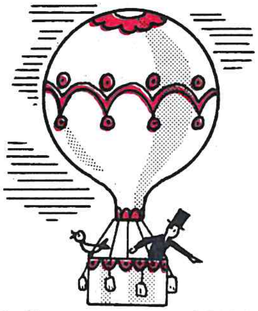
INVENTIONS


Magnetic compass 900


Color printing 1457


Thermometer 1593

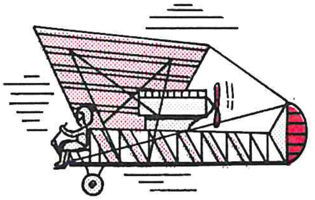
Telescope 1609


Balloon 1783

Match 1827

Bicycle 1842

Safety pin 1849


Airplane 1903

A decade is a period of 10 years.
A century is a period of 100 years.
A millennium is a period of 1000 years.

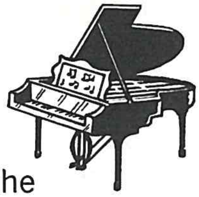
You have lived about a decade. Very few people have lived for a century.

Use the chart and your knowledge of place value to answer the questions.

1. The sewing machine was invented 3 years later than the match. When was the sewing machine invented?
2. The bicycle tire was invented 4 decades later than the safety pin. When was the bicycle tire invented?
3. The adding machine was invented 2 centuries earlier than the bicycle. When was the adding machine invented?

4. The match was invented 6 decades before the gasoline auto. When was the gasoline auto invented?

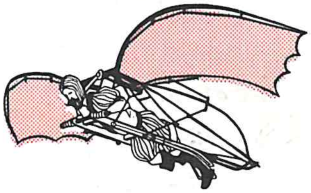
5. The piano was invented 10 decades after the telescope. Give the year in which the piano was invented.



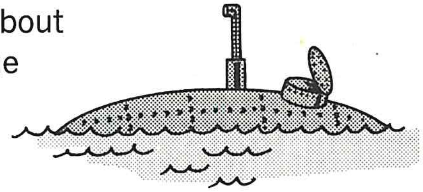
6. The fountain pen was invented 1 century and 1 year after the balloon. In what year was the fountain pen invented?

7. The movie machine (projector) was invented 4 centuries and 1 year after Columbus discovered America. When was the projector invented?

8. Leonardo Da Vinci learned about flight by watching birds. He drew a picture of an airplane about 400 years before the airplane was invented. In what year did Da Vinci draw the picture?



9. The first modern naval submarine was built about 10 centuries after Chinese sailors invented the magnetic compass. About when was the first naval submarine built?



10. What will be the date one millennium after the invention of the thermometer?

11. Suppose that when the first sheet of color printing was made it was sealed in a block of stone. If it is discovered 3000 years later, what year will that be?

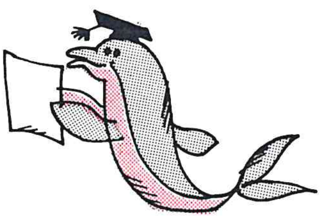
Thousands

DISCUSSION EXERCISES

1. In the numeral 4253, the number of thousands is 4. We read, "four thousand two hundred fifty-three."
   - [A] What is the number of thousands for 8156?
   - [B] Read this numeral.
2. In the numeral 34,253, the number of thousands is 34. We read, "thirty-four thousand, two hundred fifty-three."
   - [A] What is the number of thousands for 38,156?
   - [B] Read this numeral.
3. In the numeral 634,253, the number of thousands is 634. We read, "six hundred thirty-four thousand, two hundred fifty-three."
   - [A] What is the number of thousands for 438,156?
   - [B] Read this numeral.

Think

There are many times that I
Would like to be one more.
Then I would have five digits,
Instead of only four.



WHO AM I?

ORAL EXERCISES

For each numeral, tell how many thousands. Then read the numeral.

1. 8425
2. 6741
3. 2308
4. 5764
5. 8325
6. 27,681
7. 34,912
8. 73,659
9. 84,216
10. 79,843
11. 61,208
12. 60,208
13. 54,347
14. 50,347
15. 80,008
16. 70,080
17. 40,267
18. 80,000
19. 49,000
20. 84,001
21. 256,348
22. 347,286
23. 754,200
24. 654,389
25. 561,219
26. 440,285
27. 700,456
28. 803,347
29. 900,400
30. 605,985
31. 64,247
32. 8295
33. 50,000
34. 706,444
35. 25,000
36. 605,396
37. 7000
38. 82,647
39. 379,642
40. 70,007

EXERCISES

1. Give the number of thousands. For exercise 1**A** write 3.
For exercise 1**B** write 10.

[**A**] 3942
[**B**] 10,536
[**C**] 392,647
[**D**] 408,212
[**E**] 396,000
[**F**] 1852
[**G**] 765
[**H**] 439,000
[**I**] 100,005
[**J**] 33,052
[**K**] 320,003
[**L**] 999,999

2. Write the numeral for each exercise.

[**A**] eight thousands, six hundreds, three tens, and five
[**B**] twenty-eight thousands, four hundreds, two tens, and seven
[**C**] nine hundred thirty-six thousands, two hundreds, and five
[**D**] four thousands, two hundreds, five tens, and three

3. Give the number 1000 greater than

[**A**] 1342
[**B**] 23,157
[**C**] 249,361
[**D**] 399,264
[**E**] 409,317
[**F**] 300,007
[**G**] 300,876
[**H**] 950,304

4. In each numeral below, one digit is red. Give the number for which the red digit stands.

[**A**] 27,341
(Answer: 7000)
[**B**] 28,436
[**C**] 382,984
[**D**] 675,832
[**E**] 637,243
[**F**] 513,200
[**G**] 983,741
[**H**] 200,700
[**I**] 622,317
[**J**] 307,460
[**K**] 999,999
[**L**] 436,872
[**M**] 923,465
[**N**] 345,872
[**O**] 407,600

★**5.** Give the missing numbers.

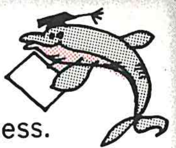
[**A**] $999 + 1 = 900 + \boxed{n} + 1$
$999 + 1 = \boxed{n}$

[**B**] $9999 + 1 = 9000 + \boxed{n} + 1$
$9999 + 1 = \boxed{n}$

[**C**] $99{,}999 + 1 = 90{,}000 + \boxed{n} + 1$
$99{,}999 + 1 = \boxed{n}$

[**D**] $999{,}999 + 1 = 900{,}000 + \boxed{n} + 1$
$999{,}999 + 1 = 900{,}000 + \boxed{n}$
$999{,}999 + 1 = \boxed{n}$

Think



Five digits I possess.
Yet were I just one less,
I'd have five no more
And be left with only four.

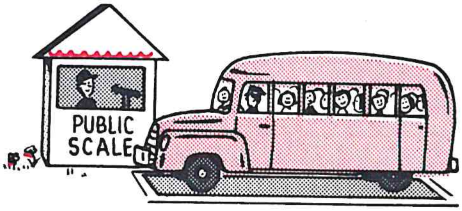
WHO AM I?

How large is a million?

If you count all the hairs on the heads of 10 classmates, you will have counted about 1,000,000 hairs.

A child must live about 5,000,000 minutes to reach his tenth birthday. Very few people live 1,000,000 hours. A person who lived this long would be 114 years old.

If you weigh 66 bus loads of fourth-grade children, the total weight will be about 1,000,000 pounds.



If this arithmetic book had 1,000,000 pages, it would take a bookshelf as long as 12 parked cars to hold the book.

Your heart beats about 1,000,000 times in 10 days.

S	M	T	W	T	F	S
●	☽	1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

The chart below shows three different ways to help you understand one million.

Ⓐ Adding thousands	Ⓑ Counting	Ⓒ Skip counting
1000	999,995	500,000
1000	999,996	600,000
1000 } 1000 addends	999,997	700,000
⋮		
1000	999,998	800,000
+ 1000	999,999	900,000
1,000,000	1,000,000	1,000,000

Millions

DISCUSSION EXERCISES

1. How many thousands does it take to make a million?

2. What whole number comes after
[A] 99? [B] 999? [C] 9999? [D] 99,999? [E] 999,999?

3. What number is 100,000 more than
[A] 300,000? [B] 700,000? [C] 900,000?

4. In the numeral 26,341,285, the number of millions is 26.
We read, "26 million, 341 thousand, 285."
[A] What is the number of millions for 75,284,367?
[B] Read this numeral.

5. In the numeral 496,624,357, the number of millions is 496.
We read, "496 million, 624 thousand, 357."
[A] What is the number of millions for 718,256,591?
[B] Read this numeral.

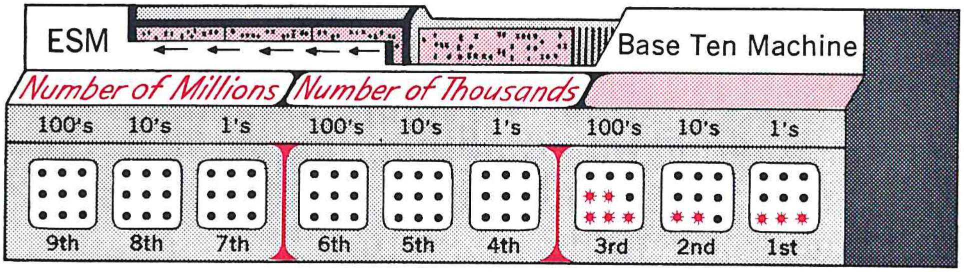
6. Read the following.
[A] 5,000,000
[B] 25,000,000
[C] 625,000,000
[D] 4,200,000
[E] 54,300,000
[F] 954,600,000
[G] 3,230,000
[H] 33,280,000
[I] 563,470,000
[J] 8,438,000
[K] 39,672,800
[L] 867,433,900
[M] 7,234,861
[N] 23,467,495
[O] 534,687,356
[P] 568,356,074

EXERCISES

1. Write the number that is
[A] 100 more than one million.
[B] 1000 more than one million.
[C] 10 more than one million.
[D] 1 more than one million.
[E] 10,000 more than one million.
[F] 100,000 more than one million.
[G] 1,000,000 more than one million.
★[H] 100 less than one million.
★[I] 1000 less than one million.
★[J] 100,000 less than one million.

2. Write the number that is
[A] three million more than 324,562,218.
[B] thirty million more than 812,469,855.
[C] five hundred million more than 283,618,962.

The place value machine

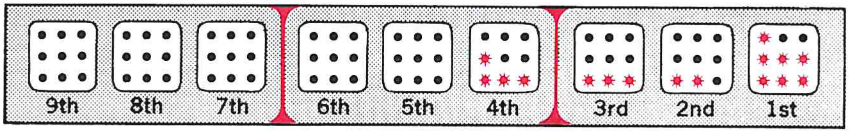

ESM
Base Ten Machine
Number of Millions
Number of Thousands
100's 10's 1's 100's 10's 1's 100's 10's 1's
9th 8th 7th 6th 5th 4th 3rd 2nd 1st

A card gave the machine a signal to remember the number 523. The red lights show how the machine works.

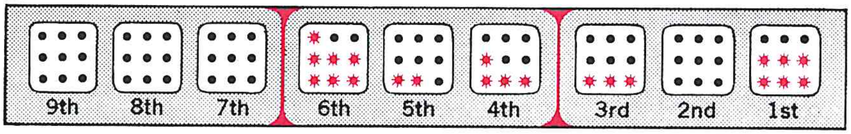
EXERCISES

1. Give the numbers that the machine was signaled to remember.

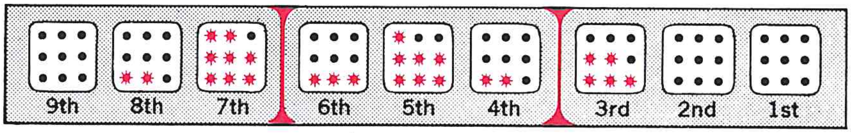
[A]



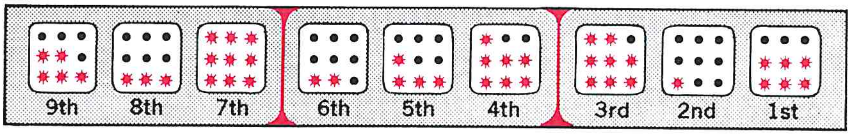
[B]



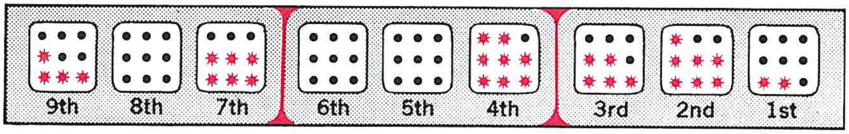
[C]



[D]



[E]

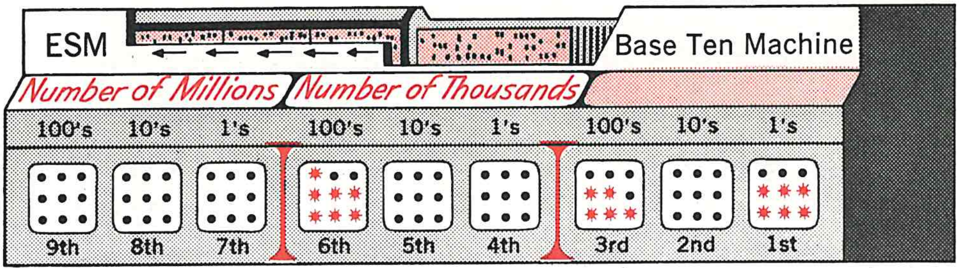


2. The machine was signaled to remember

[A] 25,234. How many lights are on in the 4th place?
[B] 32,406,871. How many lights are on in the 6th place?

3. All the lights are on in the 7th, 8th, and 9th places. No other lights are on. What number was the machine signaled to remember?

★ EXERCISES


ESM
Base Ten Machine
Number of Millions
Number of Thousands
100's 10's 1's 100's 10's 1's 100's 10's 1's
9th 8th 7th 6th 5th 4th 3rd 2nd 1st

1. Give the number the machine was signaled to remember.

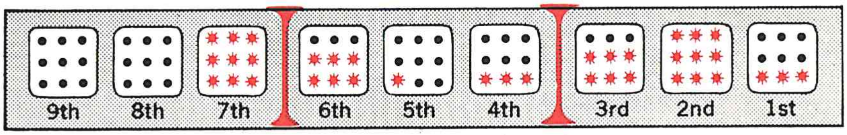
2. A card is put into the machine. It signals the machine to increase the number of ones by 4.
   [A] How many lights are now on in the 1st place?
   [B] How many lights do you think are on in the 2nd place?

3. Another card signals the machine to increase the number of hundreds by 5.
   [A] How many lights are now on in the 3rd place?
   [B] How many lights are on in the 4th place?

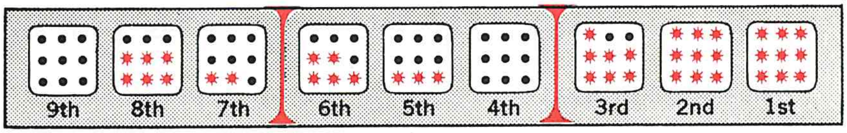
4. Another card signals the machine to increase the number of hundred thousands by 5.
   [A] How many lights are now on in the 6th place?
   [B] How many lights are now on in the 7th place?

5. Give the number that results when each signal is given to the machine shown.

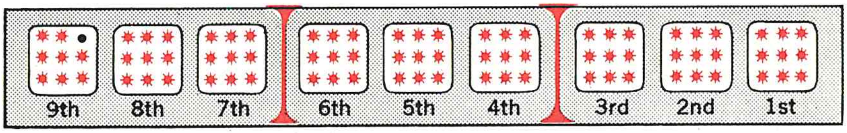
   [A] Signal: Increase the number of tens by 1.



   [B] Signal: Increase the number of ones by 1.



   [C] Signal: Increase the number of ones by 1.



Population*

The 10 United States Cities with the Largest Populations

City	Number of people
Boston, Massachusetts	3,199,000
Chicago, Illinois	6,637,000
Detroit, Michigan	3,972,000
Los Angeles, California	6,776,000
New York, New York	11,260,000
Philadelphia, Pennsylvania	4,617,000
Pittsburgh, Pennsylvania	2,367,000
San Francisco, California	2,935,000
St. Louis, Missouri	2,239,000
Washington, D.C.	2,323,000

1. Is the number of people in Chicago closer to 6,000,000 or 7,000,000?

2. Is the number of people in the Los Angeles area closer to 6,000,000 or 7,000,000?

3. Is the number of people in Pittsburgh closer to 2,000,000 or 3,000,000?

4. We say: The number of people in Philadelphia **(to the nearest million)** is 5,000,000.

   The number of people in Los Angeles (to the nearest million) is 7,000,000.

   The number of people in Boston (to the nearest million) is 3,000,000.

   Give the number of people (to the nearest million) in these cities.

   [A] Detroit
   [B] New York
   [C] Pittsburgh
   [D] Washington, D.C.
   [E] Philadelphia
   [F] San Francisco
   [G] St. Louis
   [H] Chicago

---

*All population data are based on the 1967 World Almanac. The figures are given to the nearest 1000 people and include metropolitan areas.

5. [A] Which city has the greatest number of people?
   [B] Which city has the least number of people?

6. The cities are listed in alphabetical order. List them according to population. Place the name of the city with the most people at the top of your list.

7. New York City has about 10,000,000 more people than Milwaukee, Wisconsin. What is the population of Milwaukee?

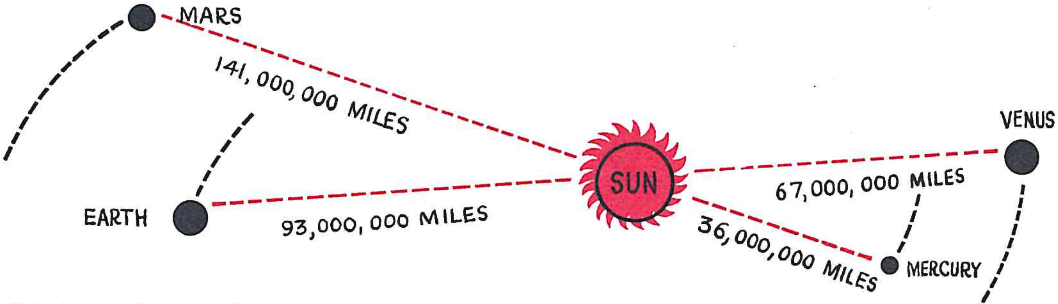
8. [A] Which cities have between 2,000,000 and 3,000,000 people?
   [B] Which cities have between 3,000,000 and 4,000,000 people?
   [C] Which city has more than 2,500,000 and less than 3,000,000 people?

9. Which two cities on the list have a population about 2,000,000 greater than Philadelphia?

10. [A] The state of New York has about 14,000,000 more people than San Francisco. About how many people live in New York State?
    [B] The state of South Carolina has about 500,000 fewer people than San Francisco. About what is the population of South Carolina?

11. The population of the United States is about 190,000,000 more than the number of people living in the New York City area. About what is the population of the United States?

Planets



1. Which of these planets is farthest from the sun?
2. Which of these planets is closest to the sun?
3. The distance of a planet from the sun is not always the same. How far would the earth be from the sun if it were a million miles closer than shown?
4. How far would the earth be from the sun if it were a million miles farther than shown?
5. Suppose Mars were ten million miles closer than shown. Then how far would it be from Mars to the sun?
6. How far would Mars be from the sun if it were 10,500,000 miles farther than shown?

One billion → 1,000,000,000

One billion is a thousand million.

1,000,000,000

One trillion → 1,000,000,000,000

One trillion is a thousand billion.

1,000,000,000,000

★ ORAL EXERCISES

Read the numbers.

1. 52,387,268,491
2. 478,291,096,503
3. 7,836,403,728,500
4. 928,304,067,392,530

Chapter review

1. Write the correct numerals.
   - [A] 7 tens and 8
   - [B] 9 tens and 3
   - [C] 8 hundreds and 9 tens
   - [D] 4 thousands and 5
   - [E] 7 thousands and 4 tens

2. Give the missing numbers.
   - [A] One hundred is ▨ tens.
   - [B] One thousand is ▨ hundreds.
   - [C] One thousand is ▨ tens.
   - [D] ▨ thousands are 1 million.
   - [E] $99 + 1 = \boxed{n}$
   - [F] $999 + 1 = \boxed{n}$

Think

When digits get together
They always count on me.
If you must find their number,
I really am the key.

WHO AM I?

Now I'm the only digit
Whose number stays the same
No matter where I am
In our place-value game.

WHO AM I?

3. Study the numeral. Then give the correct word for each blank.

$$\underbrace{5\,9\,8}_{\text{group 5}},\ \underbrace{3\,7\,2}_{\text{group 4}},\ \underbrace{4\,3\,6}_{\text{group 3}},\ \underbrace{5\,0\,7}_{\text{group 2}},\ \underbrace{2\,4\,6}_{\text{group 1}}$$

   - [A] The digits in group 2 tell how many _?_.
   - [B] The digits in group 3 tell how many _?_.
   - ★[C] The digits in group 4 tell how many _?_.
   - ★[D] The digits in group 5 tell how many _?_.

4. Write the sign (<, >) that should go in each ◍.
   - [A] 83,762 ◍ 84,672
   - [B] 342,968 ◍ 351,000
   - [C] 687,234,572 ◍ 687,243,572
   - [D] 999,999 ◍ 1,000,000

5. Give the number that is 1,000,000 more than
   - [A] 5,000,000.
   - [B] 23,560.
   - [C] 4862.
   - [D] 343,000,000.

6. [A] Give the smallest 2-digit numeral that has 3 as a digit.
   [B] Give the smallest 3-digit numeral that has the digits 1 and 9.
   [C] Give the largest 4-digit numeral that has 9 as a digit in three places.

★7. In 1960 the population of the world was estimated to be 2,971,800,000. Write the word name for this number.

CHAPTER 2 Addition and subtraction

Sets

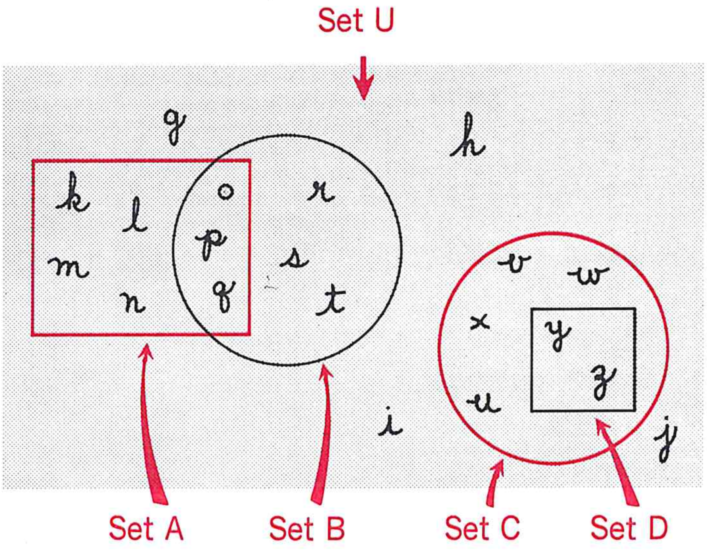
The letters inside the red box are in Set A.

The letters inside the black ring are in Set B.

The letters inside the red ring are in Set C.

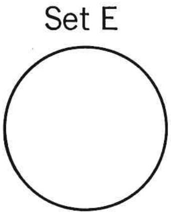
The letters inside the black box are in Set D.

Set U contains all the letters shown.



EXERCISES

1. How many letters are in these sets?
   [A] Set A [B] Set B [C] Set C [D] Set D [E] Set U
2. Which two sets contain the same number of letters?
3. How many letters are
   [A] outside Set A? [B] not in Set B?
   [C] not in the Sets A, B, C, or D?
4. Draw a ring on your paper and label it Set E as shown here. Write the letters from Set A and from Set C inside the ring. We call Set E the union of Set A and Set C. How many letters are in these sets?
   [A] Set A [B] Set C [C] The union of Set A and Set C



5. Write the letters that are in the union of Set B and Set D. Give the number of letters.
6. Write the letters that are in the union of Sets B and C. Write an addition equation to tell the number of letters in Set B, Set C, and the union of Sets B and C.
7. How many letters are in Set C but outside Set D?
8. How many letters are in **both** Set A and Set B?

Addition

Here are three ways you might think about addition.

① You can find the union of two sets.

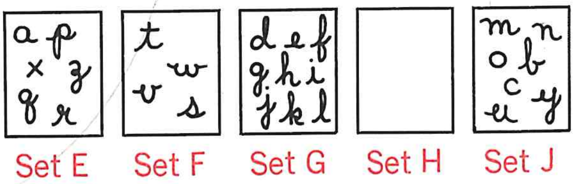
② You can count or use the number line.

③ You can discover facts you do not know by using facts you do know.

Exercises 1, 2, and 3 will help you review these ways of thinking about addition.

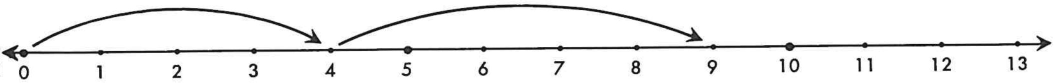
EXERCISES

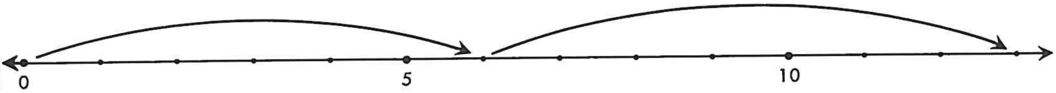
1. Answer the questions. Write an addition equation for each part.
   [A] How many letters are in Set E, Set F, the union of Sets E and F?

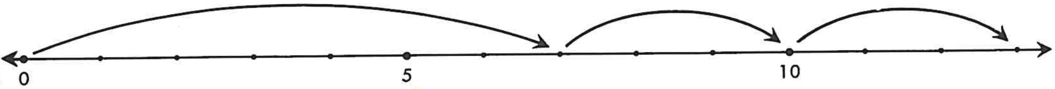


   [B] How many letters are in Set G, Set H, the union of Sets G and H?
   [C] How many letters are in Set G, Set J, the union of Sets G and J?
   [D] How many letters are in Set E, Set G, the union of Sets E and G?

2. Write an addition equation for each number-line picture.

[A]


[B]


[C]


3. Find the sums.
   [A] Since $5+5=10$, we know that $5+6=\boxed{n}$.
   [B] Since $7+7=14$, we know that $7+8=\boxed{n}$.
   [C] Since $10+8=18$, we know that $9+8=\boxed{n}$.
   [D] Since $10+5=15$, we know that $9+5=\boxed{n}$.
   [E] Since $7+8=15$, we know that $8+7=\boxed{n}$.

4. Think of the two numbers as addends. Then find their sum.

[A] 8, 3 [C] 9, 3 [E] 7, 5 [G] 6, 6 [I] 4, 8 [K] 7, 8 [M] 7, 6
[B] 9, 9 [D] 5, 8 [F] 8, 6 [H] 7, 4 [J] 8, 9 [L] 9, 6 [N] 6, 8

Subtraction

You can think of subtraction as finding the missing addend.

15 − 7

You find this difference

$\boxed{n} + 7 = 15$

$15 - 7 = \boxed{n}$

when you find this addend.

You find this difference

$\begin{array}{r} \boxed{} \\ +7 \\ \hline 15 \end{array}$ $\qquad$ $\begin{array}{r} 15 \\ -7 \\ \hline \boxed{} \end{array}$

when you find this addend.

EXERCISES

1. Find the differences by finding the missing addends.

[A] $\boxed{n} + 5 = 9$ $\quad 9 - 5 = \boxed{n}$

[B] $\boxed{n} + 6 = 11$ $\quad 11 - 6 = \boxed{n}$

[C] $\boxed{n} + 8 = 13$ $\quad 13 - 8 = \boxed{n}$

[D] $\boxed{n} + 9 = 15$ $\quad 15 - 9 = \boxed{n}$

[E] $\boxed{n} + 6 = 15$ $\quad 15 - 6 = \boxed{n}$

[F] $\boxed{n} + 8 = 17$ $\quad 17 - 8 = \boxed{n}$

[G] $\boxed{n} + 5 = 12$ $\quad 12 - 5 = \boxed{n}$

[H] $\boxed{n} + 9 = 18$ $\quad 18 - 9 = \boxed{n}$

[I] $\boxed{n} + 7 = 16$ $\quad 16 - 7 = \boxed{n}$

[J] $\boxed{n} + 9 = 16$ $\quad 16 - 9 = \boxed{n}$

[K] $\boxed{n} + 8 = 14$ $\quad 14 - 8 = \boxed{n}$

[L] $\boxed{n} + 6 = 13$ $\quad 13 - 6 = \boxed{n}$

2. Find the differences by finding the missing addends.

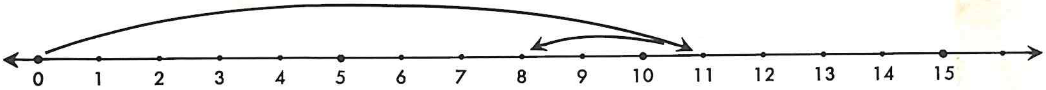
[A] $\begin{array}{r} 8 \\ -5 \\ \hline \end{array}$ [B] $\begin{array}{r} 10 \\ -7 \\ \hline \end{array}$ [C] $\begin{array}{r} 16 \\ -8 \\ \hline \end{array}$ [D] $\begin{array}{r} 11 \\ -8 \\ \hline \end{array}$ [E] $\begin{array}{r} 12 \\ -8 \\ \hline \end{array}$ [F] $\begin{array}{r} 13 \\ -7 \\ \hline \end{array}$ [G] $\begin{array}{r} 12 \\ -9 \\ \hline \end{array}$

[H] $\begin{array}{r} 15 \\ -8 \\ \hline \end{array}$ [I] $\begin{array}{r} 17 \\ -9 \\ \hline \end{array}$ [J] $\begin{array}{r} 15 \\ -7 \\ \hline \end{array}$ [K] $\begin{array}{r} 11 \\ -3 \\ \hline \end{array}$ [L] $\begin{array}{r} 13 \\ -5 \\ \hline \end{array}$ [M] $\begin{array}{r} 12 \\ -7 \\ \hline \end{array}$ [N] $\begin{array}{r} 11 \\ -4 \\ \hline \end{array}$

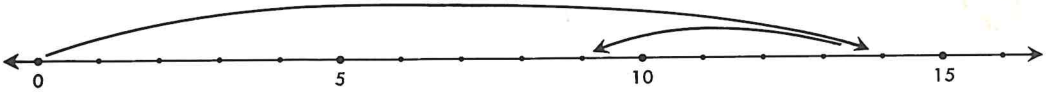
[O] $\begin{array}{r} 13 \\ -9 \\ \hline \end{array}$ [P] $\begin{array}{r} 14 \\ -7 \\ \hline \end{array}$ [Q] $\begin{array}{r} 18 \\ -0 \\ \hline \end{array}$ [R] $\begin{array}{r} 11 \\ -5 \\ \hline \end{array}$ [S] $\begin{array}{r} 16 \\ -8 \\ \hline \end{array}$ [T] $\begin{array}{r} 9 \\ -9 \\ \hline \end{array}$ [U] $\begin{array}{r} 14 \\ -6 \\ \hline \end{array}$

3. Write the subtraction equation for each number-line picture.

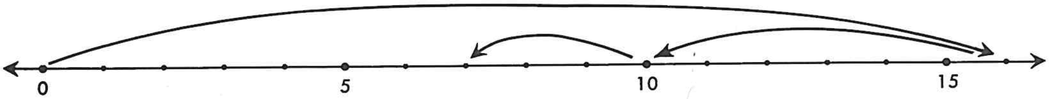
[A]



[B]



[C]



4. Find the differences.

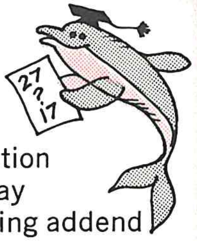
[A] Since $13-3=10$, we know that $13-4=\boxed{n}$.
[B] Since $14-4=10$, we know that $14-5=\boxed{n}$.
[C] Since $15-5=10$, we know that $15-6=\boxed{n}$.
[D] Since $16-6=10$, we know that $16-7=\boxed{n}$.
[E] Since $17-7=10$, we know that $17-8=\boxed{n}$.
[F] Since $18-8=10$, we know that $18-9=\boxed{n}$.
[G] Since $13-10=3$, we know that $13-9=\boxed{n}$.
[H] Since $14-10=4$, we know that $14-9=\boxed{n}$.
[I] Since $15-10=5$, we know that $15-9=\boxed{n}$.
[J] Since $16-10=6$, we know that $16-9=\boxed{n}$.
[K] Since $17-10=7$, we know that $17-9=\boxed{n}$.
[L] Since $8-5=3$, we know that $18-15=\boxed{n}$.
[M] Since $9-3=6$, we know that $19-3=\boxed{n}$.
[N] Since $9-5=4$, we know that $39-5=\boxed{n}$.

5. One exercise below has no whole-number answer. Find it. Then find the differences for the others.

[A] $\begin{array}{r}14\\ -8\\ \hline\end{array}$ [B] $\begin{array}{r}13\\ -5\\ \hline\end{array}$ [C] $\begin{array}{r}14\\ -6\\ \hline\end{array}$

[D] $\begin{array}{r}13\\ -0\\ \hline\end{array}$ [E] $\begin{array}{r}7\\ -11\\ \hline\end{array}$ [F] $\begin{array}{r}12\\ -7\\ \hline\end{array}$

Think



When I'm the operation
You'll never go astray
If you find the missing addend
Or think of take away.

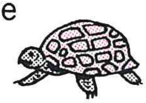
WHO AM I?

Short stories

1. 16 marbles. Lost 9.
How many left?

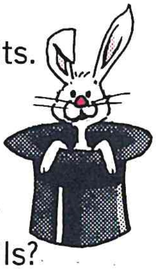
2. 5 boys in the game.
8 boys on the bench.
How many boys?

3. 13 slow turtles. 7 slow
snails. How many more
turtles than snails?



4. 16 papers. Delivered 7.
How many more
to be delivered?

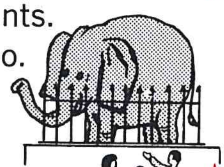
5. Magician has 12 rabbits.
7 disappear. How
many left?



6. 15 dogs. 7 tails. How
many dogs have no tails?

7. 6 baseball cards.
Need 15 in all.
How many more needed?

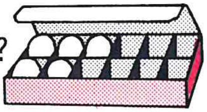
8. 13 jungle elephants.
7 taken to the zoo.
How many left?



9. 14 girls with dolls.
5 girls without dolls.
How many girls?

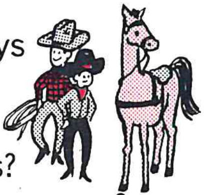
10. 4 marbles needed to make
a dozen in the box. How many
already in the box?

11. 5 eggs. How many more
needed to make a dozen?



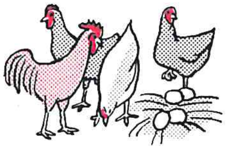
12. Had 14 cents. Spent 4 cents.
Lost 5 cents. How much left?

13. 16 cowboys. 9 horses.
How many more cowboys
than horses?

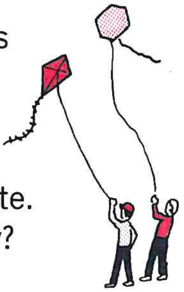


14. 4 cows. How many legs?
How many more legs than cows?

15. 14 girls.
6 go home.
How many left?



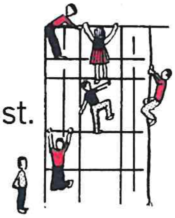
16. 19 chickens. 12 eggs.
How many more chickens
than eggs?



17. 14 kites in the air.
9 boys each flying one kite.
How many kites got away?

★18. 17 airplanes on the ground.
9 flew away. 6 others landed.
How many planes now on the
ground?

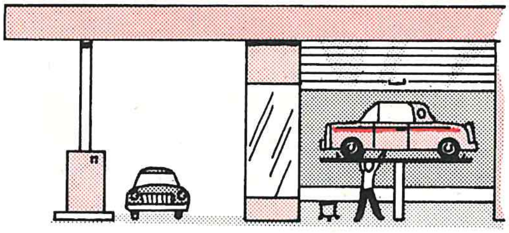
★19. Played 15 games.
Won 3 more than were lost.
How many were won?



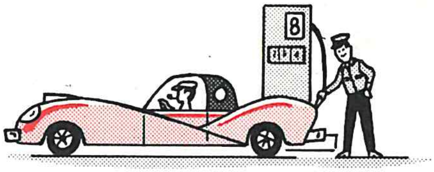
★20. 12 children. Twice as many
boys as girls. How many girls?
How many boys?

At the service station

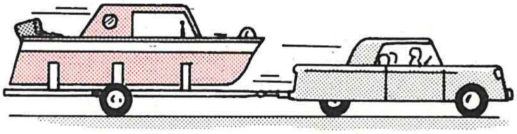
Write an equation for each exercise. Then find the solution.



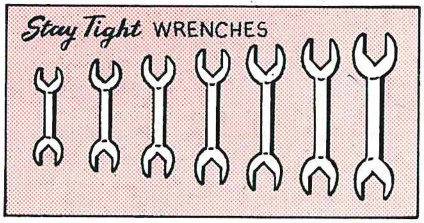
+ **1.** Mr. Blue greased 9 cars before lunch and 8 cars after lunch. How many cars did he grease?



− **2.** A gas tank on a new sports car holds 17 gallons when full. 8 gallons have been put in. How many more are needed to fill the tank?

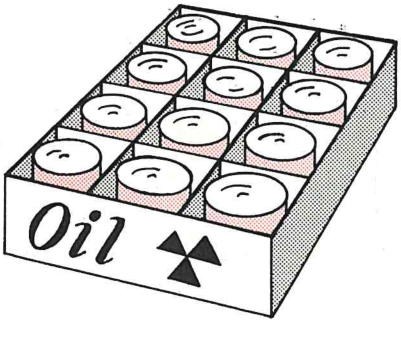


− **3.** The gas tank on Mr. Brown's boat holds 14 gallons. When 6 gallons were put in, the tank was full. How many gallons were already in the tank?



— **4.** Mr. Blue had a set of 7 wrenches. He bought a new set of 16 wrenches. How many more wrenches are in the new set than in the old?

+ **5.** On Friday 5 tires were sold, and on Saturday 9 tires were sold. How many tires were sold on these two days?

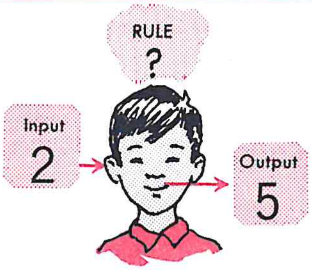


— **6.** There are 12 cans of oil in a box. 7 cans are sold. How many cans are left?

− **7.** There are 16 cans of auto polish in a box. Some are sold. 9 cans are left. How many cans were sold?

The function game

When Kay said 2, Paul answered 5.
When Kay said 3, Paul answered 7.
When Kay said 5, Paul answered 11.



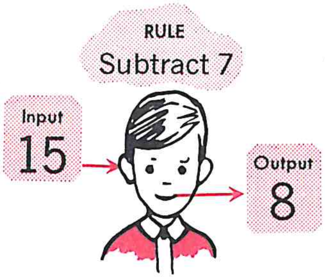
1. When Kay said 6, what do you think Paul answered?
2. What rule do you think Paul was using?

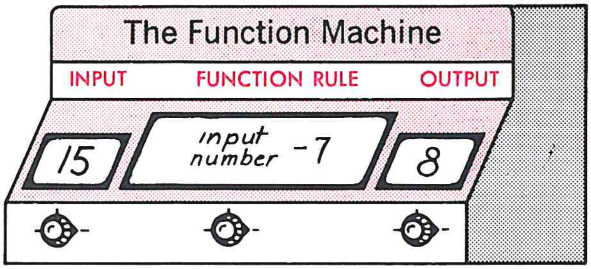
EXERCISES

Paul's game helps you understand the function machine. Study the pictures to see how it works.

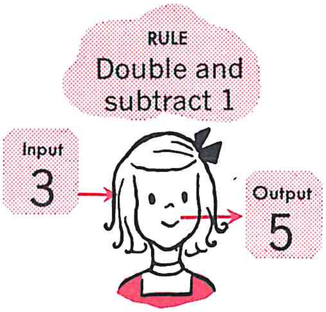
Give the output number for each input number given.

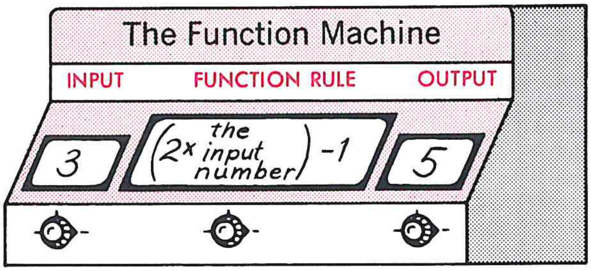
1. [A] 12
   [B] 16
   [C] 7
   [D] 17
   [E] 13



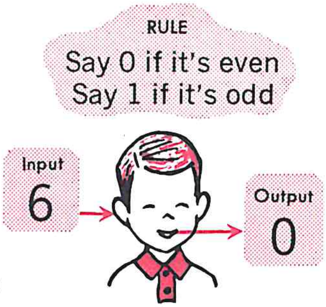


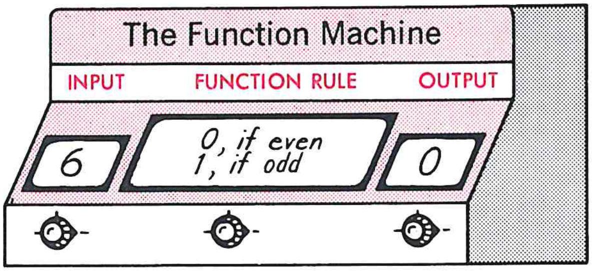
2. [A] 2
   [B] 5
   [C] 8
   [D] 10
   [E] 9





3. [A] 2
   [B] 4
   [C] 5
   [D] 37
   [E] 100





EXERCISES

Think about the function machine and tell what you think should go in each gray space.

1. Function Rule

Add 7

	Input	Output
	1	8
[A]	4	▨
[B]	7	▨
[C]	9	▨
[D]	5	▨

2. Function Rule

[A] +2

	Input	Output
	2	4
	3	6
	5	10
[B]	9	▨
[C]	▨	8

3. Function Rule

[A] ▨

	Input	Output
	16	8
	12	4
	8	0
[B]	13	▨
[C]	17	▨

4. Function Rule

[A] ▨

	Input	Output
	10	1
	18	9
	12	3
[B]	16	▨
[C]	▨	6

5. Function Rule

[A] ▨

	Input	Output
	4	10
	7	13
[B]	9	▨
[C]	▨	14
[D]	5	▨

6. Function Rule

[A] ▨

	Input	Output
	10	5
	7	2
[B]	14	▨
[C]	5	▨
[D]	▨	8

★ **7.** Function Rule

[A] ▨

	Input	Output
	2	6
	5	12
	3	8
	9	20
[B]	7	▨

★ **8.** Function Rule

Add 7 if odd
Add 8 if even

	Input	Output
[A]	5	▨
[B]	6	▨
[C]	7	▨
[D]	8	▨
[E]	9	▨

★ **9.** Function Rule

7 or less: add 8
> 7: subt. 8

	Input	Output
[A]	4	▨
[B]	15	▨
[C]	10	▨
[D]	7	▨
[E]	17	▨

The order principle for addition

Give the missing numbers. Remember the order principle for addition.

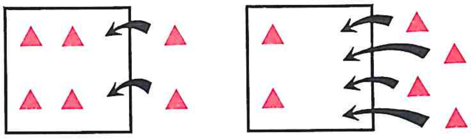
Since $8+7=15$, we know that $7+8=\boxed{n}$.
Since $13+15=28$, we know that $15+13=\boxed{n}$.
Since $324+265=589$, we know that $265+324=\boxed{n}$.
Since $5237+6435=11{,}672$, we know that $6435+5237=\boxed{n}$.

When we change the **ORDER** of the addends, we get the same sum.

EXERCISES

1. Each exercise suggests an example of the order principle. Give the example.

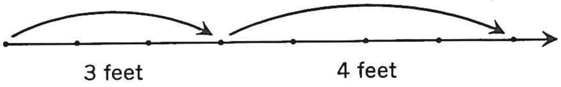
[A]



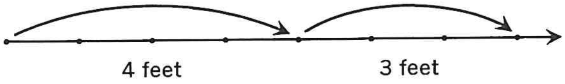
(Answer: $4+2=2+4$)

[B] Start at 9. Count forward 4.
Start at 4. Count forward 9.

[C] Jump 3 feet. Then jump 4 feet.



Jump 4 feet. Then jump 3 feet.



2. Write on your paper the equations that are examples of the order principle.

[A] $9+6=6+9$
[B] $23+45=43+25$
[C] $23+46=46+23$
[D] $15+36=15+36$
[E] $27+35=72+53$
[F] $42+35=53+42$

3. Which equations in exercise 2 are true?

4. Solve the equations.

[A] $13+7=\boxed{n}+13$
[B] $\boxed{n}+59=59+36$
[C] $364+\boxed{n}=281+364$
[D] $5280+3292=3292+\boxed{n}$
[E] $35+(17+5)=35+(\boxed{n}+17)$
[F] $(263+54)+\boxed{n}=83+(54+263)$

The grouping principle for addition

If we do not change the order when adding three numbers, we have two choices of which addends we can add first.

We could add these first, $7+3+6$ or we could add these first.	We could add these first, $\begin{array}{r}8\\4\\+6\\\hline\end{array}$ or we could add these first.
$(7+3)+6=7+(3+6)$	$(8+4)+6=8+(4+6)$

When we add, we can change the **GROUPING** and get the same sum.

EXERCISES

1. Find the sums. For some of the exercises, it will help to change the grouping.

[A] $6+(4+9)$
[B] $2+(8+9)$
[C] $(11+9)+7$
[D] $69+(1+8)$
[E] $(64+1)+9$
[F] $1+(99+72)$
[G] $1854+(8+2)$
[H] $999+(1+3424)$
[I] $3{,}465{,}824+(98+2)$
[J] $998+(2+5{,}342{,}861)$
[K] $96+(4+5862)$
[L] $999{,}999+(1+3{,}462{,}385)$

2. Find each sum. Choose the grouping that is most helpful. Do not change the order.

[A] $3+7+8$
[B] $9+6+4$
[C] $13+7+5$
[D] $6+12+8$
[E] $9+98+2$
[F] $3+7+5$
[G] $15+7+3$
[H] $8+39+1$

[I] $\begin{array}{r}7\\4\\+6\\\hline\end{array}$
[J] $\begin{array}{r}11\\9\\+6\\\hline\end{array}$
[K] $\begin{array}{r}4\\14\\+6\\\hline\end{array}$
[L] $\begin{array}{r}1\\99\\+57\\\hline\end{array}$
[M] $\begin{array}{r}8\\3\\+7\\\hline\end{array}$

★ **3.** When we subtract, we **do not** always get the same difference when we change the grouping. Write these equations with grouping marks so that the answers are correct. Exercise **A** is answered correctly.

[A] $12-(8-4)=8$
[B] $13-5-4=4$
[C] $17-8-3=6$
[D] $16-7-5=14$

Using the order and grouping principles

Because of the order and grouping principles, we can add any two addends first and get the same sum.

EXERCISES

1. For each part of exercises **A**, **B**, and **C** add the numbers given in red first. Then find the sum. Is each sum the same in exercise **A**? in exercise **B**? in exercise **C**?

[**A**] $3+4+5$
$3+4+5$
$3+4+5$

[**B**] $2+5+4+6$
$2+5+4+6$
$2+5+4+6$

[**C**] $2+5+4+6$
$2+5+4+6$
$2+5+4+6$

2. Find the sums. Add any two numbers first.

[**A**] $6+9+4$
[**B**] $20+7+30$
[**C**] $90+38+10$
[**D**] $1+3+5+7$
[**E**] $20+30+3+4$
[**F**] $20+4+30+5$
[**G**] $30+7+40+2$
[**H**] $5+3+50+40$
[**I**] $50+3+40+5$

[**J**] $\begin{array}{r}20\\30\\3\\+4\\\hline\end{array}$ [**K**] $\begin{array}{r}30\\2\\50\\+2\\\hline\end{array}$ [**L**] $\begin{array}{r}30\\3\\60\\+6\\\hline\end{array}$ [**M**] $\begin{array}{r}7\\2\\20\\+40\\\hline\end{array}$ [**N**] $\begin{array}{r}50\\3\\40\\+5\\\hline\end{array}$ [**O**] $\begin{array}{r}70\\4\\20\\+5\\\hline\end{array}$

3. Find each pair of sums.

[**A**] $30+2+20+4=\boxed{n}$
$32+24=\boxed{n}$

[**B**] $40+5+30+4=\boxed{n}$
$45+34=\boxed{n}$

[**C**] $50+3+20+2=\boxed{n}$
$53+22=\boxed{n}$

[**D**] $40+2+40+5=\boxed{n}$
$42+45=\boxed{n}$

[**E**] $20+6+70+3=\boxed{n}$
$26+73=\boxed{n}$

[**F**] $60+5+30+4=\boxed{n}$
$65+34=\boxed{n}$

[**G**] $\begin{array}{r}30\\2\\30\\+5\\\hline\end{array}$ $\begin{array}{r}32\\+35\\\hline\end{array}$

[**H**] $\begin{array}{r}40\\5\\20\\+3\\\hline\end{array}$ $\begin{array}{r}45\\+23\\\hline\end{array}$

[**I**] $\begin{array}{r}30\\8\\50\\+1\\\hline\end{array}$ $\begin{array}{r}38\\+51\\\hline\end{array}$

★**4.** Give the missing numbers.

[**A**] Since $6+7+8+9=30$, we know $6+9+7+8+5=\boxed{n}$.
[**B**] Since $2+4+6+8=20$, we know $2+2+4+4+6+6+8+8=\boxed{n}$.
[**C**] Since $3+4+5+6+7=25$, we know $3+4+5+5+7+6=\boxed{n}$.

Reasoning—addition

EXERCISES

1. Give the missing numbers.

[A] Since $3+6=9$, we know that $13+6=\boxed{n}$.
[B] Since $3+7=10$, we know that $13+7=\boxed{n}$.
[C] Since $3+8=11$, we know that $13+8=\boxed{n}$.
[D] Since $8+7=15$, we know that $18+7=\boxed{n}$.
[E] Since $6+8=14$, we know that $16+8=\boxed{n}$.
[F] Since $9+7=16$, we know that $29+7=\boxed{n}$.
[G] Since $8+9=17$, we know that $38+9=\boxed{n}$.
[H] Since $4+9=13$, we know that $44+9=\boxed{n}$.
[I] Since $5+8=13$, we know that $55+8=\boxed{n}$.
[J] Since $6+9=15$, we know that $86+9=\boxed{n}$.

2. Solve the equations.

[A] $16+6=\boxed{n}$
[B] $5+17=\boxed{n}$
[C] $\boxed{n}=16+8$
[D] $\boxed{n}=9+17$
[E] $18+7=\boxed{n}$
[F] $23+9=\boxed{n}$
[G] $\boxed{n}=28+3$
[H] $6+35=\boxed{n}$
[I] $47+6=\boxed{n}$
[J] $7+59=\boxed{n}$
[K] $\boxed{n}=68+8$
[L] $\boxed{n}=8+74$
[M] $87+8=\boxed{n}$
[N] $7+96=\boxed{n}$
[O] $\boxed{n}=18+5$
[P] $\boxed{n}=165+5$
[Q] $246+9=\boxed{n}$
[R] $357+7=\boxed{n}$
[S] $9+573=\boxed{n}$
[T] $862+9=\boxed{n}$

Think

For each exercise, give the pair of numbers that should go in the gray spaces.

	Sum	+ −	Difference
	15	9 + 6	3
	7	5 + 2	3
[A]	12	▨ + ▨	4
[B]	15	▨ + ▨	1
[C]	18	▨ + ▨	0
[D]	9	▨ + ▨	9

	Sum	+ −	Difference
[E]	30	▨ + ▨	10
[F]	16	▨ + ▨	4
[G]	14	▨ + ▨	4
[H]	80	▨ + ▨	20
[I]	100	▨ + ▨	0
[J]	100	▨ + ▨	98

Column addition

The examples will help you review a way of finding the sum of three or more addends.

Ⓐ	Ⓑ
$\begin{array}{r}\left.\begin{array}{r}8\\7\end{array}\right\}15\\+9\\\hline\end{array}$	$\begin{array}{r}\left.\begin{array}{r}\left.\begin{array}{r}9\\8\end{array}\right\}17\\7\end{array}\right\}24\\+9\\\hline\end{array}$
1. Add 8 and 7. 2. Think: $15+9=24$	1. Add 9 and 8. 2. Think: $17+7=24$ 3. Think: $24+9=33$

Because of the **order** and **grouping** principles, there are other ways to find the sums in examples A and B. Find two other ways to compute the sum in example A and explain your work.

EXERCISES

1. Find the sums. Then choose two different addends to add first and find the sum again to check your work.

[A] $\begin{array}{r}4\\6\\+3\\\hline\end{array}$
[B] $\begin{array}{r}5\\4\\+6\\\hline\end{array}$
[C] $\begin{array}{r}8\\6\\+3\\\hline\end{array}$
[D] $\begin{array}{r}9\\5\\+7\\\hline\end{array}$
[E] $\begin{array}{r}7\\6\\+8\\\hline\end{array}$
[F] $\begin{array}{r}9\\9\\+6\\\hline\end{array}$
[G] $\begin{array}{r}8\\6\\+7\\\hline\end{array}$
[H] $\begin{array}{r}7\\8\\+8\\\hline\end{array}$

[I] $\begin{array}{r}6\\9\\+9\\\hline\end{array}$
[J] $\begin{array}{r}8\\8\\+9\\\hline\end{array}$
[K] $\begin{array}{r}3\\9\\+8\\\hline\end{array}$
[L] $\begin{array}{r}9\\9\\+9\\\hline\end{array}$
[M] $\begin{array}{r}7\\9\\+4\\\hline\end{array}$
[N] $\begin{array}{r}8\\8\\+8\\\hline\end{array}$
[O] $\begin{array}{r}6\\7\\+8\\\hline\end{array}$
[P] $\begin{array}{r}7\\9\\+9\\\hline\end{array}$

[Q] $\begin{array}{r}3\\4\\6\\+8\\\hline\end{array}$
[R] $\begin{array}{r}4\\7\\8\\+6\\\hline\end{array}$
[S] $\begin{array}{r}5\\9\\3\\+7\\\hline\end{array}$
[T] $\begin{array}{r}6\\8\\4\\+8\\\hline\end{array}$
[U] $\begin{array}{r}7\\7\\9\\+9\\\hline\end{array}$
[V] $\begin{array}{r}8\\6\\8\\+7\\\hline\end{array}$
[W] $\begin{array}{r}3\\5\\6\\+9\\\hline\end{array}$
[X] $\begin{array}{r}9\\8\\8\\+8\\\hline\end{array}$

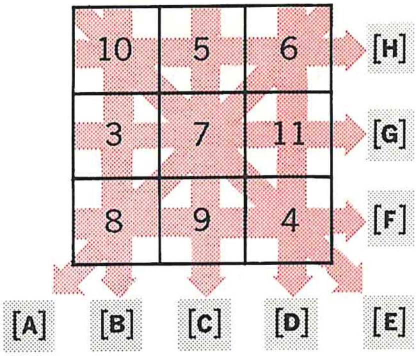
2. Solve the equations. Check your work.

[A] $7+6+8=\boxed{n}$
[B] $23+5+7=\boxed{n}$
[C] $\boxed{n}=17+8+6$
[D] $\boxed{n}=24+9+8$
[E] $4+17+9=\boxed{n}$
[F] $8+8+34=\boxed{n}$
[G] $\boxed{n}=16+9+8$
[H] $\boxed{n}=57+8+8$
[I] $7+8+9+1=\boxed{n}$
[J] $6+7+8+9=\boxed{n}$
[K] $3+4+5+6=\boxed{n}$
[L] $9+9+8+7=\boxed{n}$

EXERCISES

1. [A] Find the row, column, and diagonal sums **A** through **H** for this magic square.

[B] Why do you think it is called a magic square?



2. Add 3 to each number in the magic square.

[A] Do you still have a magic square?

[B] What is the sum?

3. Find the sums. Check your work.

[A]	[B]	[C]	[D]	[E]	[F]	[G]	[H]
6	25	8	7	9	38	56	78
9	9	37	6	8	9	7	6
+5	+3	+6	+24	+9	+7	+5	+9

[I]	[J]	[K]	[L]	[M]	[N]	[O]	[P]
6	9	4	9	6	8	9	9
7	7	8	7	5	7	2	3
7	8	6	9	9	6	7	5
8	4	9	9	5	5	3	6
+3	+4	+5	+8	+3	+4	+8	+7

4. Solve the equations.

[A] $8+6+9+3=n$

[B] $56+7+4=n$

[C] $3+40+7=n$

[D] $n=23+34+9$

[E] $62+27+8=n$

[F] $n=135+8+8$

[G] $n=7+146+5$

[H] $175+9+8=n$

[I] $n=136+5+9$

★ **5.** Give numbers for the spaces **A** through **N** so that each square will be a magic square.

4	9	[A]
3	5	7
8	1	[B]

5	[C]	5
[D]	6	6
7	[E]	7

5	[F]	[G]
13	8	[H]
6	[I]	11

4	8	9
[J]	7	[K]
[L]	[M]	[N]

Sums—2-digit numbers

The explanation below will help you understand addition of 2-digit numbers.

Step 1	Step 2	Step 3
$\begin{array}{r} 58 \\ +\ 36 \\ \hline 14 \end{array}$	$\begin{array}{r} 58 \\ +\ 36 \\ \hline 14 \\ 80 \\ \hline \end{array}$	$\begin{array}{r} 58 \\ +\ 36 \\ \hline 14 \\ 80 \\ \hline 94 \end{array}$
$8+6=14$	$50+30=80$	$14+80=94$

EXERCISES

1. Find the sums. Use the method shown above.

[A] $\begin{array}{r} 37 \\ +45 \\ \hline \end{array}$ [B] $\begin{array}{r} 67 \\ +23 \\ \hline \end{array}$ [C] $\begin{array}{r} 56 \\ +29 \\ \hline \end{array}$ [D] $\begin{array}{r} 64 \\ +19 \\ \hline \end{array}$ [E] $\begin{array}{r} 29 \\ +37 \\ \hline \end{array}$ [F] $\begin{array}{r} 49 \\ +57 \\ \hline \end{array}$ [G] $\begin{array}{r} 73 \\ +58 \\ \hline \end{array}$

The explanation below will help you remember a short cut for adding 2-digit numbers.

Step 1	Step 2
$\begin{array}{r} 1\ \ \\ 58 \\ +\ 36 \\ \hline 4 \end{array}$	$\begin{array}{r} 1\ \ \\ 58 \\ +\ 36 \\ \hline 94 \end{array}$
$8+6=14$	$10+50+30=90$

2. Find the sums.

[A] $\begin{array}{r} 16 \\ +35 \\ \hline \end{array}$ [B] $\begin{array}{r} 15 \\ +19 \\ \hline \end{array}$ [C] $\begin{array}{r} 34 \\ +18 \\ \hline \end{array}$ [D] $\begin{array}{r} 23 \\ +37 \\ \hline \end{array}$ [E] $\begin{array}{r} 49 \\ +35 \\ \hline \end{array}$ [F] $\begin{array}{r} 54 \\ +27 \\ \hline \end{array}$ [G] $\begin{array}{r} 72 \\ +28 \\ \hline \end{array}$

[H] $\begin{array}{r} 58 \\ +37 \\ \hline \end{array}$ [I] $\begin{array}{r} 76 \\ +86 \\ \hline \end{array}$ [J] $\begin{array}{r} 67 \\ +64 \\ \hline \end{array}$ [K] $\begin{array}{r} 79 \\ +56 \\ \hline \end{array}$ [L] $\begin{array}{r} 94 \\ +89 \\ \hline \end{array}$ [M] $\begin{array}{r} 86 \\ +68 \\ \hline \end{array}$ [N] $\begin{array}{r} 57 \\ +99 \\ \hline \end{array}$

EXERCISES

1. Find the sums.

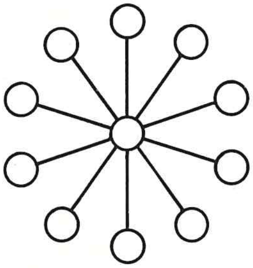
- [A] $\begin{array}{r}56\\+34\\\hline\end{array}$ [B] $\begin{array}{r}47\\+60\\\hline\end{array}$ [C] $\begin{array}{r}68\\+25\\\hline\end{array}$ [D] $\begin{array}{r}59\\+76\\\hline\end{array}$ [E] $\begin{array}{r}37\\+85\\\hline\end{array}$ [F] $\begin{array}{r}63\\+57\\\hline\end{array}$
- [G] $\begin{array}{r}45\\+79\\\hline\end{array}$ [H] $\begin{array}{r}88\\+66\\\hline\end{array}$ [I] $\begin{array}{r}63\\+49\\\hline\end{array}$ [J] $\begin{array}{r}74\\+58\\\hline\end{array}$ [K] $\begin{array}{r}85\\+67\\\hline\end{array}$ [L] $\begin{array}{r}96\\+76\\\hline\end{array}$
- [M] $\begin{array}{r}91\\+19\\\hline\end{array}$ [N] $\begin{array}{r}34\\+87\\\hline\end{array}$ [O] $\begin{array}{r}43\\+99\\\hline\end{array}$ [P] $\begin{array}{r}55\\+88\\\hline\end{array}$ [Q] $\begin{array}{r}56\\+89\\\hline\end{array}$ [R] $\begin{array}{r}99\\+89\\\hline\end{array}$

2. Find the sums.

- [A] $\begin{array}{r}11\\23\\+35\\\hline\end{array}$ [B] $\begin{array}{r}67\\20\\+31\\\hline\end{array}$ [C] $\begin{array}{r}78\\34\\+32\\\hline\end{array}$
- [D] $\begin{array}{r}38\\26\\+59\\\hline\end{array}$ [E] $\begin{array}{r}90\\19\\+78\\\hline\end{array}$ [F] $\begin{array}{r}58\\46\\+85\\\hline\end{array}$
- [G] $\begin{array}{r}67\\89\\+34\\\hline\end{array}$ [H] $\begin{array}{r}48\\67\\+16\\\hline\end{array}$ [I] $\begin{array}{r}67\\27\\+58\\\hline\end{array}$
- [J] $\begin{array}{r}59\\47\\68\\+86\\\hline\end{array}$ [K] $\begin{array}{r}77\\65\\43\\+68\\\hline\end{array}$ [L] $\begin{array}{r}95\\98\\97\\+40\\\hline\end{array}$

Think

Draw a figure like the one below on your paper. Place the numbers 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 in the circles so the sum along any line is 21.



3. Solve the equations.

[A] $54+37+28+9=\boxed{n}$

[B] $4+17+36+8=\boxed{n}$

[C] $\boxed{n}=76+58+29+7$

[D] $\boxed{n}=64+9+78+8$

[E] $\boxed{n}=85+9+7+39$

[F] $76+84+7+3=\boxed{n}$

4. Copy each problem. Give the missing digit for each ▨.

- [A] $\begin{array}{r}4▨\\+▨5\\\hline 77\end{array}$ [B] $\begin{array}{r}▨▨\\+45\\\hline 128\end{array}$ [C] $\begin{array}{r}5▨\\+35\\\hline 94\end{array}$ ★ [D] $\begin{array}{r}▨7\\+4▨\\\hline 111\end{array}$ ★ [E] $\begin{array}{r}56\\+▨▨\\\hline 145\end{array}$ ★ [F] $\begin{array}{r}7▨\\+▨7\\\hline 143\end{array}$

Sums—3-digit numbers

You can use what you know about adding 2-digit numbers to help you add 3-digit numbers. Study this example.

Adding ones	Adding tens	Adding hundreds
Step 1	Step 2	Step 3
$\begin{array}{cccc} & & 1 & \\ & 5 & 6 & 4 \\ + & 3 & 8 & 9 \\ \hline & & & 3 \end{array}$	$\begin{array}{cccc} & 1 & 1 & \\ & 5 & 6 & 4 \\ + & 3 & 8 & 9 \\ \hline & & 5 & 3 \end{array}$	$\begin{array}{cccc} & 1 & 1 & \\ & 5 & 6 & 4 \\ + & 3 & 8 & 9 \\ \hline & 9 & 5 & 3 \end{array}$
$4+9=13$	$10+60+80=150$	$100+500+300=900$

EXERCISES

1. Find the sums.

[A] $\begin{array}{r}637\\+245\\\hline\end{array}$	[B] $\begin{array}{r}152\\+439\\\hline\end{array}$	[C] $\begin{array}{r}627\\+368\\\hline\end{array}$	[D] $\begin{array}{r}384\\+463\\\hline\end{array}$	[E] $\begin{array}{r}490\\+268\\\hline\end{array}$	[F] $\begin{array}{r}307\\+892\\\hline\end{array}$
[G] $\begin{array}{r}626\\+184\\\hline\end{array}$	[H] $\begin{array}{r}939\\+384\\\hline\end{array}$	[I] $\begin{array}{r}745\\+569\\\hline\end{array}$	[J] $\begin{array}{r}785\\+368\\\hline\end{array}$	[K] $\begin{array}{r}897\\+106\\\hline\end{array}$	[L] $\begin{array}{r}814\\+696\\\hline\end{array}$
[M] $\begin{array}{r}378\\+956\\\hline\end{array}$	[N] $\begin{array}{r}625\\+785\\\hline\end{array}$	[O] $\begin{array}{r}413\\+589\\\hline\end{array}$			

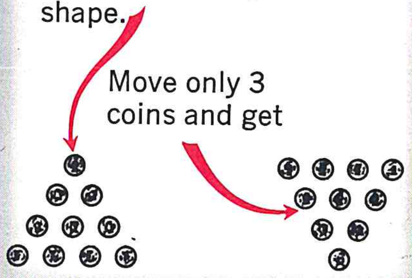
2. Find the sums.

[A] $\begin{array}{r}1789\\+7047\\\hline\end{array}$	[B] $\begin{array}{r}3346\\+2478\\\hline\end{array}$
[C] $\begin{array}{r}1567\\+9687\\\hline\end{array}$	[D] $\begin{array}{r}2027\\+8596\\\hline\end{array}$
[E] $\begin{array}{r}21{,}234\\+36{,}897\\\hline\end{array}$	[F] $\begin{array}{r}98{,}743\\+43{,}625\\\hline\end{array}$

Think

Place 10 coins (or other discs) in a triangular shape.

Move only 3 coins and get



EXERCISES

1. Find the sums.

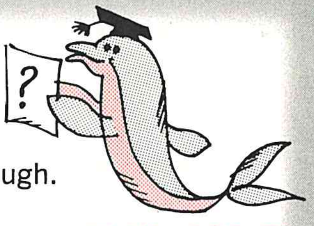
[A] $\begin{array}{r} 347 \\ +62 \\ \hline \end{array}$ [B] $\begin{array}{r} 529 \\ +55 \\ \hline \end{array}$

[C] $\begin{array}{r} 626 \\ +88 \\ \hline \end{array}$ [D] $\begin{array}{r} 39 \\ +477 \\ \hline \end{array}$

[E] $\begin{array}{r} 728 \\ +645 \\ \hline \end{array}$ [F] $\begin{array}{r} 963 \\ +758 \\ \hline \end{array}$

Think

Add me to myself.
You'll get fifty-four.
That should be enough.
I will tell no more.



WHO AM I?

[G] $\begin{array}{r} 576 \\ +948 \\ \hline \end{array}$ [H] $\begin{array}{r} 909 \\ +427 \\ \hline \end{array}$ [I] $\begin{array}{r} 638 \\ +362 \\ \hline \end{array}$ [J] $\begin{array}{r} 876 \\ +567 \\ \hline \end{array}$ [K] $\begin{array}{r} 508 \\ +967 \\ \hline \end{array}$

[L] $\begin{array}{r} 5673 \\ +29 \\ \hline \end{array}$ [M] $\begin{array}{r} 8764 \\ +297 \\ \hline \end{array}$ [N] $\begin{array}{r} 7958 \\ +6342 \\ \hline \end{array}$ [O] $\begin{array}{r} 6478 \\ +8579 \\ \hline \end{array}$ [P] $\begin{array}{r} 4007 \\ +6998 \\ \hline \end{array}$

[Q] $\begin{array}{r} 76{,}503 \\ +49{,}798 \\ \hline \end{array}$ [R] $\begin{array}{r} 67{,}002 \\ +63{,}998 \\ \hline \end{array}$ [S] $\begin{array}{r} 58{,}589 \\ +47{,}968 \\ \hline \end{array}$ [T] $\begin{array}{r} 80{,}067 \\ +68{,}975 \\ \hline \end{array}$

2. Copy each problem. Give the missing digit for each ▥.

[A] $\begin{array}{r} 38▥ \\ +▥6 \\ \hline 398 \end{array}$ [B] $\begin{array}{r} 567 \\ +▥▥ \\ \hline 580 \end{array}$ [C] $\begin{array}{r} ▥7▥ \\ +▥8 \\ \hline 404 \end{array}$ [D] $\begin{array}{r} ▥5▥ \\ +5▥2 \\ \hline 989 \end{array}$ [E] $\begin{array}{r} 634 \\ +▥▥▥ \\ \hline 962 \end{array}$ [F] $\begin{array}{r} 5▥9 \\ +67▥ \\ \hline 1219 \end{array}$

[G] $\begin{array}{r} ▥3▥5 \\ +5▥4▥ \\ \hline 9998 \end{array}$ [H] $\begin{array}{r} 6{,}▥0▥ \\ +7{,}549 \\ \hline 13{,}5▥2 \end{array}$ ★[I] $\begin{array}{r} ▥{,}607 \\ +1{,}▥▥5 \\ \hline 10{,}042 \end{array}$ ★[J] $\begin{array}{r} ▥{,}▥▥▥ \\ +6{,}967 \\ \hline 13{,}295 \end{array}$ ★[K] $\begin{array}{r} 8{,}▥8▥ \\ +▥{,}6▥7 \\ \hline 16{,}672 \end{array}$

★**3.** Here is another way to find sums. Study the steps carefully. Then use this method to work the exercises.

Step 1	Step 2
$\begin{array}{r} 58 \\ +\ 34 \\ \hline 62 \end{array}$	$\begin{array}{r} 58 \\ +\ 34 \\ \hline 62 \\ 92 \end{array}$ ← sum
58 + 4 = 62 →	62 + 30 = 92

[A] $\begin{array}{r} 27 \\ +35 \\ \hline \end{array}$ [B] $\begin{array}{r} 56 \\ +35 \\ \hline \end{array}$

[C] $\begin{array}{r} 47 \\ +69 \\ \hline \end{array}$ [D] $\begin{array}{r} 68 \\ +57 \\ \hline \end{array}$

[E] $\begin{array}{r} 466 \\ +58 \\ \hline \end{array}$ [F] $\begin{array}{r} 946 \\ +75 \\ \hline \end{array}$

Column addition—larger numbers

You can find the sum of a column of three or more numbers in the same way that you found the sum of two numbers.

Adding ones	Adding tens	Adding hundreds
Step 1	Step 2	Step 3
$\begin{array}{r}4\,\overset{2}{3}\,6\\3\,5\,8\\+\,2\,8\,9\\\hline 3\end{array}$	$\begin{array}{r}\overset{1}{4}\,\overset{2}{3}\,6\\3\,5\,8\\+\,2\,8\,9\\\hline 8\,3\end{array}$	$\begin{array}{r}\overset{1}{4}\,3\,6\\3\,5\,8\\+\,2\,8\,9\\\hline 1\,0\,8\,3\end{array}$
$6+8+9=23$	$20+30+50+80=180$	$100+400+300+200=1000$

EXERCISES

1. Find the sums.

[A] $\begin{array}{r}243\\572\\+146\\\hline\end{array}$ [B] $\begin{array}{r}568\\57\\+223\\\hline\end{array}$ [C] $\begin{array}{r}68\\322\\+417\\\hline\end{array}$ [D] $\begin{array}{r}635\\444\\+511\\\hline\end{array}$ [E] $\begin{array}{r}428\\35\\+6\\\hline\end{array}$ [F] $\begin{array}{r}465\\874\\+956\\\hline\end{array}$

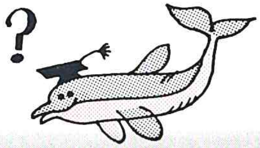
[G] $\begin{array}{r}113\\621\\+345\\\hline\end{array}$ [H] $\begin{array}{r}526\\204\\+789\\\hline\end{array}$ [I] $\begin{array}{r}456\\383\\+49\\\hline\end{array}$ [J] $\begin{array}{r}7764\\6752\\+8568\\\hline\end{array}$ [K] $\begin{array}{r}3993\\7306\\+8541\\\hline\end{array}$ [L] $\begin{array}{r}9104\\8249\\+2174\\\hline\end{array}$

2. Solve the equations.

[A] $632+75+8=\boxed{n}$
[B] $68+232+40=\boxed{n}$
[C] $\boxed{n}=9+6+34+346$
[D] $\boxed{n}=876+345+23$
[E] $693+27+8=\boxed{n}$
[F] $87+9+658=\boxed{n}$
[G] $\boxed{n}=658+26+37+9$
[H] $759+88+54+8=\boxed{n}$

Think

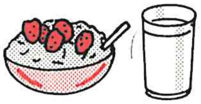
Suppose 3 squirrels can eat 3 nuts in 3 minutes. At that rate, how long will it take 100 squirrels to eat 100 nuts?

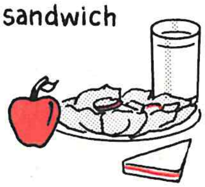


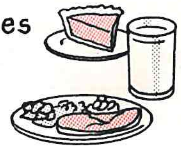
Energy

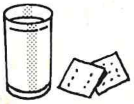
The food we eat gives us energy to work and play and keep our body warm. Some foods give us more energy than others. Foods which supply more calories give us more energy. Jack read that 9- or 10-year-old girls and boys need about 2000 to 2400 calories each day. He made this chart to record his supply of calories for one day.

Calorie Chart









	Number of Calories
BREAKFAST	
Juice	85
Cereal	95
Strawberries	43
1/2 cup milk	85
Sugar	15
LUNCH	
Ham and cheese sandwich	395
Lettuce salad	48
Baked Beans	147
Apple	75
Milk, 1 glass	166
DINNER	
Round Steak	235
Hash Brown Potatoes	197
Peas and Carrots	60
Bread and Butter	157
Pumpkin Pie	198
Milk, 1 glass	166
SNACK	
Milk, 1 glass	166
2 soda crackers	45

1. How many calories did Jack get from
   [A] his breakfast?
   [B] his lunch?
   [C] his dinner?

2. How many calories did Jack get in all?

3. How many calories did Jack get from milk and juice?

4. How many calories did Jack get from the sandwich and the pie?

5. Jack made this chart of his calories for one week.
   [A] How many calories did Jack get during the 5 days of the school week?
   [B] How many calories did he get on the weekend?
   [C] What was Jack's total calorie supply for the week?

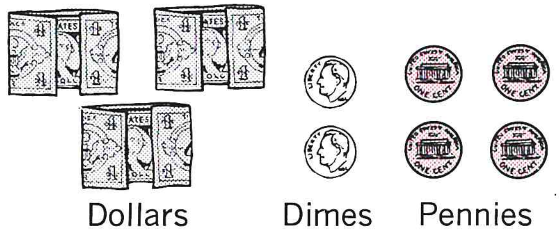
Sun.	2607
Mon.	2454
Tues.	2213
Wed.	2378
Thurs.	2185
Fri.	2732
Sat.	2578

★6. A calorie is the amount of heat needed to raise the temperature of 1 pound of water 4 Fahrenheit degrees. How many calories would be needed to raise the temperature of 50 pounds of water from 92 degrees to 100 degrees Fahrenheit?

Money—finding the total amount

We write: $3.24

We say: "three dollars | and | twenty-four cents"



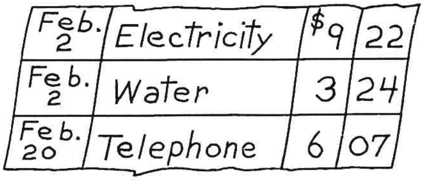
The period separates the dollars from the cents.

$3.24 was collected on Monday. $5.89 was collected on Tuesday. Study these steps to see that finding the total amount is much like adding the numbers 324 and 589.

Step 1	Step 2	Step 3
Dimes $\begin{array}{r} \$3.\overset{1}{2}4 \\ \$5.89 \\ \hline 3 \end{array}$	Dollars $\begin{array}{r} \$\overset{1}{3}.\overset{1}{2}4 \\ \$5.89 \\ \hline 13 \end{array}$	$\begin{array}{r} \$\overset{1}{3}.\overset{1}{2}4 \\ \$5.89 \\ \hline 9.13 \end{array}$
4 pennies and 9 pennies make 13 pennies.	1 dime, 2 dimes, and 8 dimes make 11 dimes.	1 dollar, 3 dollars, and 5 dollars make 9 dollars.
When we have 10 pennies, we can exchange them for 1 dime.	When we have 10 dimes, we can exchange them for 1 dollar.	We have 9 dollars, 1 dime, and 3 pennies, or 9 dollars and 13 cents.

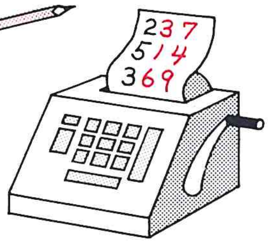
EXERCISES

1. Susan's mother keeps a record of the family expenses in a budget book. Susan figured the total amount spent for electricity, water, and telephone. How much was spent?

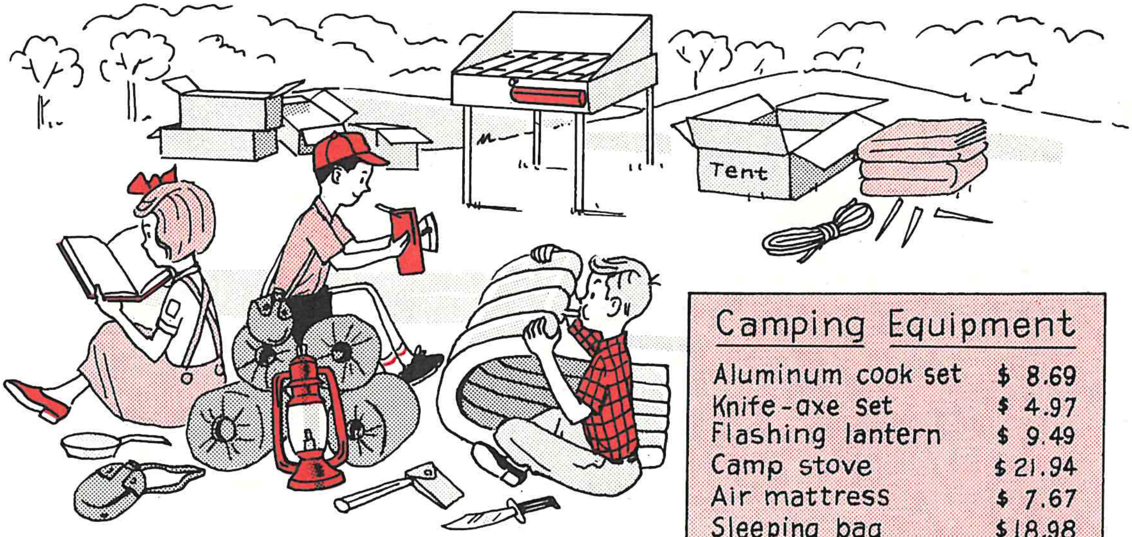


Feb. 2	Electricity	$9	22
Feb. 2	Water	3	24
Feb. 20	Telephone	6	07

2. This adding machine prints the dollars in black and the cents in red. When the "total" button is pushed, the machine will give the total amount. What will it be?



Planning the camping trip



Carol's family is getting ready for a camping trip. Here is a list of some of the things they might need and the cost of each item.

Camping Equipment

Aluminum cook set	\$ 8.69
Knife-axe set	\$ 4.97
Flashing lantern	\$ 9.49
Camp stove	\$ 21.94
Air mattress	\$ 7.67
Sleeping bag	\$ 18.98
Tent	\$ 98.50
Campground guide book	\$ 2.17
Canteen set	\$ 3.84
Hunting knife	\$ 4.27

1. Carol used part of her money to buy an air mattress and a campground guide book. How much did she spend?

2. Kent wants to earn enough money to buy a sleeping bag and a hunting knife. How much must he earn?

3. What is the total cost of the tent and the camp stove?

4. Alan has \$11. How much more does he need to buy a flashing lantern and a knife-axe set?

5. [A] Each of the five members of the family bought a canteen set. What was the total cost?

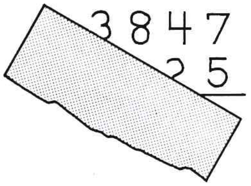
[B] Would they have paid more or less if they had bought two aluminum cook sets instead of the five canteen sets? How much more or less?

★ **6.** Find the total cost of all the equipment listed.

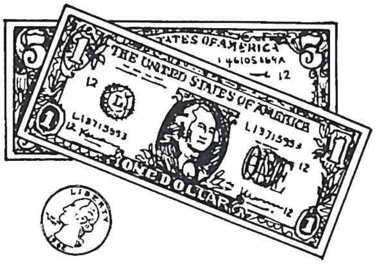
Reasoning

DISCUSSION EXERCISES

1. Ted found the sum of the two numbers that are partly covered by his paper. He told Tom that the answer was an odd number. How do you know he was wrong?

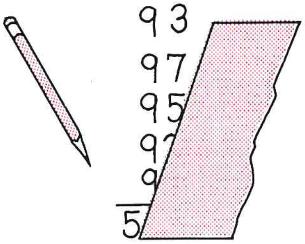


2. Helen gave the clerk a $10 bill to pay for $4.73 worth of groceries. Part of her change looked like this. Explain how you know that the clerk made a mistake.



3. John worked an addition problem. His answer was 325,463. Bill worked the same problem. His answer was 324,463. What can you say about their work?

4. Judy added five 2-digit numbers. Her answer was 534. Why is her answer incorrect?

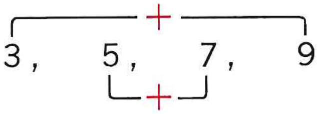


5. Here are the first 10 numbers in the sequence of odd numbers.

1 3 5 7 9 11 13 15 17 19

Pick any four of the numbers in order (like 3, 5, 7, 9) and add them as shown in this diagram.

outer sum, $3+9=12$



inner sum, $5+7=12$

[A] Are the inner and outer sums the same for any four odd numbers in a row? Try three more examples.

[B] Explain why the sums are always the same.

Sequences

A sequence is a list of numbers. Here is a sequence. Give the next four numbers.

Ⓐ 1, 7, 13, 19, ☐ ☐, ☐, ☐, . . .

Here is another sequence. Give the next four numbers.

Ⓑ 2, 4, 8, 16, ☐, ☐, ☐, ☐, . . .

Each sequence above shows a pattern that helps you find the next numbers. In some sequences the pattern is not as clear as those above. Give the next four numbers in this sequence,

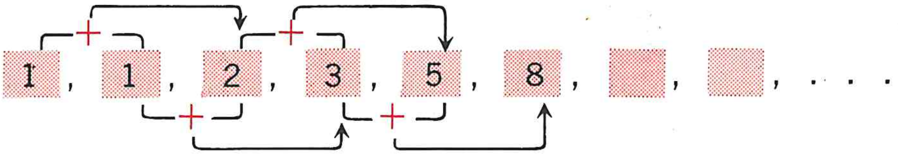
Ⓒ 1, 1, 2, 3, 5, 8, ☐, ☐, ☐, ☐, . . .

EXERCISES

1. Give the next four numbers in each sequence.

[A] 5, 10, 15, 20, . . .
[B] 2, 4, 6, 8, . . .
[C] 1, 0, 2, 0, 3, 0, . . .
[D] 1, 3, 5, 7, . . .
[E] 10, 100, 1000, . . .
[F] 12, 23, 34, 45, . . .
[G] 1, 34, 67, 100, . . .
[H] 3, 1, 1, 6, 1, 1, 9, 1, 1, . . .
★[I] 4, 3, 2, 8, 7, 6, 12, 11, . . .
★[J] 5, 5, 10, 15, 25, . . .

2. In sequence Ⓒ above, each number after the first two is found by adding the two numbers that come before.

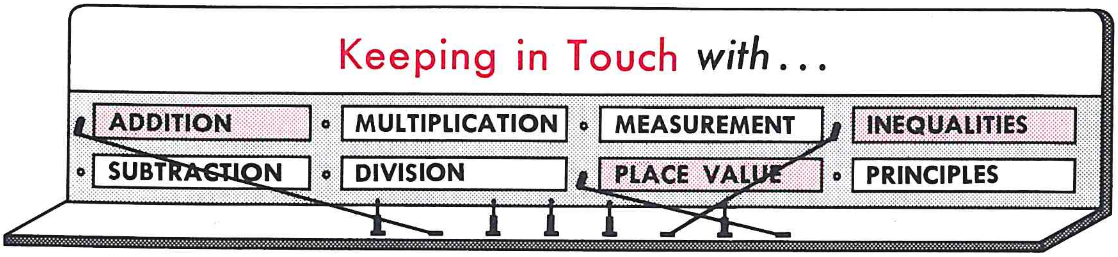


[A] Guess how many more numbers you would need to list in this sequence before you reached a number larger than 1000.

[B] List the numbers in the sequence until you reach a number larger than 1000.

★ **3.** Do you think the twentieth number in sequence B will be as large as 10,000? List as many numbers as you need to find out.

Keeping in Touch with...



1. Solve the equations.

[A] $287 = 200 + 80 + \boxed{n}$

[B] $456 = 400 + \boxed{n} + 6$

[C] $981 = \boxed{n} + 80 + 1$

[D] $342 = 300 + \boxed{n} + 2$

[E] $780 = 700 + 80 + \boxed{n}$

[F] $604 = 600 + \boxed{n} + 4$

2. Write the numeral for

[A] three thousand two hundred thirty-five.

[B] fifty-six thousand, fifty-four.

[C] fifty-six thousand, two hundred fifty-four.

[D] nine hundred thirty-six thousand, one hundred one.

[E] nine hundred fifty thousand, twelve.

[F] four million, twenty-three thousand, forty-two.

[G] thirty-nine million, one hundred seventy-eight.

3. Write each of the following numbers as in the example.

Example: $4386 = 4000 + 300 + 80 + 6$

[A] 56

[B] 342

[C] 783

[D] 5287

[E] 6043

[F] 28,642

[G] 75,480

[H] 628,492

[I] 3,482,649

4. In the numeral 43,682 the 3 stands for 3000. Give the number for each colored digit below.

[A] 34,682

[B] 34,682

[C] 34,682

[D] 34,682

[E] 6,284,563

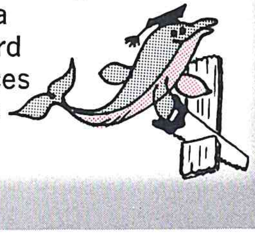
[F] 6,284,563

[G] 6,284,563

[H] 6,284,563

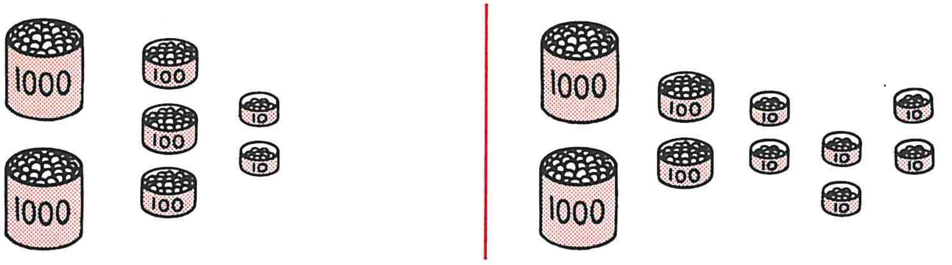
Think

If it takes 2 minutes to make each cut, how long will it take to cut a 10-foot board into 10 pieces of the same length?

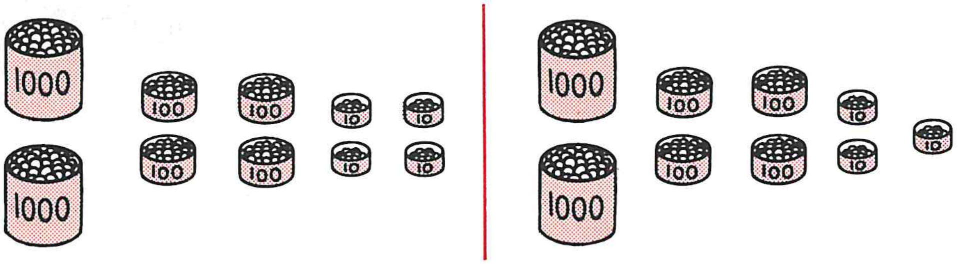


5. The number of beads in each can is labeled on the can. Which set, left or right, contains more beads?

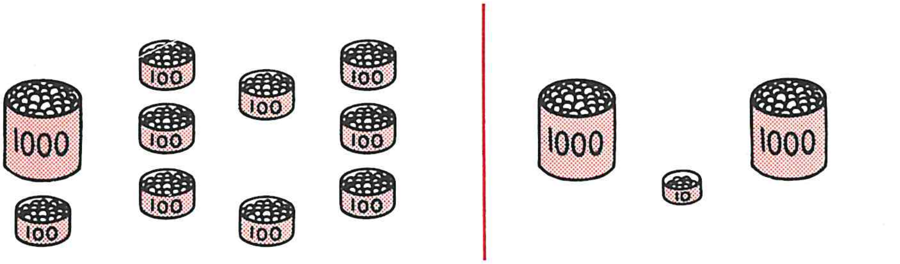
[A]



[B]



[C]



6. In each exercise, give the sign < or > for the ▥. Then give the words (greater than or less than) for the blank.

[A] 286 ▥ 276
286 is _?_ 276

[B] 304 ▥ 296
304 is _?_ 296

[C] 5279 ▥ 5280
5279 is _?_ 5280

[D] 6848 ▥ 6792
6848 is _?_ 6792

[E] 8267 ▥ 4983
8267 is _?_ 4983

[F] 2409 ▥ 2410
2409 is _?_ 2410

[G] 8663 ▥ 8636
8663 is _?_ 8636

[H] 9528 ▥ 7643
9528 is _?_ 7643

[I] 25,367 ▥ 25,267
25,367 is _?_ 25,267

[J] 83,000 ▥ 82,999
83,000 is _?_ 82,999

[K] 48,287 ▥ 48,290
48,287 is _?_ 48,290

[L] 80,080 ▥ 80,008
80,080 is _?_ 80,008

Regrouping

It is useful to think about numbers in different ways. Explain how the grouping principle is used in the examples below.

48 40 + 8 (30 + 10) + 8 30 + (10 + 8) 30 + 18	640 600 + 40 (500 + 100) + 40 500 + (100 + 40) 500 + 140	8600 8000 + 600 (7000 + 1000) + 600 7000 + (1000 + 600) 7000 + 1600

EXERCISES

1. Give the missing numbers.

[A] $32 = 30 + n$
$32 = 20 + n$

[B] $75 = n + 5$
$75 = n + 15$

[C] $56 = 50 + n$
$56 = 40 + n$

[D] $63 = n + 3$
$63 = n + 13$

[E] $64 = n + 4$
$64 = n + 14$

[F] $93 = 90 + n$
$93 = 80 + n$

[G] $36 = 30 + n$
$36 = 20 + n$

[H] $72 = n + 2$
$72 = n + 12$

[I] $450 = 400 + n$
$450 = 300 + n$

[J] $630 = 600 + n$
$630 = 500 + n$

[K] $4400 = n + 400$
$4400 = n + 1400$

[L] $7800 = 7000 + n$
$7800 = 6000 + n$

2. Instead of writing $56 = 40 + 16$, we could write: $\longrightarrow$ $\begin{array}{cc} \text{tens} & \text{ones} \\ 5 & 6 \end{array} = \begin{array}{cc} \text{tens} & \text{ones} \\ 4 & {}^{1}6 \end{array}$

Find the number of ones (O), tens (T), or hundreds (H) for each red space.

[A] $\begin{array}{cc} T & O \\ 8 & 3 \end{array} = \begin{array}{cc} T & O \\ 7 & \square \end{array}$

[B] $\begin{array}{cc} T & O \\ 6 & 4 \end{array} = \begin{array}{cc} T & O \\ \square & {}^{1}4 \end{array}$

[C] $\begin{array}{cc} T & O \\ 4 & \square \end{array} = \begin{array}{cc} T & O \\ 3 & {}^{1}6 \end{array}$

[D] $\begin{array}{cc} T & O \\ 9 & 6 \end{array} = \begin{array}{cc} T & O \\ 8 & \square \end{array}$

[E] $\begin{array}{cc} T & O \\ \square & 6 \end{array} = \begin{array}{cc} T & O \\ 4 & {}^{1}6 \end{array}$

[F] $\begin{array}{ccc} H & T & O \\ 3 & 4 & 7 \end{array} = \begin{array}{ccc} H & T & O \\ 3 & \square & {}^{1}7 \end{array}$

[G] $\begin{array}{ccc} H & T & O \\ 6 & 8 & 4 \end{array} = \begin{array}{ccc} H & T & O \\ 5 & \square & 4 \end{array}$

[H] $\begin{array}{ccc} H & T & O \\ 9 & 1 & 7 \end{array} = \begin{array}{ccc} H & T & O \\ 9 & \square & {}^{1}7 \end{array}$

[I] $\begin{array}{ccc} H & T & O \\ 4 & 5 & 7 \end{array} = \begin{array}{ccc} H & T & O \\ \square & \square & {}^{1}7 \end{array}$

[J] $\begin{array}{ccc} H & T & O \\ 5 & 0 & 3 \end{array} = \begin{array}{ccc} H & T & O \\ \square & \square & {}^{1}3 \end{array}$

★**[K]** $\begin{array}{cccc} & H & T & O \\ 6 & 2 & 9 & 6 \end{array} = \begin{array}{cccc} & H & T & O \\ 5 & \square & 9 & 6 \end{array}$

★**[L]** $\begin{array}{cccc} & H & T & O \\ 3 & 7 & 0 & 1 \end{array} = \begin{array}{cccc} & H & T & O \\ 3 & 6 & \square & {}^{1}1 \end{array}$

Subtraction—2-digit numbers

DISCUSSION EXERCISES

Explain how regrouping is used to find 75 — 29 in example A.

A

$$\begin{array}{r}75\\-29\\\hline\end{array}\longrightarrow\begin{array}{r}70\\-20\\\hline\end{array}\quad\begin{array}{r}5\\-9\\\hline ?\end{array}\longrightarrow\begin{array}{r}60\\-20\\\hline 40\end{array}\quad\begin{array}{r}15\\-9\\\hline 6\end{array}\longrightarrow\begin{array}{r}75\\-29\\\hline 46\end{array}$$

Explain how example B uses regrouping to find 75 — 29.

B Step 1	Step 2	Step 3	Step 4
$\begin{array}{r}7\ 5\\-\ 2\ 9\\\hline\end{array}$	$\begin{array}{r}\overset{6}{\cancel{7}}\ {}^{1}5\\-\ 2\ 9\\\hline\end{array}$	$\begin{array}{r}\overset{6}{\cancel{7}}\ {}^{1}5\\-\ 2\ 9\\\hline 6\end{array}$	$\begin{array}{r}\overset{6}{\cancel{7}}\ {}^{1}5\\-\ 2\ 9\\\hline 4\ 6\end{array}$
We cannot subtract 9 from 5 and get a whole number.	75 = 60 + 15	15 — 9 = 6	60 — 20 = 40

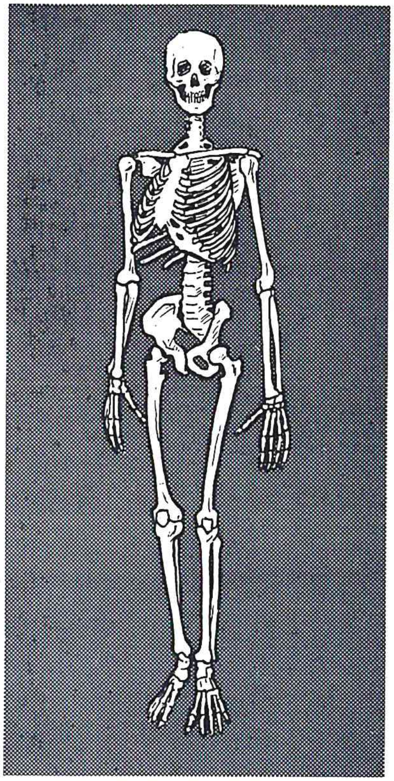
EXERCISES

Find the differences.

1. $\begin{array}{r}54\\-16\\\hline\end{array}$	**2.** $\begin{array}{r}35\\-28\\\hline\end{array}$	**3.** $\begin{array}{r}24\\-16\\\hline\end{array}$	**4.** $\begin{array}{r}90\\-45\\\hline\end{array}$	**5.** $\begin{array}{r}56\\-38\\\hline\end{array}$	**6.** $\begin{array}{r}45\\-8\\\hline\end{array}$
7. $\begin{array}{r}67\\-39\\\hline\end{array}$	**8.** $\begin{array}{r}30\\-16\\\hline\end{array}$	**9.** $\begin{array}{r}50\\-27\\\hline\end{array}$	**10.** $\begin{array}{r}46\\-28\\\hline\end{array}$	**11.** $\begin{array}{r}80\\-39\\\hline\end{array}$	**12.** $\begin{array}{r}43\\-19\\\hline\end{array}$
13. $\begin{array}{r}76\\-68\\\hline\end{array}$	**14.** $\begin{array}{r}135\\-57\\\hline\end{array}$	**15.** $\begin{array}{r}154\\-96\\\hline\end{array}$	**16.** $\begin{array}{r}183\\-28\\\hline\end{array}$	**17.** $\begin{array}{r}174\\-96\\\hline\end{array}$	**18.** $\begin{array}{r}135\\-48\\\hline\end{array}$
19. $\begin{array}{r}124\\-97\\\hline\end{array}$	**20.** $\begin{array}{r}157\\-68\\\hline\end{array}$	**21.** $\begin{array}{r}194\\-99\\\hline\end{array}$	**22.** $\begin{array}{r}173\\-69\\\hline\end{array}$	**23.** $\begin{array}{r}176\\-89\\\hline\end{array}$	**24.** $\begin{array}{r}191\\-97\\\hline\end{array}$

The human skeleton

The head	Number of bones
Cranium	8
Face	14
Ears	6
Hyoid in neck	1
The trunk	
Spinal column	26
Ribs	24
Breastbone	1
Collar bones	2
Shoulder bones	2
Pelvic bones	2
The limbs	
Arms	60
Legs	60



1. How many bones are in the head?
2. How many bones are in the trunk?
3. How many more bones does the trunk have than the head?
4. Each arm has 30 bones. The hand and wrist contain all but three of these. How many bones are in the hand and wrist?
5. Each leg has 30 bones. The ankle and foot have 26 of these. How many bones are in the rest of the leg?
6. 12 bones in the spinal column each have two ribs attached to them. How many bones in the spinal column do not have ribs attached?
7. How many bones are in the human body?

Subtraction—3-digit numbers

You can use what you have learned about subtracting 2-digit numbers to help you subtract 3-digit numbers.

Step 1	Step 2	Step 3
$\begin{array}{r} 6\,3\,4 \\ -\,3\,7\,8 \\ \hline \end{array}$	$\begin{array}{r} 6\,\overset{2}{\cancel{3}}\,{}^{1}4 \\ -\,3\,7\,8 \\ \hline \end{array}$	$\begin{array}{r} 6\,\overset{2}{\cancel{3}}\,{}^{1}4 \\ -\,3\,7\,8 \\ \hline 6 \end{array}$
There is no whole-number answer.	$34 = 20 + 14$	$14 - 8 = 6$

Step 4	Step 5	Step 6
$\begin{array}{r} \overset{5}{\cancel{6}}\,\overset{12}{\cancel{3}}\,{}^{1}4 \\ -\,3\,7\,8 \\ \hline 6 \end{array}$	$\begin{array}{r} \cancel{6}\,\overset{12}{\cancel{3}}\,{}^{1}4 \\ -\,3\,7\,8 \\ \hline 5\,6 \end{array}$	$\begin{array}{r} \overset{5}{6}\,\overset{12}{\cancel{3}}\,4 \\ -\,3\,7\,8 \\ \hline 2\,5\,6 \end{array}$
$620 = 500 + 120$	$120 - 70 = 50$	$500 - 300 = 200$

EXERCISES

Find the differences.

1. $\begin{array}{r} 762 \\ -425 \\ \hline \end{array}$ **2.** $\begin{array}{r} 847 \\ -363 \\ \hline \end{array}$ **3.** $\begin{array}{r} 934 \\ -568 \\ \hline \end{array}$

4. $\begin{array}{r} 654 \\ -257 \\ \hline \end{array}$ **5.** $\begin{array}{r} 820 \\ -316 \\ \hline \end{array}$ **6.** $\begin{array}{r} 717 \\ -209 \\ \hline \end{array}$

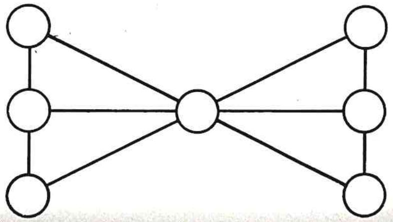
7. $\begin{array}{r} 438 \\ -59 \\ \hline \end{array}$ **8.** $\begin{array}{r} 564 \\ -209 \\ \hline \end{array}$ **9.** $\begin{array}{r} 713 \\ -307 \\ \hline \end{array}$

10. $\begin{array}{r} 876 \\ -497 \\ \hline \end{array}$ **11.** $\begin{array}{r} 521 \\ -475 \\ \hline \end{array}$ **12.** $\begin{array}{r} 923 \\ -854 \\ \hline \end{array}$

13. $\begin{array}{r} 863 \\ -429 \\ \hline \end{array}$ **14.** $\begin{array}{r} 562 \\ -367 \\ \hline \end{array}$ **15.** $\begin{array}{r} 674 \\ -289 \\ \hline \end{array}$

Think

Draw a figure like the one below. Put the digits 2, 3, 4, 5, 6, 7, and 8 in the circles so the sum along any line is 15.



Subtraction exercises

1. Find the differences.

[A] 564 −191　[B] 851 −234　[C] 323 −119　[D] 746 −246　[E] 475 −158　[F] 663 −495

[G] 524 −519　[H] 405 −208　[I] 230 −150　[J] 964 −787　[K] 787 −499　[L] 455 −277

[M] 631 −356　[N] 715 −538　[O] 376 −189　[P] 1428 −619　[Q] 1536 −657　[R] 1847 −588

2. Find the differences.

[A] 1543 −1369　[B] 2173 −1478　[C] 1467 −873　[D] 3424 −1945　[E] 6531 −3874

[F] 98,752 −75,176　[G] 86,345 −38,751　[H] 97,683 −69,794　[I] 83,745 −51,876

3. Solve the equations.

[A] $34 - 29 = \boxed{n}$　[D] $565 - 27 = \boxed{n}$　[G] $732 - 727 = \boxed{n}$

[B] $156 - 48 = \boxed{n}$　[E] $654 - 646 = \boxed{n}$　[H] $854 - 8 = \boxed{n}$

[C] $347 - 9 = \boxed{n}$　[F] $1548 - 538 = \boxed{n}$　[I] $923 - 19 = \boxed{n}$

★**4.** Copy the problems. Give the missing digit for each ▨.

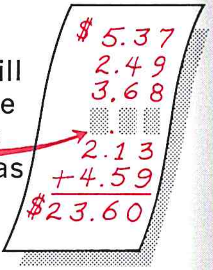
[A] ▨▨▨ −264 = 274　[B] 8▨3 −▨48 = 50▨

[C] ▨73 −1▨▨ = 348　[D] 727 −▨▨▨ = 260

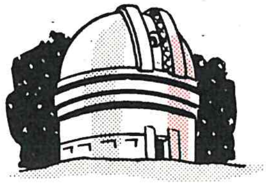
[E] 9▨3 −488 = ▨0▨　[F] 58▨ −2▨6 = ▨82

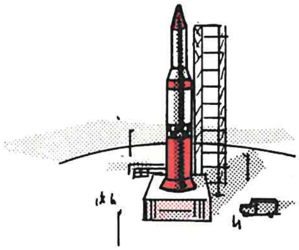
Think

Here is Mrs. Murphy's bill from the meat market. The cost of the hamburger was marked out. How much was the hamburger?



Science short stories





1. Telescope made by Galileo in 1610. Mt. Palomar 200-inch reflector telescope built 1948. How many years later?

2. Thomas Edison. Born, 1847. Died, 1931. How long did he live?

3. Edison invented the electric light bulb, 1879. How old was he then?

4. Albert Einstein. Born, 1879. Died, 1955. How long did he live?

5. Electricity generated from atomic energy, 1951. How old was Einstein then?

6. Compound microscope. Invented by Janssen in 1590. How many years ago was this?

7. Chinese rockets invented about 1232. First liquid-fuel rocket launched, 1926. How many years later?

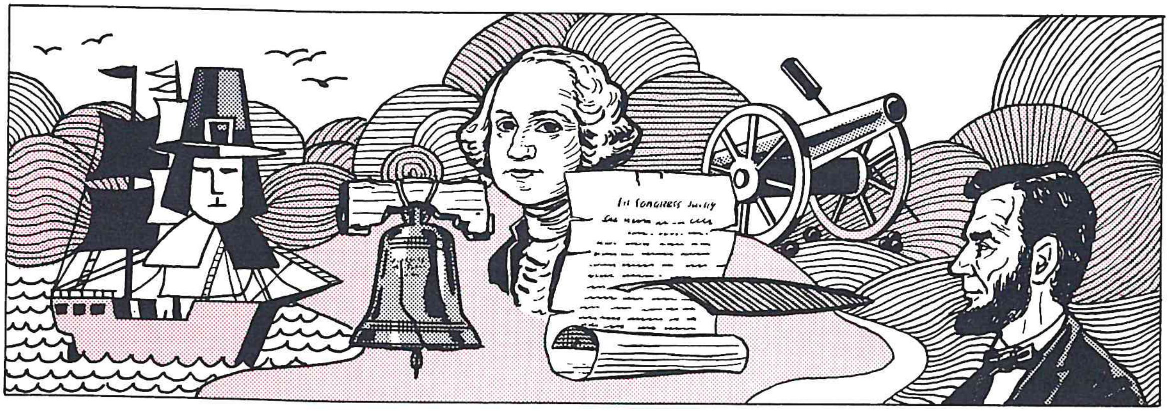
8. Artificial satellites suggested by Sir Isaac Newton, 1689. First manned satellite into orbit, 1961. How many years later?

9. Louis Pasteur prevented diseases by vaccination, 1881. Polio vaccine developed by Jonas Salk, 1953. How many years later?

10. First adding machine invented by Pascal, 1642. How many years ago?

11. Electromagnetic waves studied by James Maxwell in 1864. Some results: radio, television, radar, lasers. About how many decades ago did Maxwell study these waves?

History short stories



1. America. Discovered in 1492. How many years ago?

2. Jamestown, Virginia. Settled in 1607. How many years ago?

3. Pilgrims. Landed at Plymouth, Massachusetts, in 1620.
   [A] How many years ago?
   [B] How many years after the Jamestown settlement?

4. Declaration of Independence. Signed, 1776. How many years ago?

5. George Washington chosen first President of United States, 1789. How many years ago?

6. Civil War began, 1861. How many years ago?

7. World War I, 1914. How many years after the Civil War?

8. World War II, 1939. How many years ago?

9. American flag, 1777. 13 stars. American flag, 1960. 50 stars.
   [A] How many more stars in 1960 than in 1777?
   [B] How many years had passed?

10. George Washington. Born, 1732. Died, 1799. How long did he live?

11. Abraham Lincoln. Born, 1809. Died, 1865. How long did he live?

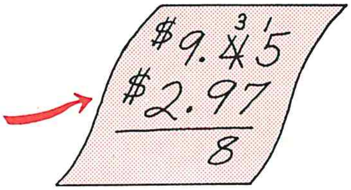
12. Gold discovered in California, 1848. How many years ago?

13. Paul Revere's ride, 1775. How many years was this after the discovery of America?

14. Hawaii annexed as a territory, 1898. Admitted as a state, 1960. How many years later?

Shopping problems

Kristy and Kathy went shopping with their mother and father. Kathy had $9.45. She wanted to buy a book about astronomy which cost $2.97. Here is the way she started to figure how much money she would have left.

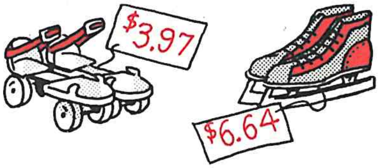


DISCUSSION EXERCISES

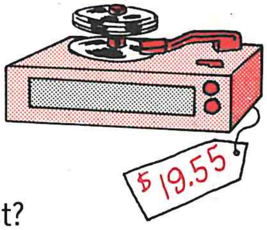
1. Explain Kathy's work.
2. Tell how Kathy should finish the problem.

EXERCISES

1. Kristy saw these roller skates and ice skates. Which cost more? How much more?



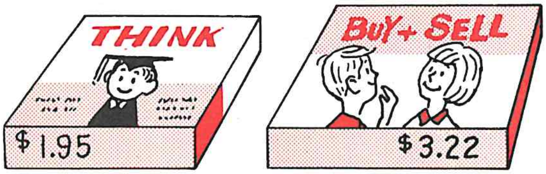
2. Kathy and Kristy are saving their money to buy a record player. They have saved $7.89. How much more must they save?



3. Kristy bought a book about flowers for $2.97, a book about horses for $1.47, and a children's dictionary for $3.73. How much did the books cost?

4. Kathy saw a birthstone ring that cost $6.49. She saw a double birthstone ring that cost $8.95. How much more did the double birthstone ring cost than the single birthstone ring?

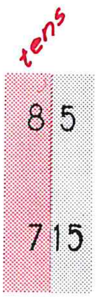
5. Kathy and Kristy each bought a game to give to friends.
   [A] Which game costs more?
   [B] How much more?

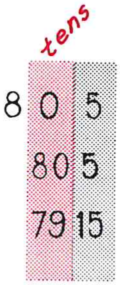


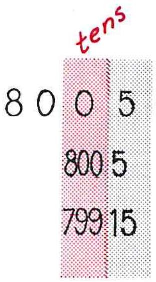
★ 6. Kathy's father gave the clerk $50 for a bicycle that cost $37.97. How much change did he receive?

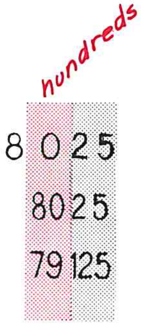
More regrouping

Here are some useful ways to think about numbers such as 85, 805, 8005, and 8025.









EXERCISES

1. Give the missing numbers.

[A] 80 ||||| tens
[B] 90 ||||| tens
[C] 100 ||||| tens
[D] 200 ||||| tens
[E] 800 ||||| tens
[F] 900 ||||| tens
[G] 1000 ||||| tens

[H] 2000 ||||| tens
[I] 5000 ||||| tens
[J] 10,000 ||||| tens
[K] 87 8 tens and |||||
[L] 92 ||||| tens and 2
[M] 109 10 tens and |||||
[N] 207 ||||| tens and 7

[O] 804 ||||| tens and 4
[P] 906 ||||| tens and 6
[Q] 1003 ||||| tens and 3
[R] 2008 200 tens and |||||
[S] 5001 ||||| tens and 1
[T] 10,009 ||||| tens and 9
[U] 12,346 ||||| tens and 6

2. Give the missing numbers.

[A] 74 ||||| tens and 4
74 ||||| tens and 14

[B] 106 ||||| tens and 6
106 ||||| tens and 16

[C] 407 40 tens and |||||
407 39 tens and |||||

[D] 705 ||||| tens and 5
705 ||||| tens and 15

[E] 1008 100 tens and |||||
1008 99 tens and |||||

[F] 5003 ||||| tens and 3
5003 ||||| tens and 13

[G] 7036 70 hundreds and |||||
7036 69 hundreds and |||||

[H] 9040 ||||| hundreds and 40
9040 ||||| hundreds and 140

More subtraction

DISCUSSION EXERCISES

1. Explain the regrouping in step 2 of example A below.
2. Explain each part of step 3 in example A.
3. Explain the regrouping in step 2 of example B below.
4. Explain each part of step 3 in example B.

Ⓐ Step 1	Step 2	Step 3
$\begin{array}{r} 507 \\ -249 \\ \hline \end{array}$	$\begin{array}{r} \overset{4}{\cancel{5}}\overset{9}{\cancel{0}}\overset{1}{7} \\ -249 \\ \hline \end{array}$	$\begin{array}{r} \overset{4}{\cancel{5}}\overset{9}{\cancel{0}}\overset{1}{7} \\ -249 \\ \hline 258 \\ \text{(c)(b)(a)} \end{array}$
There is no whole-number answer.	507: 49 tens and 17	(a) $17-9=8$ (b) $90-40=50$ (c) $400-200=200$

Ⓑ Step 1	Step 2	Step 3
$\begin{array}{r} 8004 \\ -4378 \\ \hline \end{array}$	$\begin{array}{r} \overset{7}{\cancel{8}}\overset{9}{\cancel{0}}\overset{9}{\cancel{0}}\overset{1}{4} \\ -4378 \\ \hline \end{array}$	$\begin{array}{r} \overset{7}{\cancel{8}}\overset{9}{\cancel{0}}\overset{9}{\cancel{0}}\overset{1}{4} \\ -4378 \\ \hline 3626 \\ \text{(d)(c)(b)(a)} \end{array}$
There is no whole-number answer.	8004: 799 tens and 14	(a) $14-8=6$ (b) $90-70=20$ (c) $900-300=600$ (d) $7000-4000=3000$

EXERCISES

Find the differences.

1. $\begin{array}{r} 604 \\ -327 \\ \hline \end{array}$ **2.** $\begin{array}{r} 708 \\ -439 \\ \hline \end{array}$ **3.** $\begin{array}{r} 903 \\ -317 \\ \hline \end{array}$ **4.** $\begin{array}{r} 500 \\ -278 \\ \hline \end{array}$ **5.** $\begin{array}{r} 403 \\ -181 \\ \hline \end{array}$ **6.** $\begin{array}{r} 906 \\ -386 \\ \hline \end{array}$

7. $\begin{array}{r} 8005 \\ -2837 \\ \hline \end{array}$ **8.** $\begin{array}{r} 7002 \\ -5469 \\ \hline \end{array}$ **9.** $\begin{array}{r} 6000 \\ -3874 \\ \hline \end{array}$ **10.** $\begin{array}{r} 5006 \\ -2998 \\ \hline \end{array}$ **11.** $\begin{array}{r} 4000 \\ -1687 \\ \hline \end{array}$ **12.** $\begin{array}{r} 5010 \\ -2674 \\ \hline \end{array}$

Subtraction exercises

1. Find the differences. Check your answers by adding.

[A] $\begin{array}{r} 602 \\ -38 \\ \hline \end{array}$
[B] $\begin{array}{r} 700 \\ -87 \\ \hline \end{array}$
[C] $\begin{array}{r} 406 \\ -298 \\ \hline \end{array}$
[D] $\begin{array}{r} 1507 \\ -348 \\ \hline \end{array}$
[E] $\begin{array}{r} 104 \\ -37 \\ \hline \end{array}$
[F] $\begin{array}{r} 706 \\ -389 \\ \hline \end{array}$

[G] $\begin{array}{r} 534 \\ -125 \\ \hline \end{array}$
[H] $\begin{array}{r} 607 \\ -437 \\ \hline \end{array}$
[I] $\begin{array}{r} 876 \\ -409 \\ \hline \end{array}$
[J] $\begin{array}{r} 500 \\ -347 \\ \hline \end{array}$
[K] $\begin{array}{r} 607 \\ -278 \\ \hline \end{array}$
[L] $\begin{array}{r} 832 \\ -564 \\ \hline \end{array}$

[M] $\begin{array}{r} 803 \\ -529 \\ \hline \end{array}$
[N] $\begin{array}{r} 716 \\ -438 \\ \hline \end{array}$
[O] $\begin{array}{r} 640 \\ -539 \\ \hline \end{array}$
[P] $\begin{array}{r} 902 \\ -399 \\ \hline \end{array}$
[Q] $\begin{array}{r} 803 \\ -739 \\ \hline \end{array}$
[R] $\begin{array}{r} 916 \\ -488 \\ \hline \end{array}$

2. Find the differences. Check your answers by adding.

[A] $\begin{array}{r} 604 \\ -238 \\ \hline \end{array}$
[B] $\begin{array}{r} 3604 \\ -328 \\ \hline \end{array}$

[C] $\begin{array}{r} 4503 \\ -2437 \\ \hline \end{array}$
[D] $\begin{array}{r} 8702 \\ -739 \\ \hline \end{array}$

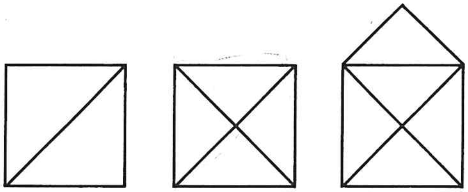
[E] $\begin{array}{r} 4300 \\ -2643 \\ \hline \end{array}$
[F] $\begin{array}{r} 5000 \\ -4365 \\ \hline \end{array}$

[G] $\begin{array}{r} 5036 \\ -2457 \\ \hline \end{array}$
[H] $\begin{array}{r} 9003 \\ -4328 \\ \hline \end{array}$

[I] $\begin{array}{r} 50{,}024 \\ -32{,}337 \\ \hline \end{array}$
[J] $\begin{array}{r} 80{,}435 \\ -23{,}629 \\ \hline \end{array}$

Think

Try to draw each figure below without lifting your pencil from the paper and without retracing. Two of them **can** be done. One of them **cannot.**



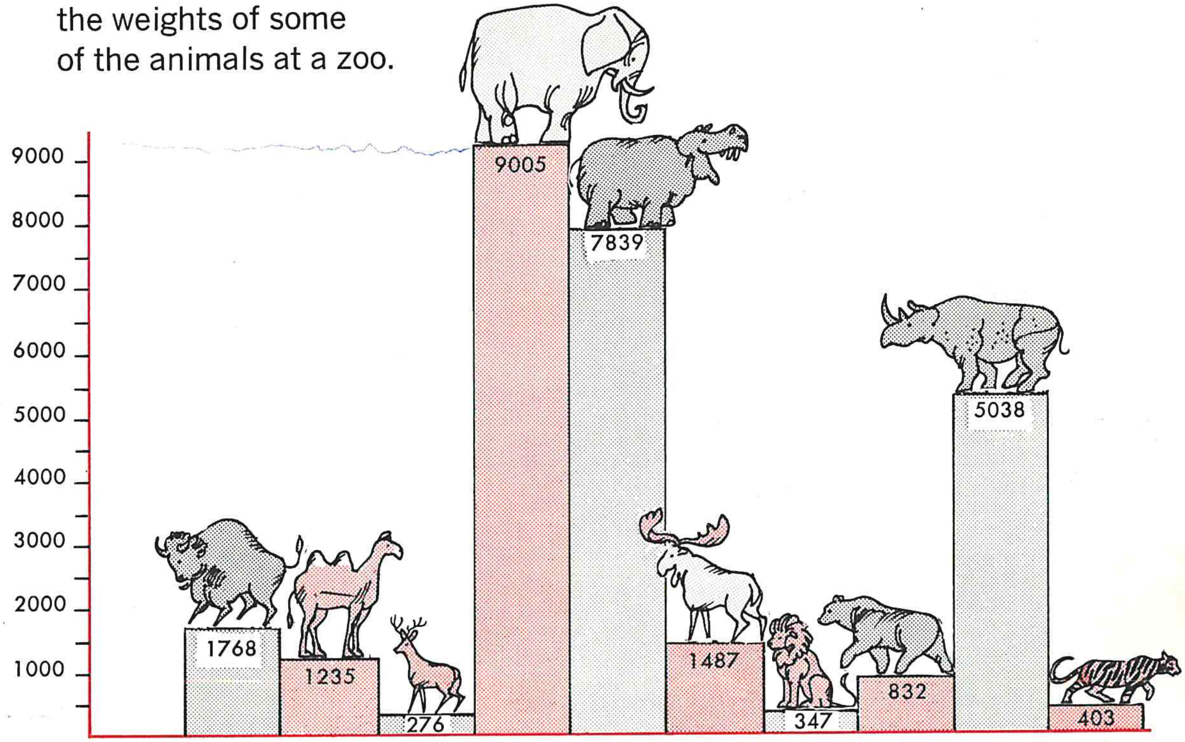
★3. Copy each problem. Write the missing digit for each ▥.

[A] $\begin{array}{r} 679 \\ -\text{▥▥▥} \\ \hline 374 \end{array}$
[B] $\begin{array}{r} \text{▥▥▥} \\ -595 \\ \hline 5 \end{array}$
[C] $\begin{array}{r} 1\text{▥}0\text{▥} \\ -8\text{▥}5 \\ \hline 702 \end{array}$
[D] $\begin{array}{r} \text{▥▥▥}5 \\ -57 \\ \hline 948 \end{array}$
[E] $\begin{array}{r} \text{▥▥}6 \\ -23\text{▥} \\ \hline 368 \end{array}$

[F] $\begin{array}{r} 5\text{▥▥}4 \\ -\text{▥}73\text{▥} \\ \hline 3004 \end{array}$
[G] $\begin{array}{r} \text{▥}705 \\ -4\text{▥▥}9 \\ \hline 4466 \end{array}$
[H] $\begin{array}{r} \text{▥▥▥▥} \\ -3658 \\ \hline 3342 \end{array}$
[I] $\begin{array}{r} 5\text{▥▥}6 \\ -\text{▥}53\text{▥} \\ \hline 1768 \end{array}$
[J] $\begin{array}{r} \text{▥}8\text{▥}7 \\ -3\text{▥}7\text{▥} \\ \hline 4828 \end{array}$

Animal weights

The bar graph gives the weights of some of the animals at a zoo.



1. How many pounds must the lion gain to weigh as much as the tiger?
2. The rhinoceros lost 699 pounds. How much did it weigh then?
3. Find the total weight of the two heaviest animals.
4. Find the total weight of the three lightest animals.
5. Find the difference in the weights of the two heaviest animals.
6. If a buffalo and a camel were weighed on the scales together, what would the scales read then?
7. How much more does the buffalo weigh than the bear?

★ 8. Some animals were on the scales. The scales read 5169 pounds. The moose got on the scales and the bear got off. What did the scales then read?

★ 9. A medium-sized car weighs 4327 pounds. What is the difference in the weight of the elephant and the weight of two cars?

Addition and subtraction exercises

1. Find the sums and differences.

[A]	[B]	[C]	[D]	[E]	[F]
567 +839	843 −456	705 −239	618 +496	583 +819	900 −347

[G]	[H]	[I]	[J]	[K]	[L]
608 −439	874 −567	767 +869	409 +786	676 −186	501 −403

[M]	[N]	[O]	[P]	[Q]	[R]
871 623 489 376 +554	304 122 304 169 +785	652 374 418 920 +513	364 905 845 367 +978	768 858 979 659 +675	896 784 599 867 +657

[S]	[T]	[U]	[V]
5076 −3489	6785 +8432	70,043 −46,528	8403 −7685

[W]	[X]	[Y]	[Z]
17,954 +98,767	8006 −4875	16,934 +8,657	35,007 −3,879

2. Give the sign (<, >, =) for each ●.

[A] 84 + 79 ● 74 + 79

[B] 567 + 35 ● 467 + 35

[C] 736 + 485 ● 736 + 385

[D] 895 + 567 ● 795 + 467

[E] 657 + 383 ● 567 + 393

[F] 876 − 634 ● 776 − 34

[G] 964 − 87 ● 964 − 97

[H] 384 − 169 ● 484 − 269

[I] 763 − 487 ● 863 − 497

487

[J] 836 − 597 ● 846 − 607

[K] 893 + 467 ● 783 + 577

[L] 475 − 257 ● 585 − 147

★ **3.** Without finding the sum or difference, tell how much more the first sum or difference is than the second.

[A] 57 + 78, 47 + 78

[B] 453 + 78, 353 + 68

[C] 817 − 39, 817 − 59

[D] 746 − 87, 646 − 87

[E] 654 + 239, 554 + 249

[F] 876 + 547, 776 + 557

[G] 943 − 378, 943 − 578

[H] 6007 − 349, 5007 − 449

More reasoning

I hope that
we dadent
do this

DISCUSSION EXERCISES

1. The number that 345 is subtracted from is covered. Explain how you can find this number.

$$\begin{array}{r} \text{▨} \\ -345 \\ \hline 597 \end{array}$$

2. Two digits have been covered.
   [A] Find the covered digits.
   [B] Explain your solution.

$$\begin{array}{r} \text{▨}6 \\ -39 \\ \hline 2\text{▨} \end{array}$$

3. One digit has been covered. Is the difference
   [A] less than 20?
   [B] 30 or less?
   [C] between 30 and 40?
   [D] Explain how you know.

$$\begin{array}{r} 5\text{▨} \\ -29 \\ \hline \end{array}$$

4. Make up a subtraction problem so that when you cover the ones digits it looks like this. →

$$\begin{array}{r} 8\text{▨} \\ -3\text{▨} \\ \hline 5\text{▨} \end{array}$$

5. Make up a subtraction problem so that when you cover the ones digits it looks like this. →

$$\begin{array}{r} 6\text{▨} \\ -2\text{▨} \\ \hline 3\text{▨} \end{array}$$

6. [A] Study the equation. Then choose the words that best complete the sentence:

   The number that is subtracted is
   greater than
   less than
   equal to
   the number that is added.

$$(53 - \text{▨}) + \text{▨} = 54$$

   [B] Explain how you decided.

Chapter review

1. Find the differences by finding the missing addends.

[A] $\boxed{n} + 49 = 52$
$52 - 49 = \boxed{n}$

[B] $\boxed{n} + 28 = 32$
$32 - 28 = \boxed{n}$

[C] $\boxed{n} + 357 = 362$
$362 - 357 = \boxed{n}$

[D] $\boxed{n} + 4349 = 4361$
$4361 - 4349 = \boxed{n}$

Think

Start with seven thousand.
Add four hundred three.
Then subtract eleven.
This will give you me.

WHO AM I?

2. Find the sums and differences.

[A] $\begin{array}{r} 36 \\ +25 \\ \hline \end{array}$ [B] $\begin{array}{r} 53 \\ -26 \\ \hline \end{array}$ [C] $\begin{array}{r} 60 \\ -37 \\ \hline \end{array}$ [D] $\begin{array}{r} 97 \\ +68 \\ \hline \end{array}$ [E] $\begin{array}{r} 79 \\ +87 \\ \hline \end{array}$ [F] $\begin{array}{r} 87 \\ -79 \\ \hline \end{array}$

[G] $\begin{array}{r} 326 \\ +497 \\ \hline \end{array}$ [H] $\begin{array}{r} 547 \\ -293 \\ \hline \end{array}$ [I] $\begin{array}{r} 603 \\ -329 \\ \hline \end{array}$ [J] $\begin{array}{r} 700 \\ -287 \\ \hline \end{array}$ [K] $\begin{array}{r} 754 \\ +869 \\ \hline \end{array}$ [L] $\begin{array}{r} 1565 \\ +978 \\ \hline \end{array}$

[M] $\begin{array}{r} 8046 \\ -3779 \\ \hline \end{array}$ [N] $\begin{array}{r} 5008 \\ +6053 \\ \hline \end{array}$ [O] $\begin{array}{r} 9002 \\ -5437 \\ \hline \end{array}$ [P] $\begin{array}{r} 5037 \\ -2649 \\ \hline \end{array}$ [Q] $\begin{array}{r} 8703 \\ -5827 \\ \hline \end{array}$

[R] $\begin{array}{r} 67 \\ 85 \\ +49 \\ \hline \end{array}$ [S] $\begin{array}{r} 385 \\ 277 \\ +969 \\ \hline \end{array}$ [T] $\begin{array}{r} 243 \\ 166 \\ +437 \\ \hline \end{array}$ [U] $\begin{array}{r} 5471 \\ 2364 \\ +6575 \\ \hline \end{array}$ [V] $\begin{array}{r} 56{,}732 \\ 85{,}968 \\ +97{,}879 \\ \hline \end{array}$

3. Copy each problem on your paper. Then find the sum.

[A] $356 + 49 + 8 + 6$
[B] $57 + 476 + 387 + 8$
[C] $5439 + 367 + 28$
[D] $395 + 2007 + 66$
[E] $6975 + 3428 + 26$
[F] $875 + 15{,}693 + 78$
[G] $9 + 892 + 3467 + 12$
[H] $54{,}937 + 6843 + 58$

4. Solve these problems about baseball batting records.

[A] One of the longest home runs on record was hit by Babe Ruth. The ball went 587 feet. If the longest baseball throw is 445 feet, how much longer was the home run?

[B] Ty Cobb had a lifetime record of 4191 hits. Babe Ruth made 2873 hits. How many more hits did Cobb make than Ruth?

[C] Babe Ruth hit 714 home runs during his major-league play. He also hit 506 doubles and 136 triples. How many extra base hits did Babe Ruth make?

5. Give the sign (<, >, =) that should go in each ●.

[A] 57 + 34 ● 54 + 37
[B] 532 + 256 ● 256 + 532
[C] 633 + 45 ● 533 + 55
[D] 857 + 384 ● 867 + 374
[E] 8324 + 637 ● 8224 + 647
[F] 8007 − 678 ● 8007 − 578
[G] 3967 − 1888 ● 3867 − 1888
[H] 6595 + 2387 ● 6387 + 2595
[I] 234 + (317 + 58) ● (234 + 316) + 58
[J] 5365 + 4579 ● 5465 + 4589

6595
2387
8182

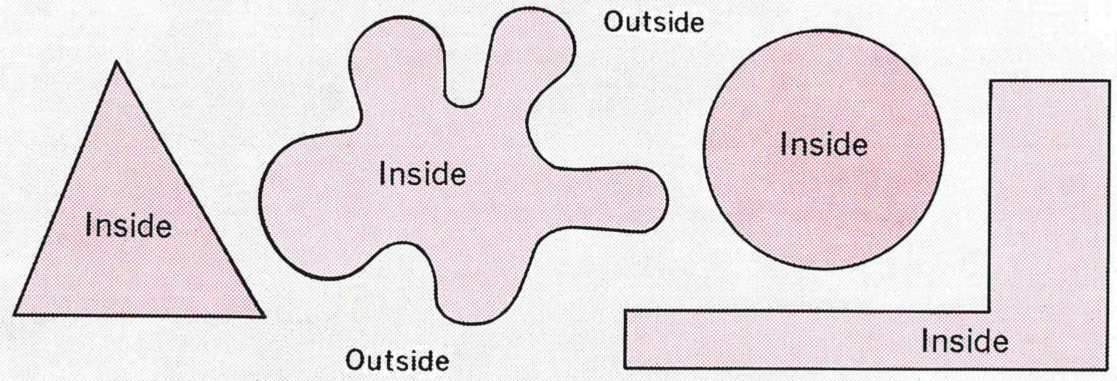
638
25
895

★ **6.** For each exercise, tell how much more the first sum or difference is than the second sum or difference.

[A] 376 + 423, 276 + 123
[B] 658 + 49, 638 + 59
[C] 865 + 378, 875 + 358
[D] 593 + 476, 493 + 366
[E] 847 + 599, 647 + 699
[F] 603 − 37, 503 − 37
[G] 7004 − 295, 7004 − 395
[H] 876 − 498, 776 − 598
[I] 738 − 299, 538 − 199
[J] 993 − 489, 793 − 499

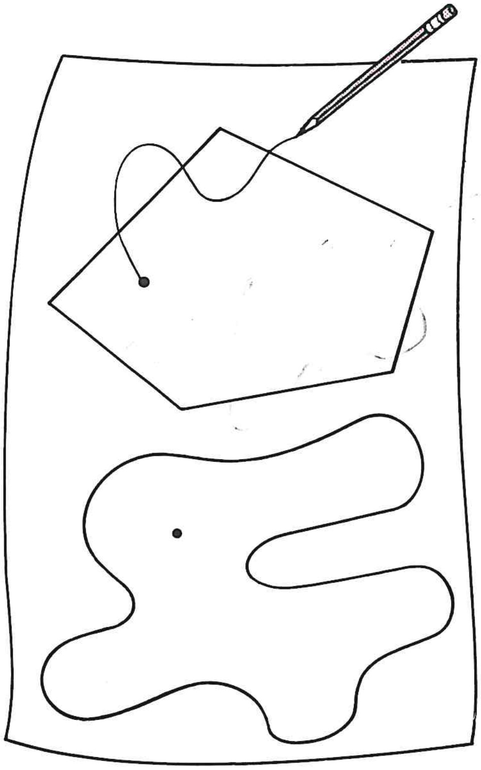
GEOMETRY Unit 1

Simple closed curves



SOME SPECIAL SIMPLE CLOSED CURVES

Each black outline above illustrates a simple closed curve. Notice that each figure has an inside (red) and an outside (gray). You can think of a simple closed curve as a loop of string that does not cross itself.



1. Draw one simple closed curve using your ruler. Draw another without using a ruler.

[**A**] Mark a point inside each curve. Starting at any one point, draw a line crossing the curve 6 times. Is your pencil point inside or outside?

[**B**] On the other figure, do the same thing you did in part **A** only cross 7 times. Where is your pencil point?

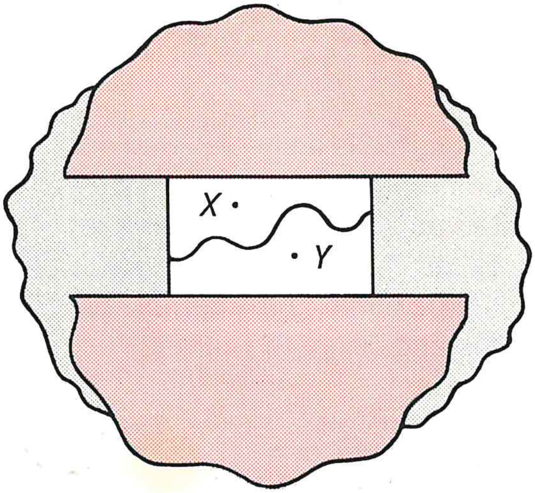
[**C**] Complete each sentence with the word inside or outside.

- ▶ If you start inside a simple closed curve and cross the curve an **even number of times,** you will be __?__.
- ▶ If you start inside a simple closed curve and cross the curve an **odd number of times,** you will be __?__.

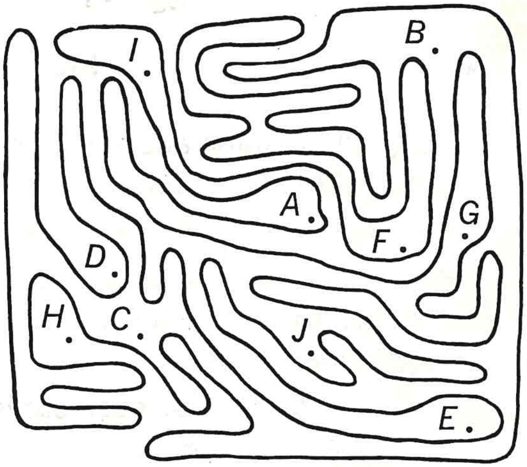
2. The red and gray screens are hiding all but a small part of a simple closed curve.

[**A**] If *X* is inside the curve, what do you know about *Y*?

[**B**] If *X* is outside the curve, what do you know about *Y*?



3. The figure on the right is a simple closed curve. Some of the points are inside the curve and some are outside. If *A* is outside and *B* is inside, give the location (inside or outside) of each point.



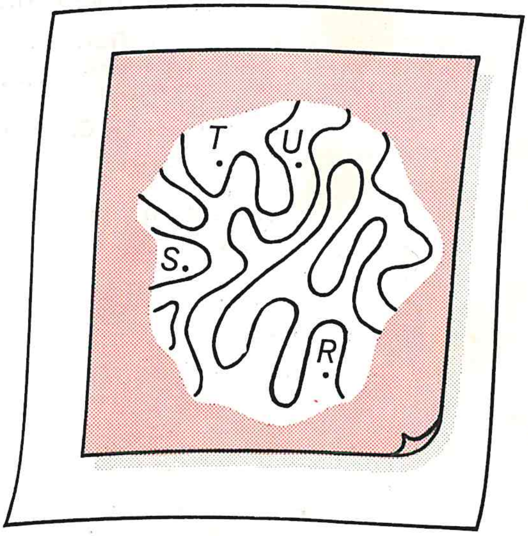
★ **4.** The hole torn in the red paper shows part of a simple closed curve.

[**A**] If *R* is inside the curve, where is *S*?

[**B**] Is it possible that both *T* and *U* are inside the curve?

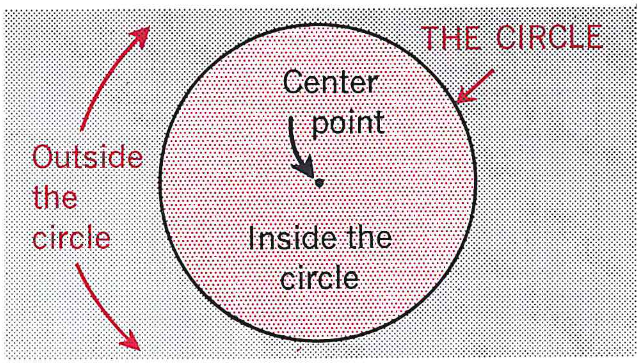
[**C**] If *S* is outside, where is *T*?

[**D**] If *S* is outside, where is *U*?

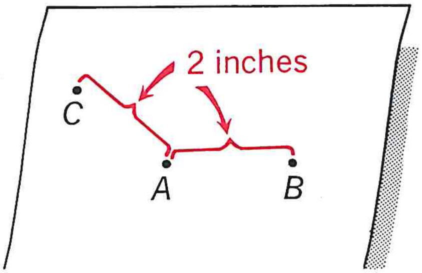


Circles and points

A circle is a special kind of simple closed curve. One of the points inside the curve is called the center. Each point on the circle is the same distance from the center.

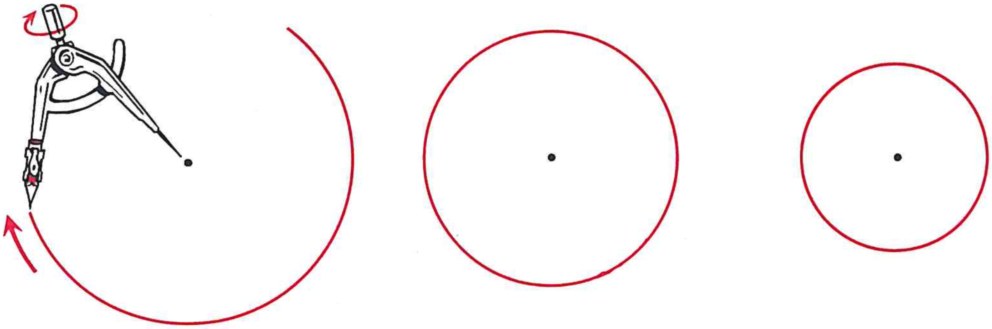


1. [A] On your paper, mark a point and label it *A*. Now find a point that is 2 inches from point *A*. Label it *B*. Label another point *C* that is 2 inches from *A*.



[B] Continue marking points that are 2 inches from *A* until you have enough points so you can easily sketch the circle suggested by these points.

2. You can mark center points and use a compass to draw circles. Use your compass to draw 3 circles of different sizes.

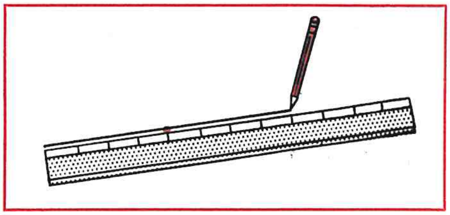


3. Mark a point on each circle. Label these points *A*, *B*, *C*, so that *A* is on the largest circle and *C* on the smallest.

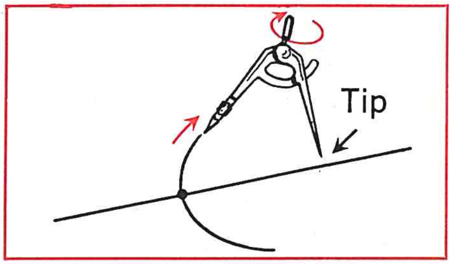
4. It is easy to mark a **point** on a **circle.** Can you draw a **circle** through a **point?** Mark a point *A* on your paper. Draw 6 different circles through this point.

Here are some interesting ways to draw circles through points.

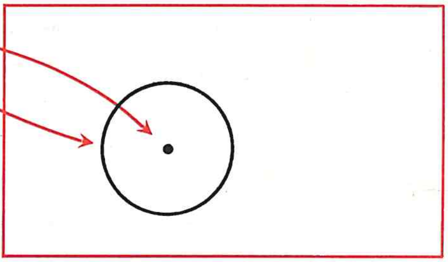
5. [A] Mark a point on your paper. Use your ruler to draw a line that goes through this point.



   [B] Now draw 6 different-sized circles through the point. Do this by putting the tip of your compass at different places **on the line.**



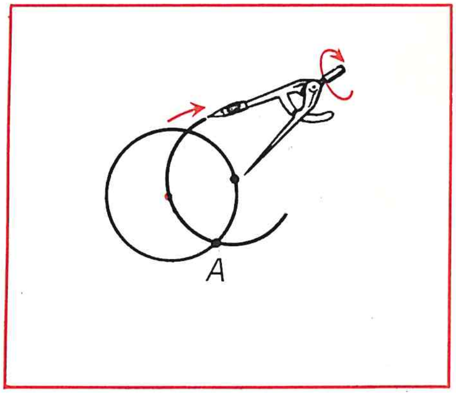
6. [A] Mark a point on your paper. Use your compass to draw a circle with this point as the center.



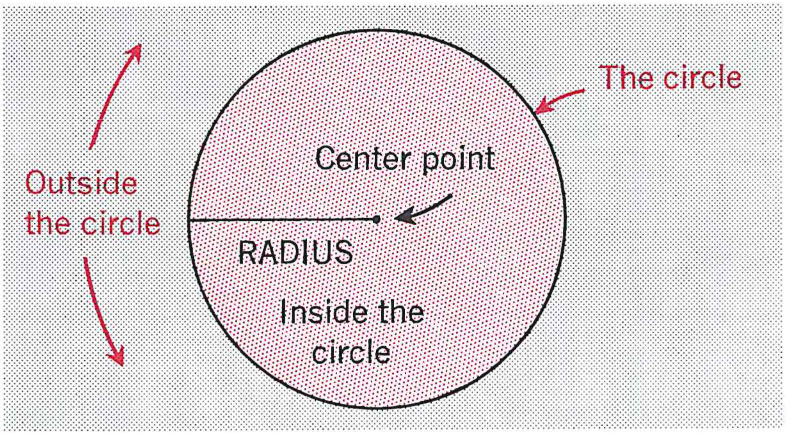
   [B] Now draw 6 circles through this point. Do this by putting the tip of your compass at different places **on the circle.**

   [C] Are all the circles the same size?

   [D] Do this again. Each time, put the tip of your compass **where the circles meet** (for example, at point *A* in the picture).

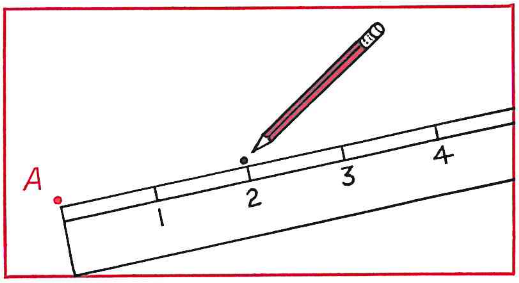


Circles and pairs of points

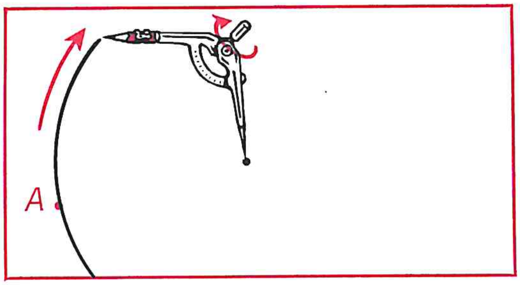


This circle has a 1-inch radius.

1. [A] Mark point *A* on your paper. Then mark a second point 2 inches from *A*.



[B] Now use your compass to draw a circle through point *A*. Your circle should have a **2-inch radius.**



2. Mark points *B, C,* and *D* on your paper.

[A] Draw a circle with a 3-inch radius that goes through point *B*.

[B] Draw a circle with a $1\frac{1}{2}$-inch radius that goes through point *C*.

[C] Draw a circle with a 6-centimeter radius that goes through point *D*.

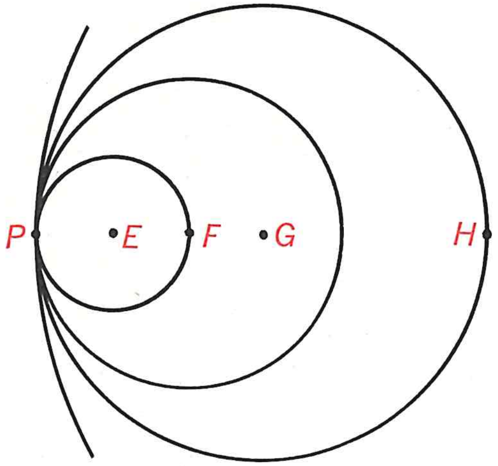
3. Use your centimeter ruler to measure

[**A**] the radius of the circle with center *E*,

[**B**] the radius of the circle with center *F*,

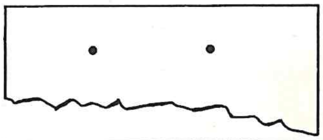
[**C**] the radius of the circle with center *G*,

[**D**] the radius of a circle that has center *H* and goes through point *P*.

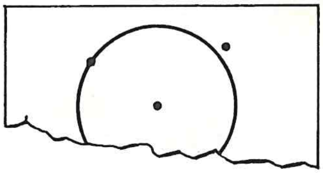


It is easy to draw a circle through **one** point. Can you draw a circle that goes through **two** points?

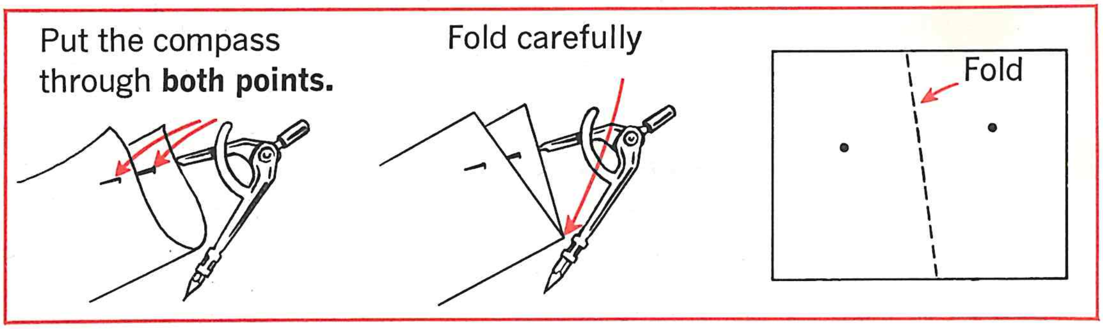
4. Draw 2 points on your paper. Now try to draw some circles that go through both points.



If your drawing looks like this, you are close but not close enough.



Here is a way to draw a circle through 2 points without guessing.



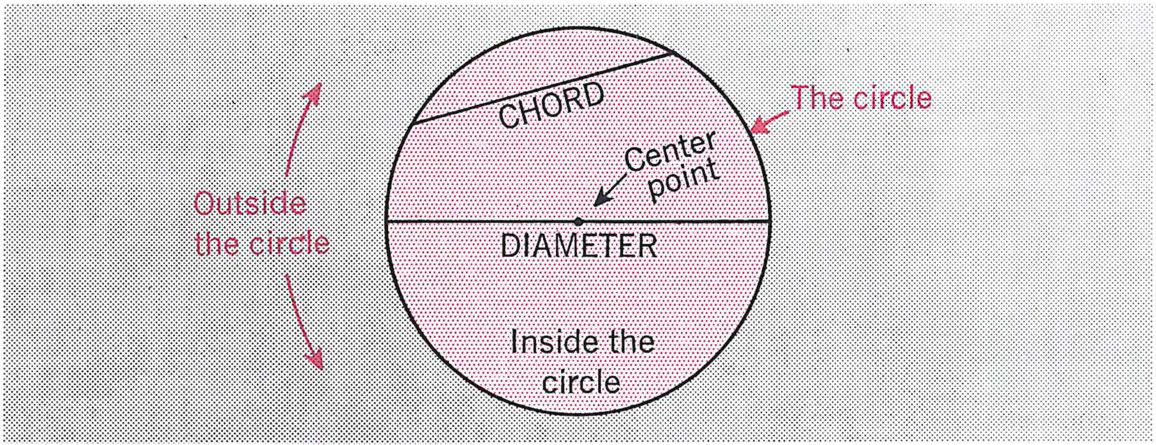
Put the compass through **both points.**

Fold carefully

5. Mark 2 points on your paper. Draw a circle through the points as shown above. Put the tip of your compass at different places on the fold, and draw 5 different-sized circles through the 2 points.

★ 6. Draw the circle that goes through the 2 points of exercise 5 and has the smallest possible radius. Where did you put the tip of your compass?

3 points and a circle

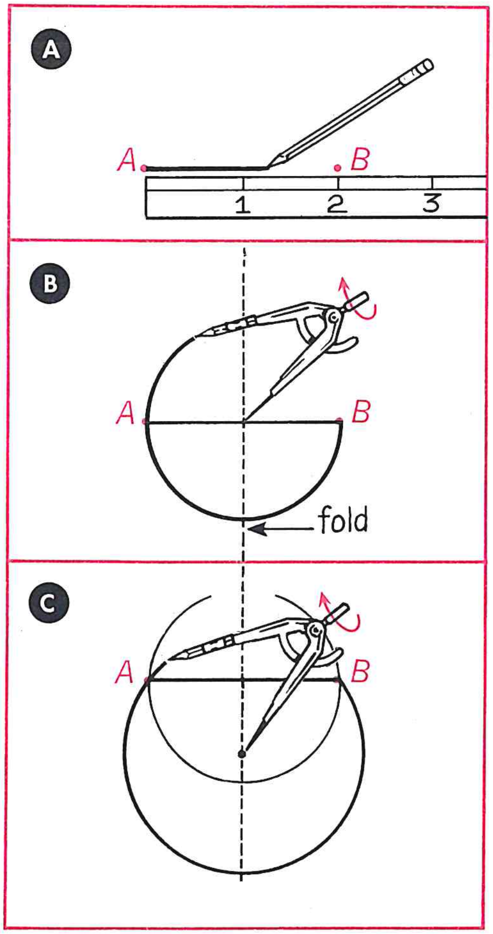


Here is one way to draw a circle with a 2-inch diameter.

1. [A] Mark points *A* and *B* 2 inches apart. Then draw the segment from *A* to *B*.

   [B] Now use your compass as shown in the picture to draw a circle through points *A* and *B*. Your circle should have a 2-inch **diameter.**

   [C] Put the tip of your compass at a different point on the fold. Now you can draw a circle that has a 2-inch **chord** through points *A* and *B*.



2. Draw a circle with a 3-inch diameter.

3. Draw a circle with a 3-inch chord.

4. Draw a circle with a 10-centimeter diameter.

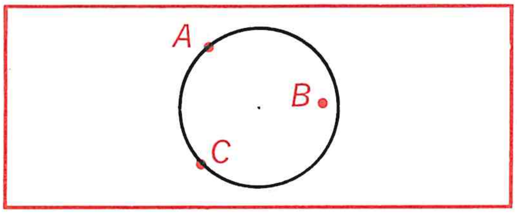
5. Draw a circle with a 10-centimeter chord.

★6. Find a different way to draw a circle with a 2-inch diameter.

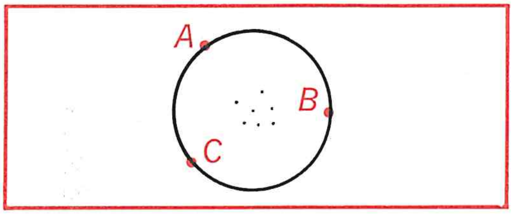
You know how to draw a circle through 2 points. Can you draw a circle that goes through 3 points?

7. Mark 3 points (not lined up) on your paper. Try to draw a circle that goes through all 3 points.

Does your paper look like either of these?

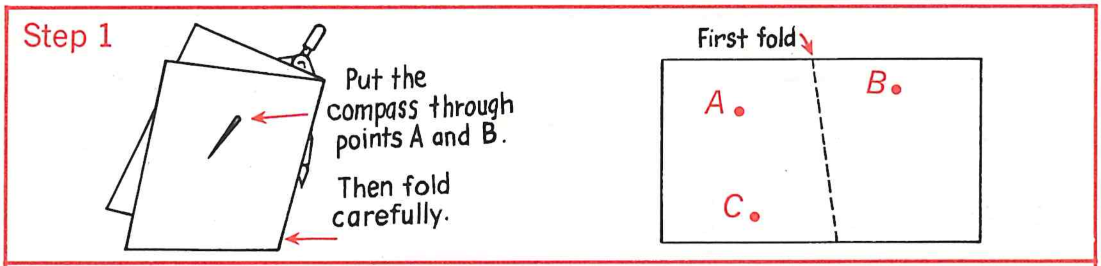


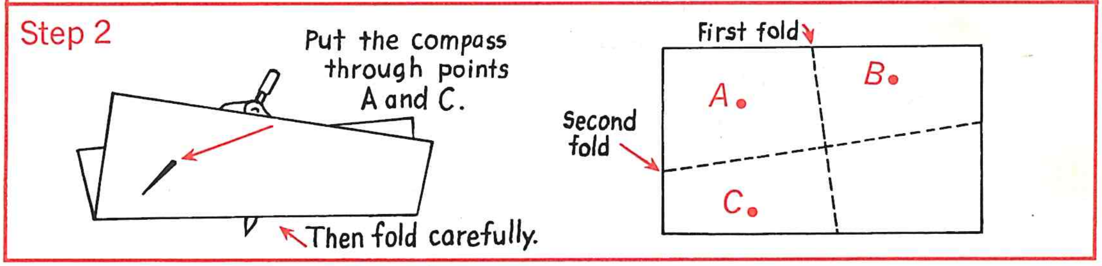
This is not close enough.



This took a lot of guessing.

Here is a way to draw a circle through 3 points **without guessing.**





When you place the tip of your compass on the first fold, you can draw a circle through points *A* and *B*. When you place your compass on the second fold, you can draw a circle through points *A* and *C*. Where could you place the tip of your compass to draw a circle through all 3 points?

8. Mark 3 points on your paper. Use the steps shown above to draw a circle through all 3 points.

9. How many different circles do you think can be drawn through all 3 points?

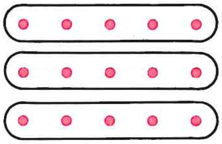
CHAPTER 3 Multiplication and division

Multiplication and sets

Here is a way to think about multiplication.

A

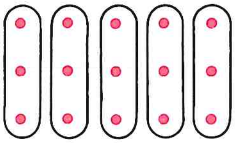
Think:
3 sets of 5



Write: $3 \times 5 = 15$

B

Think:
5 sets of 3



Write: $5 \times 3 = 15$

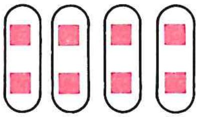
3 and 5 are **factors** of 15.

15 is the **product** of 3 and 5.

EXERCISES

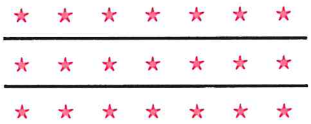
1. Study the sets. Then solve the equation.

[A]



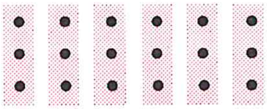
4 sets of 2
$4 \times 2 = \boxed{n}$

[B]



3 sets of 7
$3 \times 7 = \boxed{n}$

[C]



6 sets of 3
$6 \times 3 = \boxed{n}$

2. Draw the sets of dots. Then solve the equation.

[A] 4 sets: 3 dots in each
$4 \times 3 = \boxed{n}$

[B] 5 sets: 2 dots in each
$5 \times 2 = \boxed{n}$

[C] 4 sets: 5 dots in each
$4 \times 5 = \boxed{n}$

[D] 3 sets: 6 dots in each
$3 \times 6 = \boxed{n}$

3. Give the missing numbers.

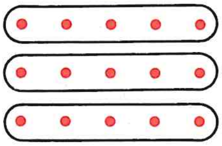
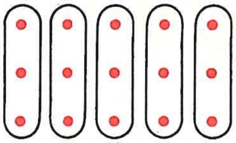
[A] 6 twos are ▨.
$6 \times 2 = \boxed{n}$

[B] 4 fives are ▨.
$4 \times 5 = \boxed{n}$

[C] 3 fours are ▨.
$3 \times 4 = \boxed{n}$

Division and sets

Here is a way to think about division.

A	**B**
Think: There are 3 sets of 5 in a set of 15.  Write: $15 \div 5 = 3$	Think: There are 5 sets of 3 in a set of 15.  Write: $15 \div 3 = 5$
5 is the divisor. 3 is the quotient.	3 is the divisor. 5 is the quotient.

EXERCISES

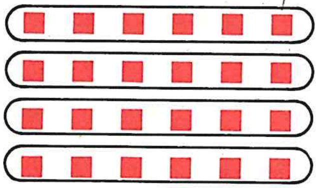
1. Draw a set of 18 dots on your paper. Ring as many sets of 6 as you can.
   [A] How many did you find? [B] Solve the equation, $18 \div 6 = \boxed{n}$

2. Draw a set of 24 dots on your paper. Ring as many sets of 4 as you can. Solve the equation, $24 \div 4 = \boxed{n}$.

3. Draw a set of 32 dots on your paper. Ring as many sets of 8 as you can. Solve the equation, $32 \div 8 = \boxed{n}$.

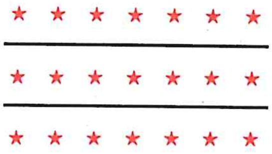
4. Study the sets. Then solve the equation.

[A] 4



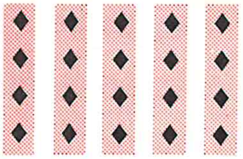
$24 \div 6 = \boxed{n}$

[B] 3



$21 \div 7 = \boxed{n}$

[C] 5

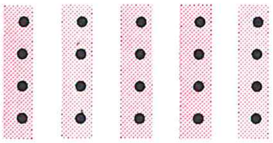


$20 \div 4 = \boxed{n}$

5. Give the missing numbers.
   [A] There are ▥ sixes in 36.
   $36 \div 6 = \boxed{n}$
   [B] There are ▥ sevens in 28.
   $28 \div 7 = \boxed{n}$

Multiplication and addition

You can use addition to find products.



Since $4+4+4+4+4=20$, we know that $5\times4=20$.

EXERCISES

1. Solve the equations.

[A] $5+5+5=n$
$3\times5=n$

[B] $2+2+2+2=n$
$4\times2=n$

[C] $6+6+6+6=n$
$4\times6=n$

[D] $7+7+7+7=n$
$4\times7=n$

[E] $8+8+8+8+8=n$
$5\times8=n$

[F] $1+1+1+1+1=n$
$5\times1=n$

[G] $4+4+4+4+4+4=n$
$6\times4=n$

[H] $9+9+9=n$
$3\times9=n$

[I] $7+7+7=n$
$3\times7=n$

[J] $0+0+0+0+0=n$
$5\times0=n$

[K] $8+8+8+8=n$
$4\times8=n$

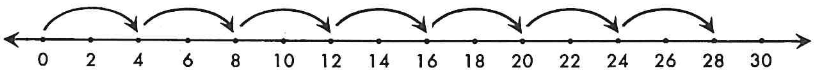
[L] $9+9+9+9=n$
$4\times9=n$

[M] $6+6+6+6+6+6+6+6=n$
$8\times6=n$

[N] $9+9+9+9+9+9+9=n$
$7\times9=n$

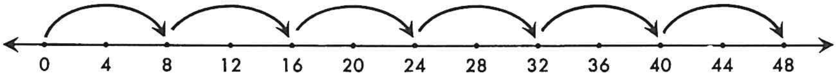
2. Use the number lines to help you solve the equations.

[A]



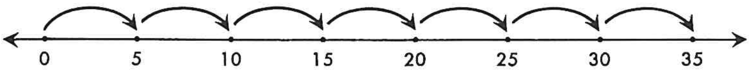
$7\times4=n$

[B]



$6\times8=n$

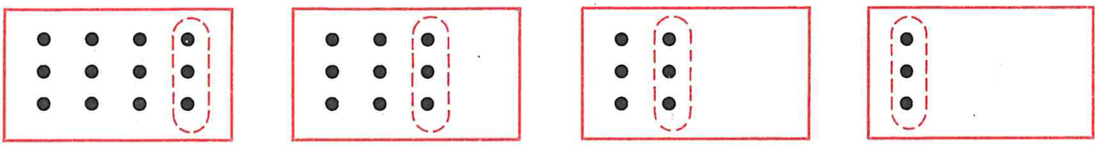
[C]



$7\times5=n$

Division and subtraction

You can use subtraction to find quotients.



$$12-3=9 \longrightarrow 9-3=6 \longrightarrow 6-3=3 \longrightarrow 3-3=0$$

There are 4 threes in 12.

$12 \div 3 = 4$

EXERCISES

1. Copy the equations. Write the missing numbers instead of the $\boxed{n}$.

[A]
$24-8=\boxed{n}$
$16-8=\boxed{n}$
$8-8=\boxed{n}$
$24 \div 8=\boxed{n}$

[B]
$48-8=\boxed{n}$
$40-8=\boxed{n}$
$32-8=\boxed{n}$
$24-8=\boxed{n}$
$16-8=\boxed{n}$
$8-8=\boxed{n}$
$48 \div 8=\boxed{n}$

[C]
$45-9=\boxed{n}$
$36-9=\boxed{n}$
$27-9=\boxed{n}$
$18-9=\boxed{n}$
$9-9=\boxed{n}$
$45 \div 9=\boxed{n}$

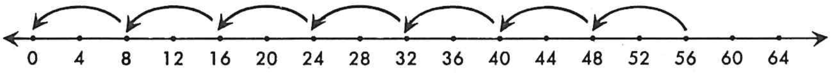
[D]
$18-3=\boxed{n}$
$15-3=\boxed{n}$
$12-3=\boxed{n}$
$9-3=\boxed{n}$
$6-3=\boxed{n}$
$3-3=\boxed{n}$
$18 \div 3=\boxed{n}$

[E]
$28-7=\boxed{n}$
$21-7=\boxed{n}$
$14-7=\boxed{n}$
$7-7=\boxed{n}$
$28 \div 7=\boxed{n}$

[F]
$42-6=\boxed{n}$
$36-6=\boxed{n}$
$30-6=\boxed{n}$
$24-6=\boxed{n}$
$18-6=\boxed{n}$
$12-6=\boxed{n}$
$6-6=\boxed{n}$
$42 \div 6=\boxed{n}$

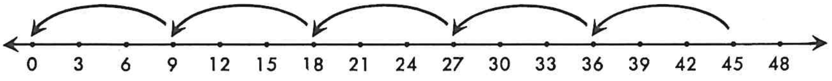
2. Use the number lines to help you solve the equations.

[A]



$56 \div 8=\boxed{n}$

[B]



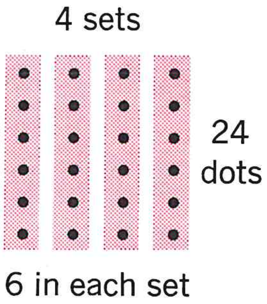
$45 \div 9=\boxed{n}$

Multiplication and division are related

Multiplication

How many are
4 sets of 6?

$4 \times 6 = 24$

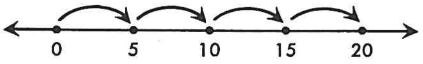


Division

How many sets
of 6 are in 24?

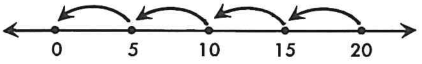
$24 \div 6 = 4$

Multiplication



$4 \times 5 = 20$

Division

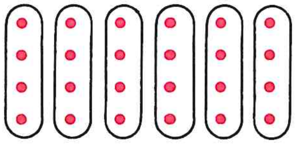


$20 \div 5 = 4$

EXERCISES

1. Study the sets. Then solve the equations.

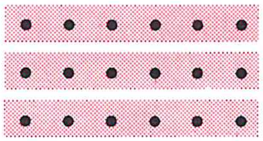
[A]



$6 \times 4 = \boxed{n}$

$24 \div 4 = \boxed{n}$

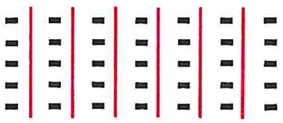
[B]



$3 \times 6 = \boxed{n}$

$18 \div 6 = \boxed{n}$

[C]

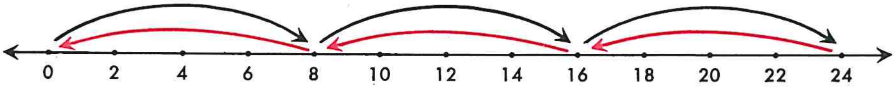


$7 \times 5 = \boxed{n}$

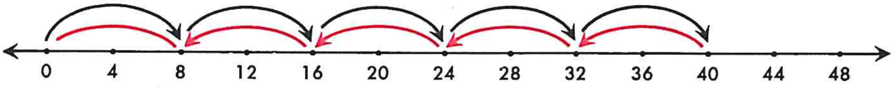
$35 \div 5 = \boxed{n}$

2. Think about the black arrows and write a multiplication equation.
Think about the red arrows and write a division equation.

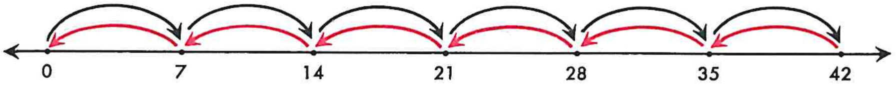
[A]



[B]



[C]

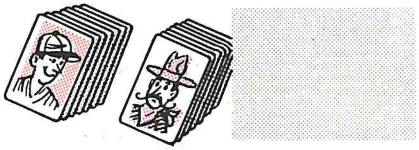


Short stories

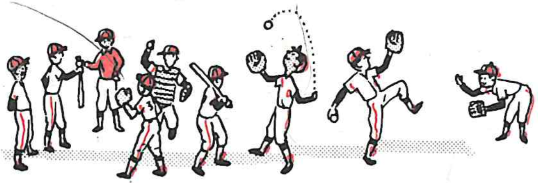
Write a multiplication or division equation for each exercise. Then find the solution.

1. 4 boxes. 8 crayons in each box. How many crayons?

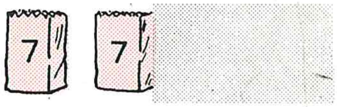
2. 24 marbles. 3 in each bag. How many bags?



3. 3 sets. 7 cards in each set. How many cards?

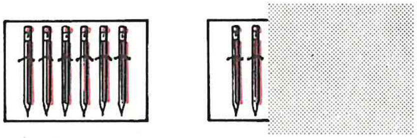


4. 36 boys. 9 on each team. How many teams?



5. 5 bags of candy. 7 pieces in each bag. How many pieces?

6. 30 band members. 5 in each row. How many rows?



7. 6 cards. 6 pencils on each card. How many pencils?

8. 32 children. Divided into groups of 4. How many groups?

9. 7 boxes. 4 tennis balls in each box. How many tennis balls?

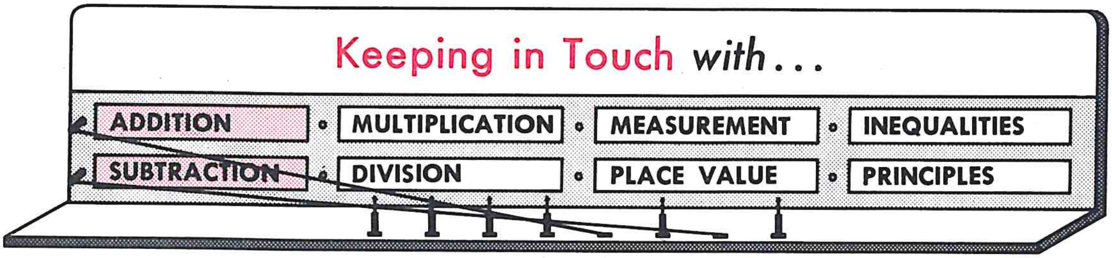
Think

$587 \times 7285 = 4{,}276{,}295$

Use this equation and addition or subtraction to find these products.

$588 \times 7285 = \boxed{n}$

$586 \times 7285 = \boxed{n}$



Keeping in Touch *with*...

1. Find the sums and differences.

[A] $\begin{array}{r}32\\+57\\\hline\end{array}$ [B] $\begin{array}{r}87\\-24\\\hline\end{array}$ [C] $\begin{array}{r}54\\+29\\\hline\end{array}$ [D] $\begin{array}{r}53\\-26\\\hline\end{array}$ [E] $\begin{array}{r}134\\-67\\\hline\end{array}$ [F] $\begin{array}{r}87\\+69\\\hline\end{array}$

2. Find the sums.

[A] $\begin{array}{r}7\\6\\+8\\\hline\end{array}$ [B] $\begin{array}{r}9\\7\\+3\\\hline\end{array}$ [C] $\begin{array}{r}29\\37\\+53\\\hline\end{array}$ [D] $\begin{array}{r}54\\67\\+36\\\hline\end{array}$ [E] $\begin{array}{r}78\\37\\+85\\\hline\end{array}$ [F] $\begin{array}{r}89\\97\\+68\\\hline\end{array}$

[G] $\begin{array}{r}347\\+652\\\hline\end{array}$ [H] $\begin{array}{r}978\\+867\\\hline\end{array}$ [I] $\begin{array}{r}763\\+898\\\hline\end{array}$

[J] $\begin{array}{r}386\\294\\+651\\\hline\end{array}$ [K] $\begin{array}{r}418\\690\\+789\\\hline\end{array}$ [L] $\begin{array}{r}916\\807\\+398\\\hline\end{array}$

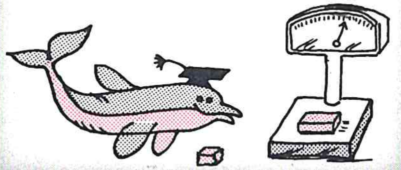
3. Find the differences.

[A] $\begin{array}{r}592\\-347\\\hline\end{array}$ [B] $\begin{array}{r}823\\-458\\\hline\end{array}$ [C] $\begin{array}{r}937\\-386\\\hline\end{array}$

[D] $\begin{array}{r}5832\\-2567\\\hline\end{array}$ [E] $\begin{array}{r}506\\-219\\\hline\end{array}$ [F] $\begin{array}{r}8007\\-4234\\\hline\end{array}$

Think

Suppose a brick weighs 8 pounds more than half a brick. How much does the brick weigh? =16



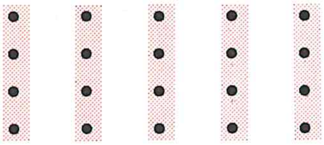
4. Find the sums and differences.

[A] $5+3+7+6=\boxed{n}$
[B] $385-234=\boxed{n}$
[C] $51-48=\boxed{n}$
[D] $53+47=\boxed{n}$
[E] $32+33+34=\boxed{n}$
[F] $403-395=\boxed{n}$
[G] $613+56+9=\boxed{n}$
[H] $485+386+67=\boxed{n}$
[I] $5678+934+7=\boxed{n}$
[J] $807-467=\boxed{n}$

The order principle for multiplication

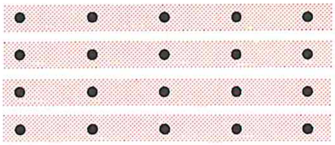
Here are two ways to think about a set of 20 dots.

In columns



We see 5 sets of 4.
We write: $5 \times 4 = 20$

In rows



We see 4 sets of 5.
We write: $4 \times 5 = 20$

The factors in these examples are 4 and 5. The product is 20.

For any factors, we find that:

When we change the **ORDER** of the **FACTORS,** we get the same **PRODUCT.**

EXERCISES

1. Find the missing numbers.

[A] Since $4 \times 3 = 12$, we know that $3 \times 4 = n$.
[B] Since $3 \times 6 = 18$, we know that $6 \times 3 = n$.
[C] Since $5 \times 3 = 15$, we know that $3 \times 5 = n$.
[D] Since $8 \times 7 = 56$, we know that $7 \times 8 = n$.
[E] Since $9 \times 6 = 54$, we know that $6 \times 9 = n$.

2. Find the products. Use the table if you need help.

[A] 14×8
[B] 56×38
[C] 49×36
[D] 16×27
[E] 15×19
[F] 28×37
[G] 39×28
[H] 57×49

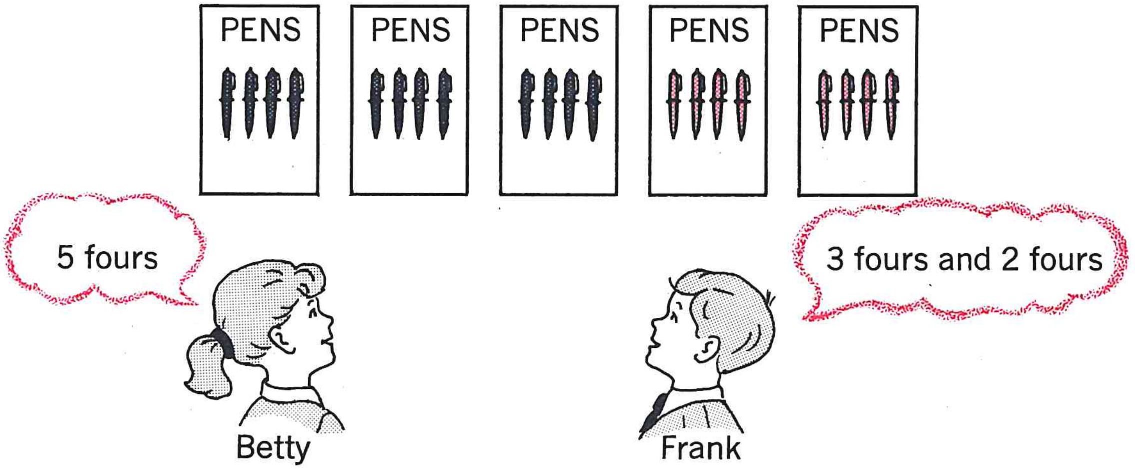
$38 \times 56 = 2128$
$49 \times 57 = 2793$
$37 \times 28 = 1036$
$36 \times 49 = 1764$
$27 \times 16 = 432$
$28 \times 39 = 1092$
$19 \times 15 = 285$
$8 \times 14 = 112$

3. Solve the equations.

[A] $8 \times 3 = n \times 8$
[B] $n \times 5 = 5 \times 8$
[C] $1 \times 0 = 0 \times n$
[D] $59 \times n = 863 \times 59$

The multiplication-addition principle

Betty and Frank are thinking about the sets of pens in different ways.



DISCUSSION EXERCISES

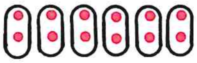
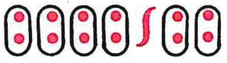
1. Explain how Betty is thinking about the pens.
2. Explain how Frank is thinking about the pens.
3. Solve: [A] $5 \times 4 = \boxed{n}$ [B] $(3 \times 4) + (2 \times 4) = \boxed{n}$
4. Is it true that $5 \times 4 = (3 \times 4) + (2 \times 4)$?
5. Which factor is being "broken apart," the 5 or the 4?

You might think about the multiplication-addition principle like this:

When we multiply, we can "break apart" a factor.

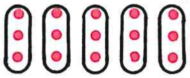
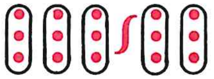
EXERCISES

1. Solve the equations.

[A] We see →  ⟷ 

We think → 6 twos ⟷ 4 twos and 2 twos

We write → $6 \times 2 = \boxed{n}$ ⟷ $(4 \times 2) + (2 \times 2) = \boxed{n}$

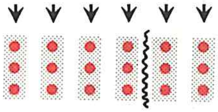
[B] We see →  ⟷ 

We think → 5 threes ⟷ 3 threes and 2 threes

We write → $5 \times 3 = \boxed{n}$ ⟷ $(3 \times 3) + (2 \times 3) = \boxed{n}$

2. Find the missing number of threes.

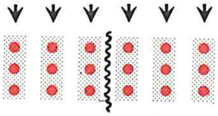
[A] 6 sets of 3



For 6 sets of 3, we can think

4 threes and ▥ threes.

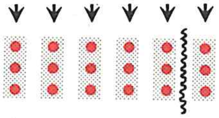
[B] 6 sets of 3



For sets of 3, we can think

3 threes and ▥ threes.

[C] 6 sets of 3



For 6 sets of 3, we can think

5 threes and ▥ three.

3. Find the missing number. Then solve the equation.

[A] 7 twos → 4 twos and ▥ twos

$7 \times 2 = (4 \times 2) + (\boxed{n} \times 2)$

[B] 7 twos → 5 twos and ▥ twos

$7 \times 2 = (5 \times 2) + (\boxed{n} \times 2)$

[C] 7 twos → 6 twos and ▥ two

$7 \times 2 = (6 \times 2) + (\boxed{n} \times 2)$

[D] 5 sixes → 3 sixes and ▥ sixes

$5 \times 6 = (3 \times 6) + (\boxed{n} \times 6)$

[E] 5 sixes → 4 sixes and ▥ six

$5 \times 6 = (4 \times 6) + (\boxed{n} \times 6)$

[F] 6×4 → 3 fours and ▥ fours

$6 \times 4 = (3 \times 4) + (\boxed{n} \times 4)$

4. Solve the equations.

[A] $5 \times 6 = (4 \times 6) + (\boxed{n} \times 6)$

[B] $7 \times 3 = (4 \times 3) + (\boxed{n} \times 3)$

[C] $4 \times 5 = (2 \times 5) + (\boxed{n} \times 5)$

[D] $6 \times 7 = (\boxed{n} \times 7) + (2 \times 7)$

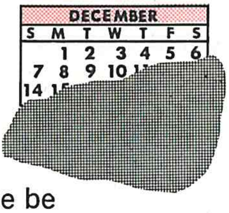
[E] $8 \times 5 = (\boxed{n} \times 5) + (2 \times 5)$

[F] $9 \times 3 = (\boxed{n} \times 3) + (2 \times 3)$

[G] $7 \times 6 = (3 \times 6) + (\boxed{n} \times 6)$

[H] $5 \times 9 = (\boxed{n} \times 9) + (1 \times 9)$

Think



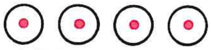
1. If today is Wednesday, the 10th of December, what will the date be in a week? In 2 weeks? In 3 weeks?

2. What day of the week is January 1 of the next year? December has 31 days.

0 and 1 in multiplication

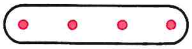
One

4 sets. 1 dot in each.



$4 \times 1 = 4$

1 set of 4 dots.



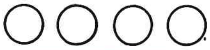
$1 \times 4 = 4$

4 is a factor. 4 is the product.
When 1 is a factor, the product is the other factor.

Choose a number. Multiply by 1.
The product is the number you chose.

Zero

4 sets. 0 dots in each.



$4 \times 0 = 0$

0 sets of 4 dots.

$0 \times 4 = 0$

0 is a factor. 0 is the product.
When 0 is a factor, the product is 0.

Choose a number. Multiply by 0. The product is 0.

EXERCISES

1. Find the missing numbers.

[A] Since $9 \times 1 = 9$, we know that $1 \times 9 = \boxed{n}$.
[B] Since $5 \times 0 = 0$, we know that $0 \times 5 = \boxed{n}$.
[C] Since $1 \times 7 = 7$, we know that $7 \times 1 = \boxed{n}$.
[D] Since $0 \times 8 = 0$, we know that $8 \times 0 = \boxed{n}$.

2. Find the equation that has no solution. Find the equation that has many solutions. Solve the other equations.

[A] $8 \times 0 = \boxed{n}$
[B] $6 \times 1 = \boxed{n}$
[C] $1 \times \boxed{n} = 5$

[D] $\boxed{n} \times 7 = 0$
[E] $\boxed{n} \times 0 = 24$
[F] $9 \times \boxed{n} = 9$

[G] $\boxed{n} \times 1 = 78$
[H] $0 \times \boxed{n} = 0$
[I] $586 \times 0 = \boxed{n}$

Reviewing multiplication facts

You can review the 100 multiplication facts by completing a multiplication table like the one shown here.

"0" facts Since the product is always 0 when 0 is a factor, you can find the products in the **0 row** and **0 column** easily.

"1" facts Since the product of 1 and a number is that number, you can find the products in the **1 row** and **1 column.**

"2" facts You can use addition facts to find the products in the **2 row.** For example, since $7+7=14$, you know that $2\times7=14$.

You can use the order principle to find the products in the **2 column.** For example, since $2\times7=14$, you know that $7\times2=14$.

Multiplication Table

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2								
3	0	3								
4	0	4								
5	0	5								
6	0	6								
7	0	7								
8	0	8								
9	0	9								

×	0	1	2	3	4	5	6	7	8	9
0										
1										
2			[A]	[B]	[C]	[D]	[E]	14	[F]	[G]
3			[H]							
4			[I]							
5			[J]							
6			[K]							
7			14							
8			[L]							
9			[M]							

EXERCISES

1. Write the multiplication facts (twos) for entries **A** through **M** in the table.
   Example: [A] $2\times2=4$
2. How many of the 100 facts have you reviewed so far?
3. Make a multiplication table and write in your table the products you have reviewed in this lesson.
   Save your table for a later lesson.

Threes, fours, and fives

As you work these exercises, continue to fill in the table you started in the last lesson.

1. "3" facts Solve the equations to find the products in the **3 row.**

[A] 3×3: $(2 \times 3) + 3 = n$
[B] 3×4: $(2 \times 4) + 4 = n$
[C] 3×5: $(2 \times 5) + 5 = n$
[D] 3×6: $(2 \times 6) + 6 = n$
[E] 3×7: $(2 \times 7) + 7 = n$
[F] 3×8: $(2 \times 8) + 8 = n$
[G] 3×9: $(2 \times 9) + 9 = n$

Use the order principle to find the products in the **3 column** (**H** through **M** in the table).

x	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3				[A]	[B]	[C]	[D]	[E]	[F]	[G]
4				[H]	[N]	[O]	[P]	[Q]	[R]	[S]
5				[I]	[T]					
6				[J]	[U]					
7				[K]	[V]					
8				[L]	[W]					
9				[M]	[X]					

"4" facts Solve the equations to find the products in the **4 row.**

[N] 4×4: $(2 \times 4) + (2 \times 4) = n$
[O] 4×5: $(2 \times 5) + (2 \times 5) = n$
[P] 4×6: $(2 \times 6) + (2 \times 6) = n$
[Q] 4×7: $(2 \times 7) + (2 \times 7) = n$
[R] 4×8: $(2 \times 8) + (2 \times 8) = n$
[S] 4×9: $(2 \times 9) + (2 \times 9) = n$

Use the order principle to find products in the **4 column** (**T** through **X**).

2. "5" facts The missing numbers are the products in the **5 row.** Find them.

[A] 5×5: 5, 10, 15, 20, ▨
[B] 5×6: $(4 \times 6) + 6 = n$
[C] 5×7: $(4 \times 7) + 7 = n$
[D] 5×8: $(4 \times 8) + 8 = n$
[E] 5×9: $(4 \times 9) + 9 = n$

x	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5						[A]	[B]	[C]	[D]	[E]
6						[F]				
7						[G]				
8						[H]				
9						[I]				

Use the order principle to find the products in the **5 column.**

Sixes, sevens, eights, and nines

Use the exercises in this lesson to complete your multiplication table.

1. **"6" facts** Solve the equations to find the products in the **6 row.**

[A] 6 × 6: (3 × 6) + (3 × 6) = n
[B] 6 × 7: (3 × 7) + (3 × 7) = n
[C] 6 × 8: (3 × 8) + (3 × 8) = n
[D] 6 × 9: (3 × 9) + (3 × 9) = n

Use the order principle to find the products in the **6 column** (E, F, G in the table).

x	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6							[A]	[B]	[C]	[D]
7							[E]	[H]	[I]	[J]
8							[F]	[K]		
9							[G]	[L]		

"7" facts Solve the equations to find the products in the **7 row.**

[H] 7 × 7: (6 × 7) + 7 = n
[I] 7 × 8: (6 × 8) + 8 = n
[J] 7 × 9: (6 × 9) + 9 = n

Find the products in the **7 column** (K, L in the table).

2. **"8" facts** Solve the equations to find products in the **8 row.**

[A] 8 × 8: (4 × 8) + (4 × 8) = n
[B] 8 × 9: (4 × 9) + (4 × 9) = n

Use the order principle to find 9 × 8 (**C** in the table).

"9" facts Solve either of the equations to find 9 × 9.

[D] 9 × 9: (8 × 9) + 9 = n
9 × 9: (10 × 9) − 9 = n

x	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6										
7										
8									[A]	[B]
9									[C]	[D]

Multiplication exercises

1. Find the products.

[A] 7×0
[B] 7×1
[C] 7×2
[D] 7×3
[E] 7×5
[F] 7×6
[G] 7×7
[H] 7×8
[I] 7×9
[J] 8×3
[K] 8×4
[L] 8×5
[M] 8×6
[N] 8×7
[O] 8×8
[P] 8×9
[Q] 9×2
[R] 9×3
[S] 9×4
[T] 9×5
[U] 9×6
[V] 9×7
[W] 9×8
[X] 9×9

2. Find the products.

[A] $\begin{array}{r}5\\ \times 6\\ \hline\end{array}$
[B] $\begin{array}{r}0\\ \times 7\\ \hline\end{array}$
[C] $\begin{array}{r}4\\ \times 4\\ \hline\end{array}$
[D] $\begin{array}{r}4\\ \times 9\\ \hline\end{array}$
[E] $\begin{array}{r}6\\ \times 6\\ \hline\end{array}$
[F] $\begin{array}{r}8\\ \times 6\\ \hline\end{array}$
[G] $\begin{array}{r}9\\ \times 7\\ \hline\end{array}$

[H] $\begin{array}{r}3\\ \times 9\\ \hline\end{array}$
[I] $\begin{array}{r}6\\ \times 8\\ \hline\end{array}$
[J] $\begin{array}{r}7\\ \times 9\\ \hline\end{array}$
[K] $\begin{array}{r}10\\ \times 9\\ \hline\end{array}$
[L] $\begin{array}{r}6\\ \times 9\\ \hline\end{array}$
[M] $\begin{array}{r}6\\ \times 7\\ \hline\end{array}$
[N] $\begin{array}{r}6\\ \times 8\\ \hline\end{array}$

3. Find the products.

[A] 6×0
[B] 9×1
[C] 6×6
[D] 6×8
[E] 5×9
[F] 7×8
[G] 6×7
[H] 5×5
[I] 5×8
[J] 0×9
[K] 0×0
[L] 8×9

★**4.** Write two 1-digit numbers (other than 1) whose product is one less than the product of the numbers given.

[A] 3, 3
[B] 6, 6
[C] 8, 8
[D] 4, 4
[E] 7, 7
[F] 5, 5

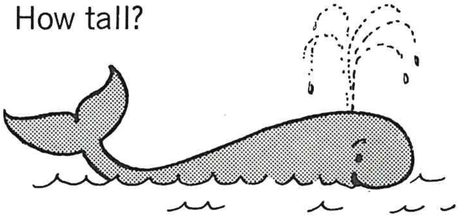
★**5.** Give the pair of numbers that could serve as both addends and factors in the gray spaces.

	Sums		+ ×		Products
[A]	5	▨	+ ×	▨	6
[B]	7	▨	+ ×	▨	12
[C]	10	▨	+ ×	▨	24
[D]	11	▨	+ ×	▨	24
[E]	6	▨	+ ×	▨	9
[F]	4	▨	+ ×	▨	4
[G]	2	▨	+ ×	▨	1

	Sums		+ ×		Products
[H]	14	▨	+ ×	▨	45
[I]	14	▨	+ ×	▨	49
[J]	15	▨	+ ×	▨	56
[K]	16	▨	+ ×	▨	64
[L]	17	▨	+ ×	▨	72
[M]	15	▨	+ ×	▨	54
[N]	13	▨	+ ×	▨	42

Animal short stories

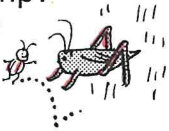
1. Tallest man on record: about 9 feet tall. Tallest giraffe: about two times as tall. How tall?



2. Pike fish: 3 feet long. Baby whale: 8 times as long. How long?

3. Rowboat: 9 feet long. Longest crocodile: almost 4 times as long. About how long?

4. Longest flea jump: 1 foot. Longest grasshopper jump: 8 times as far. How far?

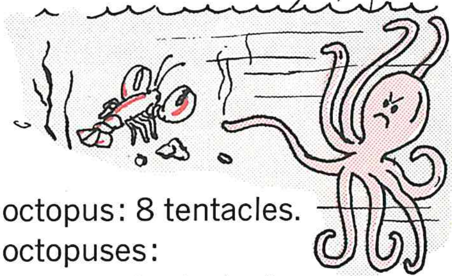


5. Ostrich egg weighs 3 pounds. How much do 9 ostrich eggs weigh?

6. Diamondback rattlesnake: 6 feet long. Longest python snake: over 5 times as long. About how long?

7. Fastest measured speed for a snake: 7 mph. Fastest measured speed for an insect: 5 times as fast. How fast?

8. Porpoise: 7 feet long. Whale: 9 times as long. How long?



9. 1 octopus: 8 tentacles. 7 octopuses: How many tentacles?

10. A fairly good frog jump: 6 feet. Long kangaroo jump. 7 times as far. How far?

11. Tall ostrich: 8 feet tall. Small jungle tree: 6 times as tall. How tall?

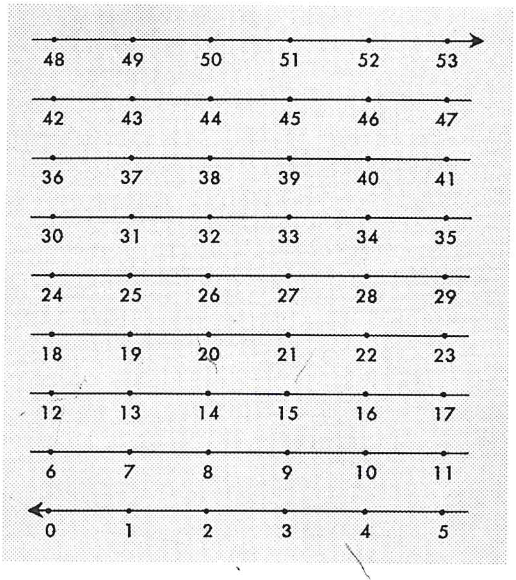
12. Slow bird: flies 9 miles per hour. Fastest duck: 8 times as fast. How fast?

A number-line game

Here are some new number-line exercises. This number line looks different from those you have used before. It is stacked up in pieces because it would be too long to fit on the page.

When a number line is stacked in a certain way, we need to agree on how to show the jumps. Instead of showing jumps on the number-line picture, we will use new symbols to describe the jumps.

Study the table to decide how these symbols tell the jumps for the number-line picture above.



Symbol	0 $\overset{\uparrow}{\textcircled{1}}$	16 $\underset{\downarrow}{\textcircled{2}}$	32 $\leftarrow\textcircled{2}$	50 $\textcircled{3}\rightarrow$	5 $\textcircled{1}\rightarrow$	48 $\leftarrow\textcircled{2}$	2 $\overset{\uparrow}{\textcircled{1}}$ $\textcircled{2}\rightarrow$	28 $\leftarrow\textcircled{3}$ $\underset{\downarrow}{\textcircled{4}}$
Start	0	16	32	50	5	48	2	28
Land	6	4	30	53	6	46	10	1

EXERCISES

1. Give the landing point for each jump.

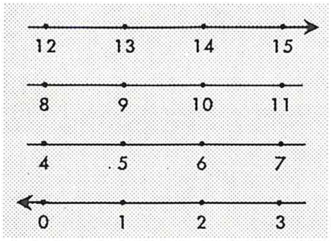
[A] 2 $\overset{\uparrow}{\textcircled{1}}$
[B] 18 $\underset{\downarrow}{\textcircled{1}}$
[C] 26 $\textcircled{1}\rightarrow$
[D] 13 $\leftarrow\textcircled{1}$
[E] 4 $\overset{\uparrow}{\textcircled{1}}$ $\underset{\downarrow}{\textcircled{1}}$
[F] 3 $\overset{\uparrow}{\textcircled{2}}$
[G] 0 $\textcircled{4}\rightarrow$
[H] 24 $\underset{\downarrow}{\textcircled{4}}$
[I] 9 $\overset{\uparrow}{\textcircled{3}}$ $\underset{\downarrow}{\textcircled{4}}$
[J] 9 $\leftarrow\textcircled{1}$ $\textcircled{1}\rightarrow$
[K] 44 $\textcircled{3}\rightarrow$ $\underset{\downarrow}{\textcircled{3}}$
[L] 15 $\underset{\downarrow}{\textcircled{1}}$ $\textcircled{1}\rightarrow$
[M] 5 $\textcircled{1}\rightarrow$
[N] 23 $\textcircled{2}\rightarrow$
[O] 18 $\leftarrow\textcircled{1}$
[P] 20 $\underset{\downarrow}{\textcircled{1}}$ $\textcircled{1}\rightarrow$ $\overset{\uparrow}{\textcircled{1}}$ $\leftarrow\textcircled{1}$
[Q] 43 $\underset{\downarrow}{\textcircled{7}}$
[R] 5 $\overset{\uparrow}{\textcircled{7}}$
[S] 0 $\overset{\uparrow}{\textcircled{6}}$ $\textcircled{1}\rightarrow$
[T] 37 $\textcircled{3}\rightarrow$ $\underset{\downarrow}{\textcircled{3}}$ $\leftarrow\textcircled{3}$ $\overset{\uparrow}{\textcircled{3}}$

★ **2.** Give the missing numbers.

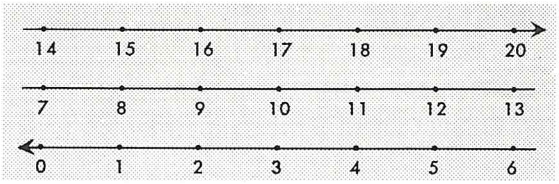
[A] ▥ $\overset{\uparrow}{\textcircled{1}}$: lands at 11
[B] ▥ $\underset{\downarrow}{\textcircled{2}}$: lands at 10
[C] ▥ $\overset{\uparrow}{\textcircled{1}}$ $\underset{\downarrow}{\textcircled{1}}$: lands at 35
[D] ▥ $\overset{\uparrow}{\textcircled{1}}$ $\textcircled{3}\rightarrow$: lands at 9
[E] 2 $\overset{\uparrow}{◍}$: lands at 20
[F] 35 ◍$\rightarrow$: lands at 37

On this page use the number-line pictures A, B, and C to work the exercises. For each picture, **imagine** as many additional rows as you need to answer the questions.

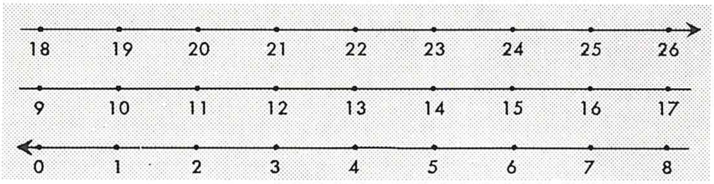
A



B



C



3. For picture A, give the landing point for each jump.

[A] 0 $\overset{\uparrow}{\textcircled{1}}$
[B] 0 $\overset{\uparrow}{\textcircled{2}}$
[C] 0 $\overset{\uparrow}{\textcircled{3}}$
[D] 0 $\overset{\uparrow}{\textcircled{4}}$
[E] 0 $\overset{\uparrow}{\textcircled{7}}$
[F] 1 $\overset{\uparrow}{\textcircled{2}}$
[G] 1 $\overset{\uparrow}{\textcircled{5}}$
[H] 1 $\overset{\uparrow}{\textcircled{9}}$
[I] 1 $\overset{\uparrow}{\textcircled{8}}$
[J] 3 $\overset{\uparrow}{\textcircled{7}}$
[K] 8 $\underset{\downarrow}{\textcircled{1}}$
[L] 17 $\underset{\downarrow}{\textcircled{2}}$
[M] 24 $\underset{\downarrow}{\textcircled{6}}$
[N] 2 $\leftarrow\textcircled{2}$ $\overset{\uparrow}{\textcircled{5}}$

4. For picture B, give the landing point for each jump.

[A] 0 $\overset{\uparrow}{\textcircled{2}}$
[B] 0 $\overset{\uparrow}{\textcircled{4}}$
[C] 0 $\overset{\uparrow}{\textcircled{9}}$
[D] 0 $\textcircled{6}\rightarrow$
[E] 13 $\leftarrow\textcircled{3}$
[F] 6 $\overset{\uparrow}{\textcircled{1}}$
[G] 10 $\overset{\uparrow}{\textcircled{4}}$
[H] 1 $\overset{\uparrow}{\textcircled{5}}$
[I] 2 $\overset{\uparrow}{\textcircled{7}}$
[J] 3 $\overset{\uparrow}{\textcircled{3}}$
[K] 0 $\overset{\uparrow}{\textcircled{9}}$
[L] 4 $\overset{\uparrow}{\textcircled{9}}$
[M] 6 $\overset{\uparrow}{\textcircled{6}}$
[N] 2 $\leftarrow\textcircled{2}$ $\overset{\uparrow}{\textcircled{8}}$
[O] 9 $\underset{\downarrow}{\textcircled{1}}$
[P] 20 $\underset{\downarrow}{\textcircled{1}}$
[Q] 56 $\underset{\downarrow}{\textcircled{2}}$
[R] 21 $\underset{\downarrow}{\textcircled{3}}$
[S] 460 $\overset{\uparrow}{\textcircled{1}}$
[T] 347 $\underset{\downarrow}{\textcircled{1}}$
[U] 26 $\underset{\downarrow}{\textcircled{2}}$

5. For picture C, give the landing point for each jump.

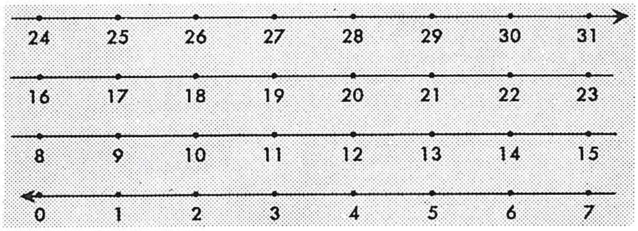
[A] 1 $\overset{\uparrow}{\textcircled{1}}$
[B] 11 $\underset{\downarrow}{\textcircled{1}}$
[C] 2 $\overset{\uparrow}{\textcircled{2}}$
[D] 0 $\textcircled{8}\rightarrow$
[E] 8 $\leftarrow\textcircled{8}$
[F] 0 $\overset{\uparrow}{\textcircled{3}}$
[G] 0 $\overset{\uparrow}{\textcircled{6}}$
[H] 1 $\overset{\uparrow}{\textcircled{6}}$
[I] 5 $\overset{\uparrow}{\textcircled{5}}$
[J] 89 $\overset{\uparrow}{\textcircled{1}}$
[K] 532 $\overset{\uparrow}{\textcircled{2}}$
[L] 17 $\underset{\downarrow}{\textcircled{1}}$
[M] 27 $\underset{\downarrow}{\textcircled{3}}$
[N] 456 $\underset{\downarrow}{\textcircled{1}}$ $\overset{\uparrow}{\textcircled{1}}$
[O] 13 $\overset{\uparrow}{\textcircled{6}}$ $\underset{\downarrow}{\textcircled{4}}$
[P] 4 $\overset{\uparrow}{\textcircled{8}}$ $\textcircled{2}\rightarrow$
[Q] 6 $\overset{\uparrow}{\textcircled{9}}$ $\leftarrow\textcircled{6}$
[R] 36 $\leftarrow\textcircled{1}$ $\underset{\downarrow}{\textcircled{3}}$ $\leftarrow\textcircled{8}$ $\overset{\uparrow}{\textcircled{4}}$

★ **6.** Give the missing numbers. Use number-line picture B.

[A] ▨ $\underset{\downarrow}{\textcircled{1}}$: lands at 3
[B] 7 $\underset{\downarrow}{\textcircled{▨}}$: lands at 0
[C] 12 $\underset{\downarrow}{\textcircled{▨}}$: lands at 5
[D] 56 $\underset{\downarrow}{\textcircled{▨}}$: lands at 0
[E] 50 $\underset{\downarrow}{\textcircled{7}}$: lands at ▨
[F] 46 $\underset{\downarrow}{\textcircled{▨}}$: lands at 4
[G] 63 $\underset{\downarrow}{\textcircled{▨}}$: lands at 0
[H] 65 $\underset{\downarrow}{\textcircled{▨}}$: lands at 2
[I] ▨ $\overset{\uparrow}{\textcircled{5}}$: lands at 35
[J] 60 $\underset{\downarrow}{\textcircled{▨}}$: lands at 4
[K] 3 $\overset{\uparrow}{\textcircled{6}}$ $\underset{\downarrow}{\textcircled{▨}}$: lands at 10
[L] 30 $\underset{\downarrow}{\textcircled{▨}}$ $\textcircled{4}\rightarrow$: lands at 20

More number-line games

Study the table below to decide how these new symbols are used to describe jumps on the number line shown at the right.



Symbol	9 ↙①	2 ↖②	0 ③↗	27 ①↘	8 ↖①	15 ①↗	23 ①↘	16 ↙①
Start	9	2	0	27	8	15	23	16
Land	0	16	27	20	15	24	16	7

EXERCISES

1. Give the landing point for each jump. Use the number line above.

[A] 3 ①↗
[B] 20 ①↗
[C] 18 ↙①
[D] 30 ↙①
[E] 17 ①↘
[F] 13 ↖①
[G] 14 ①↘
[H] 20 ↖①
[I] 12 ↖① ①↘
[J] 26 ↙① ①↗
[K] 30 ↖①
[L] 30 ←① ①↑
[M] 0 ④↗
[N] 0 ⑦↗
[O] 1 ⑥↗
[P] 8 ⑤↗
[Q] 7 ↖④
[R] 7 ↖⑥
[S] 6 ↖⑤
[T] 0 ⑦↑
[U] 50 ②↗
[V] 36 ↙③
[W] 0 ⑧↑
[X] 49 ⑥↓

★**2.** Give the landing point for each jump. Use the number line above.

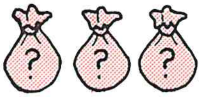
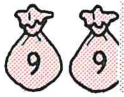
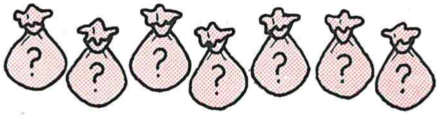
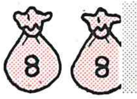
[A] 19 ④→ ←④
[B] 42 ←② ②→
[C] 19 ↙① ↖①
[D] 19 ←②
[E] 2 ⑥↗
[F] 2 ④↑
[G] 32 ④
[H] 6 ⑥↑
[I] 4 ⑧↑
[J] 31 ①↗
[K] 40 ↙①
[L] 0 ⑧↗
[M] 1326 ③↗
[N] 843 ↙④
[O] 672 ⑤↘
[P] 68 ⑧↓ ←④
[Q] 63 ⑤↓ ⑧↑
[R] 165 ④→ ②↑
[S] 72 ④↗ ⑨↓
[T] 224 ③↘ ⑤↑

★**3.** Give the missing numbers. Use the number line above.

[A] 47 ◍↓: lands at 7
[B] 64 ◍↓: lands at 0
[C] 75 ⑨↓: lands at ▥
[D] 59 ⑦↓: lands at ▥
[E] ▥ ⑥↓: lands at 0
[F] ▥ ⑤↑: lands at 46
[G] ▥ ⑦↑: lands at 61
[H] 2 ◍↗: lands at 38
[I] 48 ◍↘: lands at 6

Missing-factor exercises

1. The bags in each exercise have the same number of marbles.

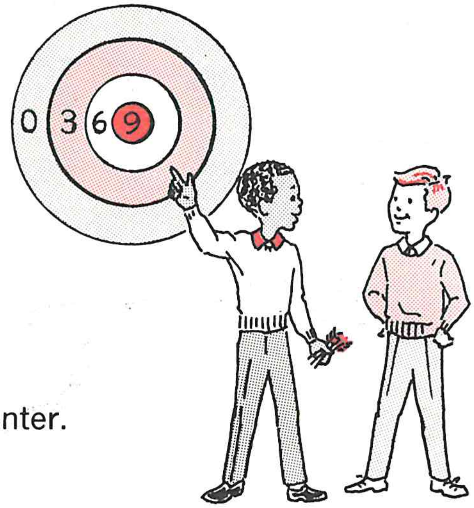
[A]		How many marbles in each bag?	27 marbles
[B]	 How many bags?	9 marbles in each bag.	54 marbles
[C]		How many marbles in each bag?	56 marbles
[D]	 How many bags?	8 marbles in each bag.	72 marbles

2. [A] How many darts must Don throw in the red center to get a score of 36?

[B] How many darts must Bob throw in the red center to get a score of 63?

[C] How many darts must Glen throw in the white ring to get a score of 48?

★ [D] Bill threw 12 darts. Some went in the red center. The others went in the white ring. If his score was 87, tell how many darts went in the red center.



3. Find the missing factors.

[A] $3 \times \boxed{n} = 18$

[B] $24 = 8 \times \boxed{n}$

[C] $\boxed{n} \times 5 = 35$

[D] $4 \times \boxed{n} = 32$

[E] $45 = 5 \times \boxed{n}$

[F] $\boxed{n} \times 8 = 40$

[G] $\boxed{n} \times 6 = 42$

[H] $49 = \boxed{n} \times 7$

[I] $48 = 8 \times \boxed{n}$

[J] $\boxed{n} \times 6 = 54$

[K] $56 = 7 \times \boxed{n}$

[L] $63 = 9 \times \boxed{n}$

[M] $64 = 8 \times \boxed{n}$

[N] $\boxed{n} \times 8 = 0$

[O] $\boxed{n} \times 9 = 72$

[P] $7 \times \boxed{n} = 42$

[Q] $\boxed{n} \times 8 = 72$

[R] $63 = \boxed{n} \times 7$

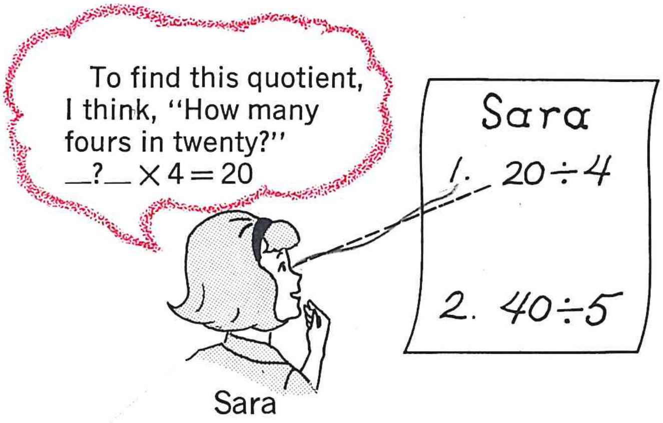
Quotients and missing factors

DISCUSSION EXERCISES

1. Explain how Sara is thinking about the quotient $20 \div 4$.

2. Explain how Sara might think about $40 \div 5$.

3. Find each quotient on Sara's paper.


To find this quotient, I think, "How many fours in twenty?" $\underline{\ ?\ } \times 4 = 20$
Sara
1. 20÷4
2. 40÷5
Sara

EXERCISES

In each exercise, when you find the **missing factor** in equation A, you will have found the **quotient** in equation B. Write equation B with the correct quotient.

1. [A] $\boxed{n} \times 6 = 30$
[B] $30 \div 6 = \boxed{n}$

2. [A] $\boxed{n} \times 6 = 36$
[B] $36 \div 6 = \boxed{n}$

3. [A] $\boxed{n} \times 6 = 42$
[B] $42 \div 6 = \boxed{n}$

4. [A] $\boxed{n} \times 6 = 48$
[B] $48 \div 6 = \boxed{n}$

6 **5.** [A] $\boxed{n} \times 6 = 54$
[B] $54 \div 6 = \boxed{n}$

6. [A] $\boxed{n} \times 5 = 35$
[B] $35 \div 5 = \boxed{n}$

7. [A] $\boxed{n} \times 7 = 42$
[B] $42 \div 7 = \boxed{n}$

8. [A] $\boxed{n} \times 7 = 49$
[B] $49 \div 7 = \boxed{n}$

9. [A] $\boxed{n} \times 7 = 56$
[B] $56 \div 7 = \boxed{n}$

10. [A] $\boxed{n} \times 7 = 63$
[B] $63 \div 7 = \boxed{n}$

11. [A] $\boxed{n} \times 8 = 40$
[B] $40 \div 8 = \boxed{n}$

12. [A] $\boxed{n} \times 8 = 48$
[B] $48 \div 8 = \boxed{n}$

13. [A] $\boxed{n} \times 8 = 56$
[B] $56 \div 8 = \boxed{n}$

14. [A] $\boxed{n} \times 8 = 64$
[B] $64 \div 8 = \boxed{n}$

15. [A] $\boxed{n} \times 8 = 72$
[B] $72 \div 8 = \boxed{n}$

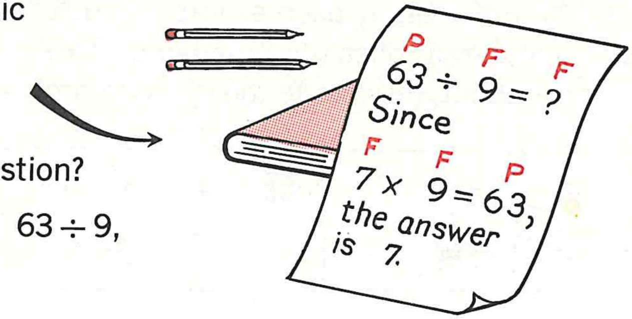
16. [A] $\boxed{n} \times 9 = 54$
[B] $54 \div 9 = \boxed{n}$

17. [A] $\boxed{n} \times 9 = 63$
[B] $63 \div 9 = \boxed{n}$

18. [A] $\boxed{n} \times 9 = 72$
[B] $72 \div 9 = \boxed{n}$

Products and factors

Tim saw Fred's arithmetic paper. It looked like this. Tim asked, "What do the red letters mean?" Can you answer Tim's question?



Fred explained, "To find 63 ÷ 9, I look for the factor that tells how many **nines** it takes to give the product 63."

A In **multiplication** you find the product of the factors.

$$\overset{F}{9}\times\overset{F}{8}=\overset{P}{\boxed{n}}$$

B In **division** you find one of the factors of the product.

$$\overset{P}{72}\div\overset{F}{8}=\overset{F}{\boxed{n}}\qquad\qquad\overset{P}{72}\div\overset{F}{9}=\overset{F}{\boxed{n}}$$

EXERCISES

1. Write the **product** of the two **factors** given.

[A] 9 × 1
[B] 5 × 9
[C] 3 × 8
[D] 6 × 4
[E] 6 × 9
[F] 7 × 4
[G] 3 × 9
[H] 4 × 9
[I] 3 × 7
[J] 4 × 8
[K] 8 × 4
[L] 4 × 7
[M] 9 × 4
[N] 7 × 9
[O] 8 × 9
[P] 9 × 9
[Q] 7 × 5
[R] 9 × 6
[S] 8 × 7
[T] 7 × 7

2. The **product** and **one factor** are given. For each exercise write an equation that gives the missing factor. Your work for exercise 1 will help.

[A] 9 ÷ 1
(Answer: 9 ÷ 1 = 9)
[B] 24 ÷ 8
[C] 24 ÷ 4
[D] 54 ÷ 9
[E] 28 ÷ 4
[F] 27 ÷ 9
[G] 36 ÷ 9
[H] 21 ÷ 7
[I] 32 ÷ 8
[J] 32 ÷ 4
[K] 28 ÷ 7
[L] 36 ÷ 4
[M] 63 ÷ 9
[N] 72 ÷ 9
[O] 81 ÷ 9

0 and 1 in division

0 in division Study each example. Look for the missing factor in the multiplication equations. This will help you think about answers to the division problems.

A	B	C
$\boxed{n} \times 0 = 5$ $5 \div 0 = \boxed{n}$	$\boxed{n} \times 5 = 0$ $0 \div 5 = \boxed{n}$	$\boxed{n} \times 0 = 0$ $0 \div 0 = \boxed{n}$

DISCUSSION EXERCISES

1. Which equations suggest **dividing by 0?**
2. Which equation suggests **dividing by 5?**
3. Which equations have no solution?
4. Which equations have exactly one solution?
5. Which equations have many solutions?

> Since an equation like $5 \div 0 = \boxed{n}$ does not have any solution and the equation $0 \div 0 = \boxed{n}$ does not have any one solution, we never divide by zero.

1 in division When you find the missing factor in the multiplication equation, you will have found the quotient in the division equation.

A	B
$\boxed{n} \times 1 = 7$ $7 \div 1 = \boxed{n}$	$\boxed{n} \times 7 = 7$ $7 \div 7 = \boxed{n}$

DISCUSSION EXERCISES

1. Choose any number. Divide it by 1. What number do you get?
2. Choose any number (except 0). Divide it by itself. What number do you get?
3. Find the quotients.

[A] $0 \div 8$
[B] $8 \div 1$
[C] $0 \div 9$
[D] $9 \div 9$
[E] $0 \div 7$
[F] $8 \div 8$
[G] $35 \div 1$
[H] $39 \div 39$
[I] $18 \div 1$
[J] $0 \div 387$
[K] $986 \div 986$
[L] $986 \div 1$
[M] $457 \div 1$
[N] $1 \div 1$
[O] $0 \div 1$

Practice finding quotients

DIVIDING BY 1, 2, 3, and 4

1. Find the quotients.

[A] $8 \div 2 = \boxed{n}$
[B] $9 \div 1 = \boxed{n}$
[C] $24 \div 3 = \boxed{n}$
[D] $16 \div 4 = \boxed{n}$
[E] $14 \div 2 = \boxed{n}$
[F] $9 \div 3 = \boxed{n}$
[G] $28 \div 4 = \boxed{n}$
[H] $18 \div 3 = \boxed{n}$
[I] $36 \div 4 = \boxed{n}$
[J] $18 \div 2 = \boxed{n}$
[K] $12 \div 3 = \boxed{n}$
[L] $21 \div 3 = \boxed{n}$
[M] $12 \div 2 = \boxed{n}$
[N] $32 \div 4 = \boxed{n}$
[O] $27 \div 3 = \boxed{n}$
[P] $20 \div 4 = \boxed{n}$
[Q] $16 \div 2 = \boxed{n}$
[R] $15 \div 3 = \boxed{n}$
[S] $6 \div 2 = \boxed{n}$
[T] $8 \div 1 = \boxed{n}$
[U] $10 \div 2 = \boxed{n}$

DIVIDING BY 5, 6, and 7

2. Find the quotients.

[A] $25 \div 5 = \boxed{n}$
[B] $24 \div 6 = \boxed{n}$
[C] $14 \div 7 = \boxed{n}$
[D] $36 \div 6 = \boxed{n}$
[E] $28 \div 7 = \boxed{n}$
[F] $30 \div 5 = \boxed{n}$
[G] $12 \div 6 = \boxed{n}$
[H] $42 \div 7 = \boxed{n}$
[I] $30 \div 6 = \boxed{n}$
[J] $35 \div 5 = \boxed{n}$
[K] $48 \div 6 = \boxed{n}$
[L] $21 \div 7 = \boxed{n}$
[M] $45 \div 5 = \boxed{n}$
[N] $35 \div 7 = \boxed{n}$
[O] $56 \div 7 = \boxed{n}$
[P] $40 \div 5 = \boxed{n}$
[Q] $49 \div 7 = \boxed{n}$
[R] $15 \div 5 = \boxed{n}$
[S] $54 \div 6 = \boxed{n}$
[T] $63 \div 7 = \boxed{n}$
[U] $20 \div 5 = \boxed{n}$

DIVIDING BY 8 and 9

3. Find the quotients.

[A] $24 \div 8$
[B] $27 \div 9$
[C] $54 \div 9$
[D] $40 \div 8$
[E] $45 \div 9$
[F] $48 \div 8$
[G] $32 \div 8$
[H] $81 \div 9$
[I] $64 \div 8$
[J] $63 \div 9$
[K] $36 \div 9$
[L] $72 \div 8$
[M] $56 \div 8$
[N] $72 \div 9$

Think

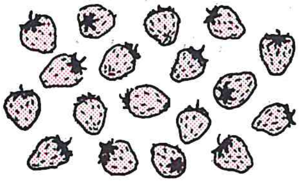
Ann's age is 4 times Bill's. Fred's age is twice Bill's. The sum of their ages is 21. How old is each child?

Begin by making a trial guess.

Division and sets again

DISCUSSION EXERCISES

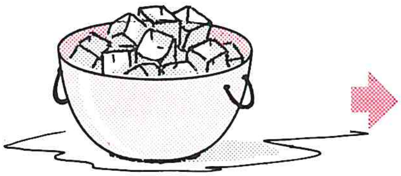
1. You can use division to find **how many sets.**
Solve the equation to find out.



18 strawberries

→ How many bowls of 6 strawberries can you serve? → $18 \div 6 = \boxed{n}$

2. You can use division to find **how many in each set.**
Solve the equation to find out.

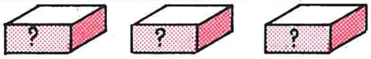
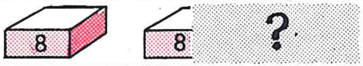
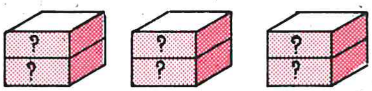
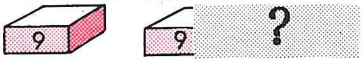
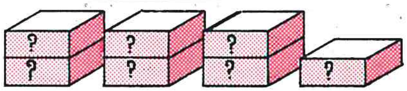
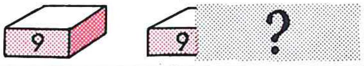


24 ice cubes

→ How many can you put in each glass if you divide them equally among 6 glasses? → $24 \div 6 = \boxed{n}$

EXERCISES

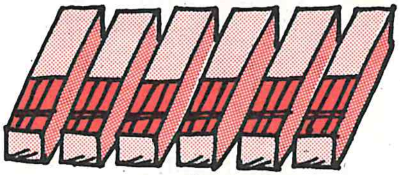
Use division to find the number of sets or the number in each set. Check by multiplication. All the boxes in each exercise have the same number of checkers.

1.	24 checkers		How many in each box?
2.	32 checkers		How many boxes?
3.	48 checkers		How many in each box?
4.	54 checkers		How many boxes?
5.	56 checkers		How many in each box?
6.	72 checkers		How many boxes?

Short stories

1. 32 buttons. 8 on each shirt. How many shirts?

2. 6 boxes of pencils.
48 pencils in all.
How many in each box?



3. 5 tires per car. 8 cars.
How many tires?

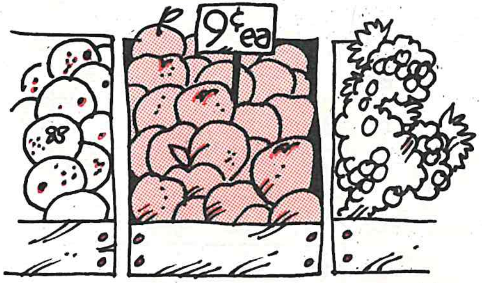
4. 16 children. Same number of girls as boys. How many of each?

5. 6 weeks. 7 days per week.
How many days?

6. 36 players divided equally into 6 teams.
How many per team?

7. 42 chairs put into 6 rows.
Same number in each row.
How many in each row?

8. Apples: 9 cents each. You have 45 cents. How many can you buy?



9. Airmail stamp: 8 cents. How many can you buy for 40 cents?

10. 9 miles to the park and back.
Three trips to the park and back.
How many miles?

11. Bicycling: 6 miles each hour.
24 miles to go. How long will it take?

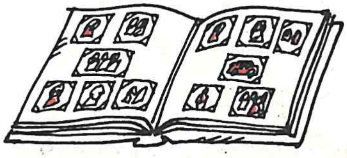
12. 40 cookies. 8 hungry children.
Divide the cookies equally.
How many for each child?

13. 9 jet airplanes. 4 jet engines per plane. How many jet engines?

14. 54 books on 6 shelves. Same number on each shelf. How many on each?

15. Had 50 cents. Lost a dime and spent the rest on nickel candy bars. How many candy bars were bought?

16. 6 photographs on a page.
How many on 8 pages?

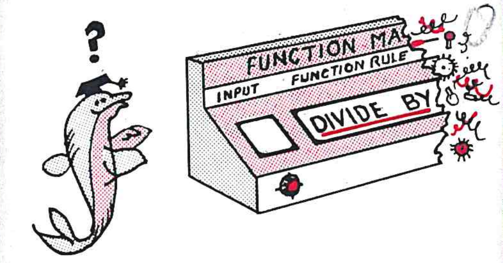


Division exercises

1. Find the quotients.

[A] 27 ÷ 3
[B] 36 ÷ 4
[C] 35 ÷ 5
[D] 28 ÷ 4
[E] 42 ÷ 7
[F] 45 ÷ 9
[G] 81 ÷ 9
[H] 72 ÷ 8
[I] 56 ÷ 7
[J] 54 ÷ 6
[K] 48 ÷ 8
[L] 63 ÷ 9
[M] 42 ÷ 6
[N] 49 ÷ 7

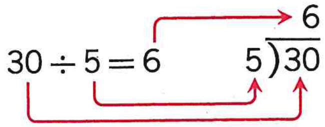
Think



I'm a special number,
As special as can be.
Use me as a factor
But don't divide by me.

WHO AM I?

2. The figure below will help you review a different way to write division exercises.



Find the quotients.

[A] $4\overline{)16}$
[B] $8\overline{)32}$
[C] $8\overline{)8}$
[D] $6\overline{)42}$
[E] $7\overline{)0}$
[F] $7\overline{)56}$
[G] $9\overline{)63}$
[H] $6\overline{)54}$
[I] $8\overline{)72}$
[J] $8\overline{)48}$
[K] $5\overline{)40}$
[L] $1\overline{)9}$
[M] $6\overline{)24}$
[N] $6\overline{)6}$
[O] $7\overline{)42}$

★**3.** Find the equation that has no solution. Find the equations that have many solutions. Solve the other equations.

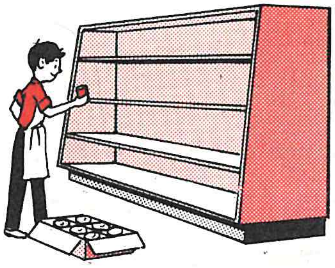
[A] $9 \times 0 = \boxed{n}$
[B] $0 \div 8 = \boxed{n}$
[C] $\boxed{n} \div 8 = 4$
[D] $0 \times 0 = \boxed{n}$
[E] $0 \times \boxed{n} = 18$
[F] $9 \div 9 = \boxed{n}$
[G] $36 \div \boxed{n} = 4$
[H] $\boxed{n} \times 0 = 0$
[I] $0 \times 5 = \boxed{n}$
[J] $0 \div 5 = \boxed{n}$
[K] $0 \times \boxed{n} = 0$
[L] $28 \div 7 = \boxed{n}$

★**4.** Give the pair of numbers (in order) for the gray spaces.

	Products		− ×		Differences
[A]	6	▨	×	▨	1
[B]	15	▨	×	▨	2
[C]	36	▨	×	▨	0
[D]	72	▨	×	▨	1
[E]	54	▨	×	▨	3
[F]	0	▨	×	▨	9

At the grocery store

1. Jeff helps at his father's grocery store after school. He put 32 cans of peaches on the shelf in 8 rows. How many cans were in each row?



2. Juice comes to the store packed in boxes. There are 6 cans in each box. Jeff's father ordered 54 cans. How many boxes was this?

3. Jeff put 72 apples into 8 plastic bags. He put the same number in each bag. How many apples did he put in each bag?

4. Jeff made these sale signs.
   [A] How much does 1 can of tomato soup cost?
   [B] How much does 1 pound of sugar cost?

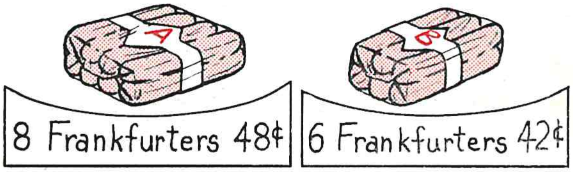
Bargains!
Tomato Soup
7 cans for 56¢

SALE PRICE
Sugar
5 pounds for
45¢

5. Ted, Jeff's friend, came to the store. He had 45 cents. How many 5-cent candy bars could he buy?

6. [A] How much does one brand A frankfurter cost?

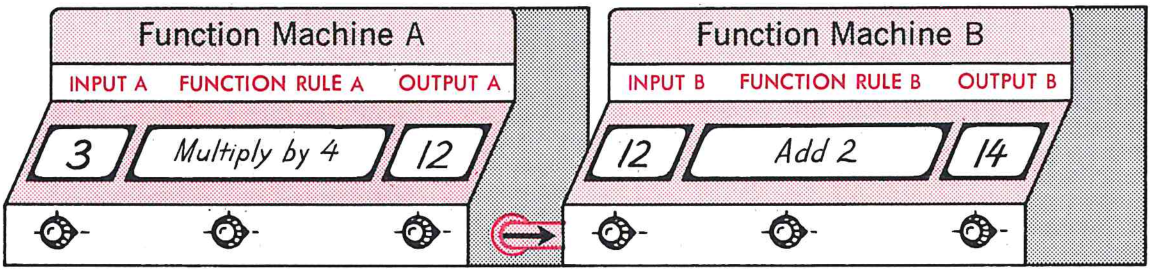
   [B] How much does one brand B frankfurter cost?



★[C] If the frankfurters are of equal quality, which would you buy to save money: 3 packages of brand A or 4 packages of brand B?

★[D] How many packages of brand A must you buy to serve one hot dog each to 56 people? How much will it cost?

The function machine



Function machine A is connected to function machine B. The output number from machine A becomes the input number for machine B. We put in 3. Machine A operates. Machine B operates. We get 14.

EXERCISES

Think about connected function machines and give the numbers you think should go in the gray spaces.

1. RULE
A Multiply by 2
B Add 1

	Input A	Output B
	3	7
[A]	6	
[B]	9	
[C]	10	

2. RULE
A Multiply by 8
B Add 47

	Input A	Output B
[A]	4	
[B]	6	
[C]	7	
[D]	9	

3. RULE
A Add 37
B Subtract 37

	Input A	Output B
[A]	25	
[B]	88	
[C]	100	
[D]	0	

4. RULE
A Multiply by 7
B [A]

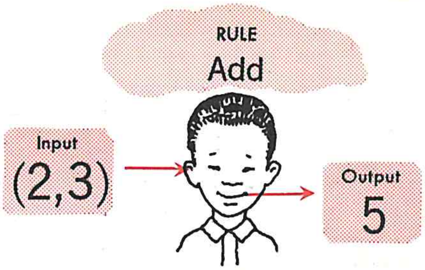
	Input A	Output B
	4	4
	9	9
[B]	6	
[C]	8	

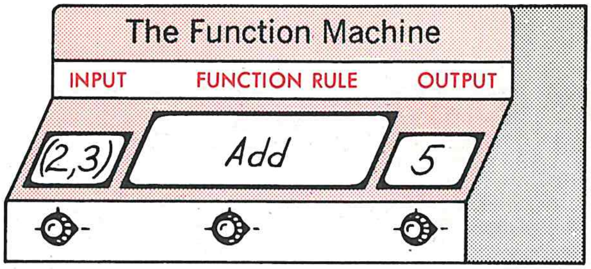
★ **5.** RULE
A Add 8
B [A] – 6

	Input A	Output B
	4	6
	6	7
[B]	2	
[C]	10	

★ **6.** RULE
A [A]
B Add 1

	Input A	Output B
	3	10
	5	26
[B]	4	
[C]	8	





You can put a pair of numbers into the machine and get a single number.

EXERCISES

Give what you think should go in the gray spaces.

1. Function Rule: Multiply

	Input	Output
[A]	(3, 7)	░
[B]	(4, 9)	░
[C]	(9, 9)	░
[D]	(7, 6)	░
[E]	(9, ▥)	45
[F]	(▥, 8)	72

2. Function Rule: Divide

	Input	Output
	(12, 4)	3
[A]	(18, 2)	░
[B]	(49, 7)	░
[C]	(0, 3)	░
[D]	(48, ▥)	8
[E]	(▥, 6)	9

3. Function Rule: Choose the greater

	Input	Output
	(39, 93)	93
	(107, 170)	170
[A]	(99, 999)	░
[B]	(384, 834)	░
[C]	(1005, 999)	░
[D]	(617, 716)	░

4. Function Rule: Double the first
Then add

	Input	Output
	(3, 4)	10
	(9, 3)	21
[A]	(7, 8)	░
[B]	(9, 18)	░
[C]	(8, 94)	░
[D]	(10, 10)	░

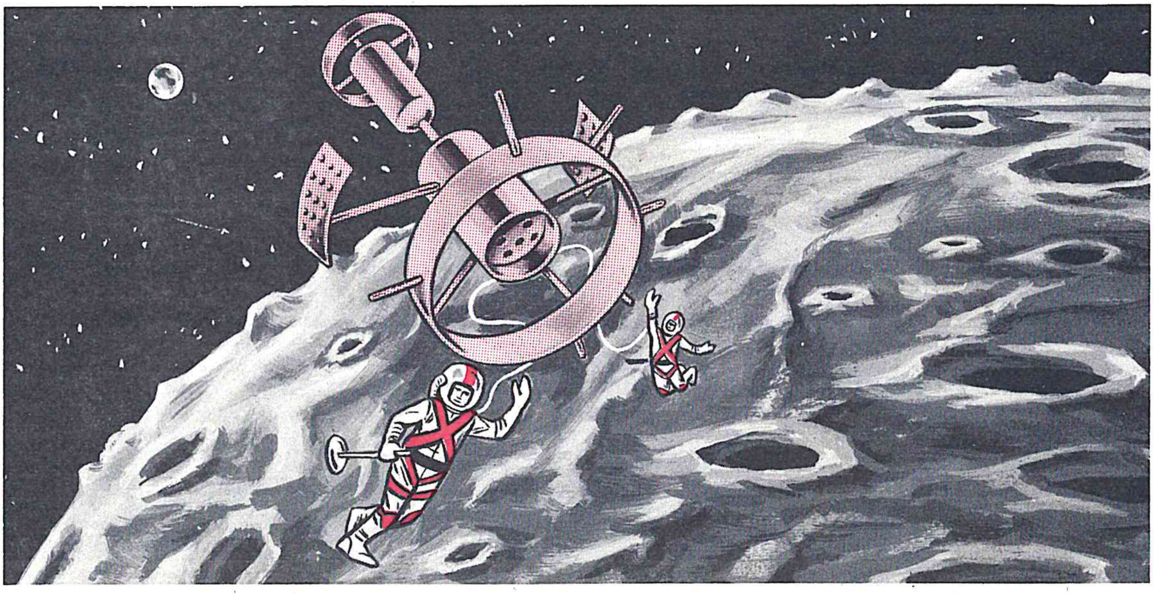
5. Function Rule: [A] ░

	Input	Output
	(6, 4)	10
	(9, 7)	16
	(14, 8)	22
[B]	(19, 8)	░
[C]	(79, 67)	░
[D]	(37, 63)	░

★**6.** Function Rule: [A] ░

	Input	Output
	(6, 4)	5
	(4, 10)	7
	(12, 8)	10
	(9, 9)	9
[B]	(7, 9)	░
[C]	(4, 8)	░

Weights on the moon



Objects weigh 6 times as much on earth as they do on the moon.

Some children at Franklin school made this table to compare their earth weight with their moon weight.

Name	Weight in pounds	
	Moon	Earth
Jean	[**A**] 6	36
Kristin	[**B**] 7	42
Tom	8	[**C**] 48
John	9	[**D**] 54
Fred	[**E**] 10	60

1. Complete the table by giving the weight that should go in each space.

2. Find the total moon weight of all the children.

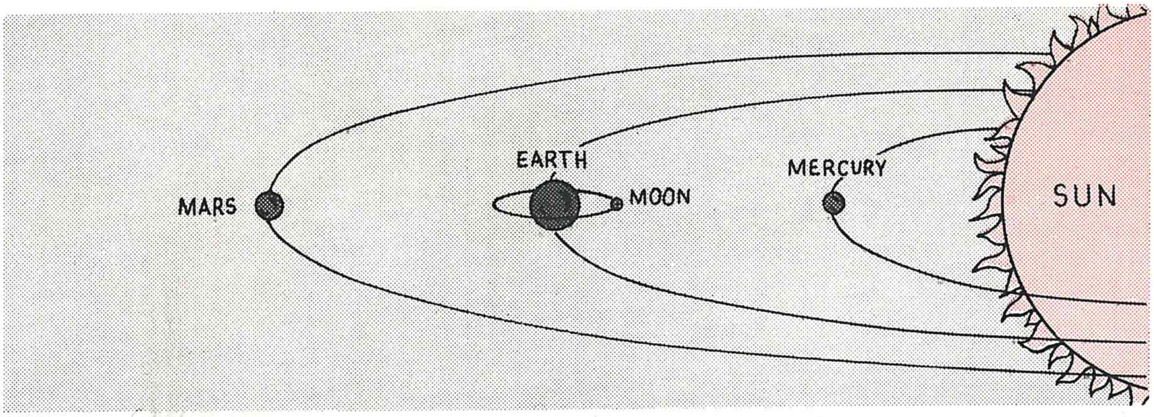
3. Find the total earth weight of all the children.

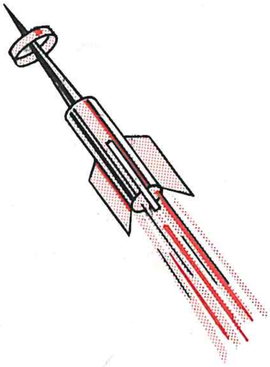
4. Bill's earth weight is 54 pounds. His dog has a moon weight of 4 pounds. What is the total earth weight of Bill and his dog?

5. Karen's bicycle has a moon weight of 6 pounds. Karen's earth weight is 52 pounds. On earth, how much more does Karen weigh than her bicycle?

★**6.** If Glen weighs $8\frac{1}{2}$ pounds on the moon, what does he weigh on earth?

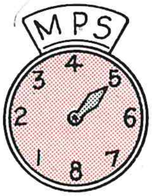
Short space stories





1. Rocket to the moon. 63 miles in 9 seconds. How many miles per second?

2. Closest distance of moon from earth: 221,463 miles. Farthest distance: 252,710 miles. How much change in distance?



3. Jupiter has 12 moons. Saturn: 9 moons. Uranus: 5 moons. Neptune: 2 moons. Mars: 2 moons. Earth: 1 moon. How many moons in all?

4. Rocket going into earth orbit. Speed: 5 miles per second. How far in 8 seconds?

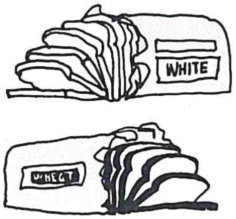
5. Diameter of moon: 2160 miles. Diameter of earth: 7920 miles. How much greater is the earth's diameter?

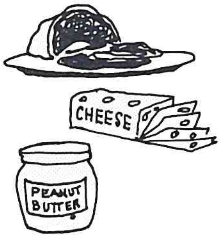
6. Weight. About 3 times as much on earth as on Mars. Object weighs 27 pounds on earth. How much on Mars?

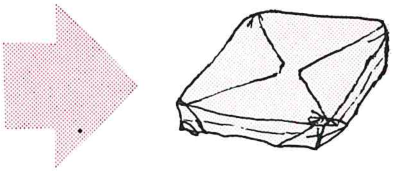
7. Weight. About 4 times as much on earth as on Mercury. 9 pounds on Mercury. How many pounds on earth?

Multiplication and pairing

Pam is making a sandwich. She plans to use white or brown bread (not both). She will use one of the following: beef, cheese, or peanut butter.







What kind of sandwich do you think Pam made?

DISCUSSION EXERCISES

1. Which kind of bread would you choose?
2. Which of the three would you put on your sandwich?
3. Name all the different sandwiches that might be made.
4. Solve the equation.

Number of kinds of bread

Number of things to put between the bread

Number of different sandwiches

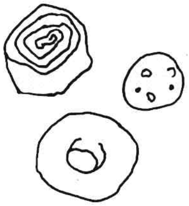
$$2 \times 3 = \boxed{n}$$

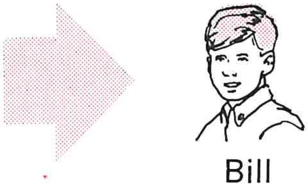
EXERCISES

1. Bill can have milk, soda or cocoa.

Bill can have a doughnut, cookie or roll.

What do you think Bill ate and drank?



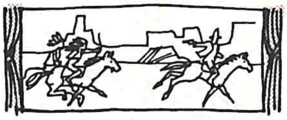


Bill

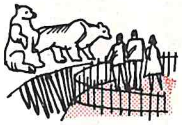
[A] Name all the different choices Bill might have made. How many choices are there?

[B] Solve: $3 \times 3 = \boxed{n}$

2. For his birthday, Jack gets to go to a movie, the zoo, or a ballgame. He can go on any day of the week except Saturday or Sunday.



Movie



Zoo

Ballgame

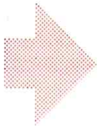
Monday

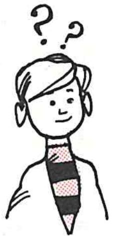
Tuesday

Wednesday

Thursday

Friday





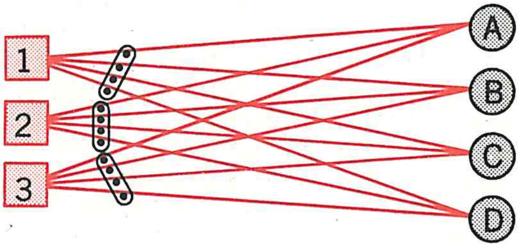
Jack

What do you think
Jack will choose?

[A] Name all the choices (like the zoo on Tuesday) that Jack could make. How many choices are there?

[B] Solve: $3 \times 5 = \boxed{n}$

3. Here is a pairing of a set of squares and a set of circles. The small dots help you count the matching lines.



[A] How many squares are there?

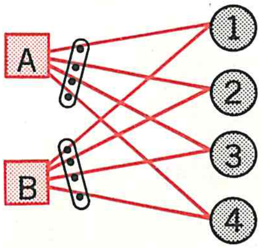
[B] How many circles are there?

[C] How many matching lines are there?

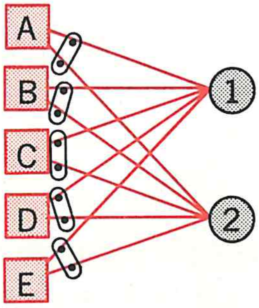
[D] Solve: $3 \times 4 = \boxed{n}$

4. Write a multiplication equation for each figure.

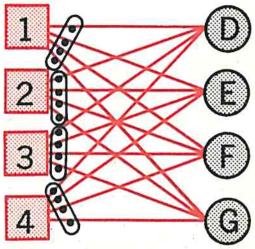
[A]



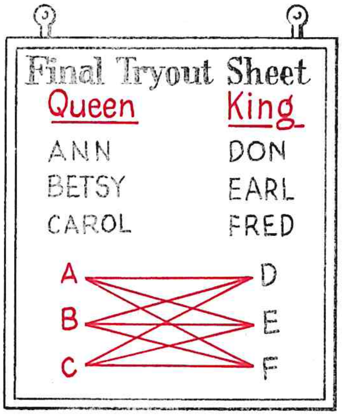
[B]



[C]

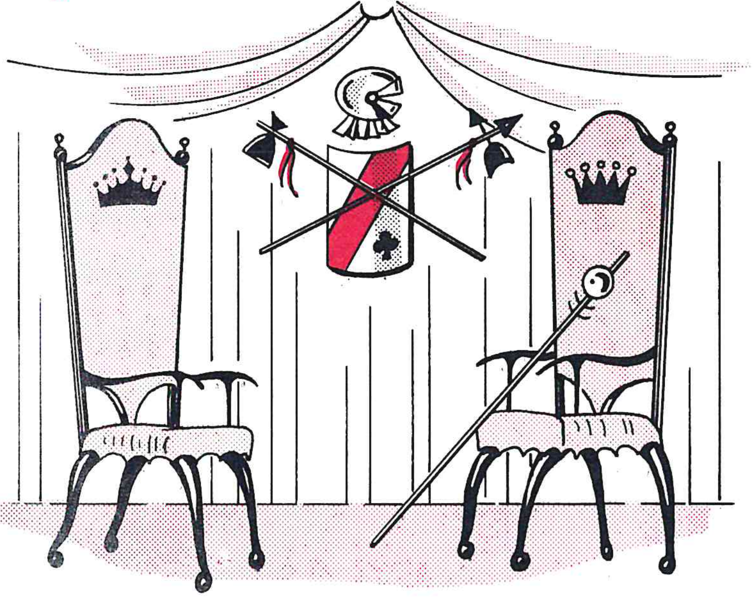


Trying out for the play



Final Tryout Sheet

Queen	King
ANN	DON
BETSY	EARL
CAROL	FRED



1. [A] Here are 2 of the possible queen-king pairs.

   A, D (This is a short way of writing Ann and Don.)

   C, E (What does this represent?)

   [B] How many different queen-king pairs are there?

   [C] Write a multiplication equation that tells how many pairs.

2. The children decided that Betsy should be a princess and not the queen. Now list the possible queen-king pairs.

3. Earl had been reading about knights. He wanted very much to be a knight, so the children agreed. How many queen-king pairs were left?

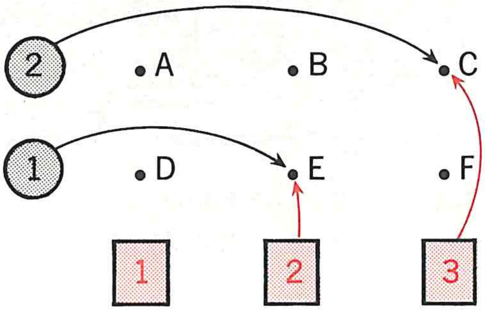
4. Fred moved away before the play was presented. Who was the king?

5. Ann was not chosen to be the queen. Who was the queen?

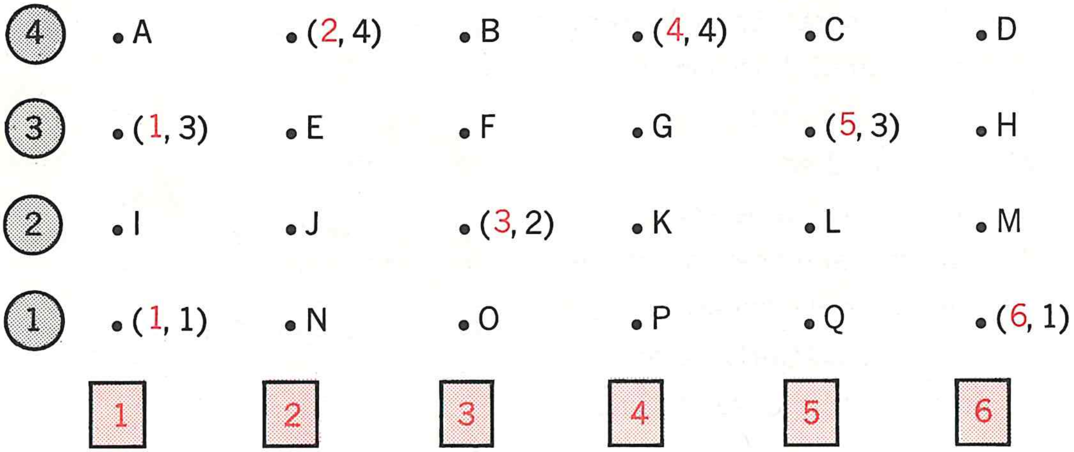
★ 6. If 8 girls want to be queen and 7 boys want to be king, how many queen-king pairs would be possible?

More about pairing

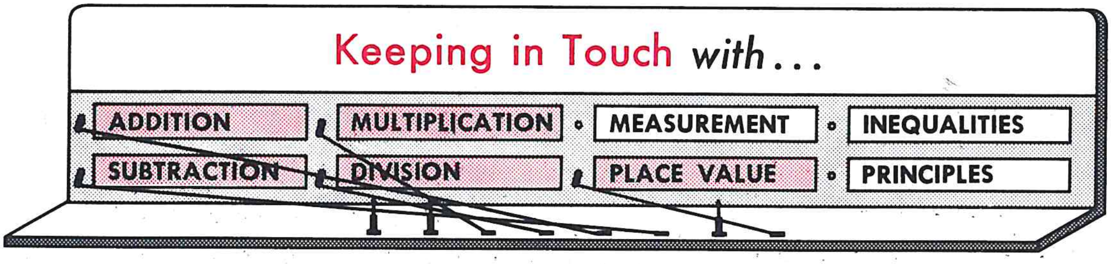
1. Here is another way to think about pairing. The dots show pairings of the squares with circles. The arrows help show why the number pair for point E is (2, 1) and the number pair for point C is (3, 2). Give the number pair for each of the other points.



2. Some of the pairs that show the matching of the red numerals with the black numerals are given.



[**A**] List the number pairs for **A** through **Q**.
[**B**] How many numerals are in circles?
[**C**] How many numerals are in squares?
[**D**] How many number pairs are there in all?
[**E**] Write a multiplication equation about this matching.
[**F**] List the pairs in which the two numbers are equal.
[**G**] List all the pairs in which the sum of the numbers is 5; greater than 5.
★[**H**] List all the pairs of numbers in which the product of the numbers is 4; less than 4.



1 1
256
256
512
1
128
128
256
64
64
128

1. Copy and find the sums.

[**A**] 538 + 24 + 365 + 4720
[**B**] 4768 + 232 + 5496
[**C**] 6437 + 35 + 932
[**D**] 375 + 893 + 5672 + 38

2. Find the sums and differences.

[**A**] 4635 + 2819
[**B**] 1386 − 539
[**C**] 8207 − 3649
[**D**] 5003 − 2216
[**E**] 5739 + 8654

32
32
64
36
13
49
1
49
15
64
1
64
17
91
1
81
19
100

3. Write the numeral for each exercise.

[**A**] five hundred thirty-seven
[**B**] nine hundred fifty
[**C**] seven hundred six
[**D**] four hundred nineteen
[**E**] eight thousand seven hundred
[**F**] one hundred sixty-six thousand
[**G**] six thousand thirty-seven
[**H**] ninety-eight thousand seventy

Think

To find how many sets in all,
I'm surely your best bet.
Or use me if you need to give
The number in each set.

WHO AM I?

4. Give the next five numbers in each sequence.

[**A**] 1, 4, 9, 16, 25, . . .
[**B**] 1, 3, 5, 7, 9, . . .
[**C**] 1, 2, 4, 8, 16, . . .

5. [**A**] Add the first 2 numbers in exercise 4**B**. Which number in exercise 4**A** do you get?

[**B**] Add the first 4 odd numbers in exercise 4**B**. You should get 4 × 4.

[**C**] Add the first 6 odd numbers. Do you get 6 × 6?

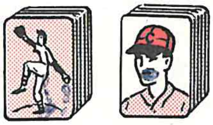
[**D**] Add the first 7 odd numbers. Do you get 7 × 7?

[**E**] What do you think is the sum of the first 8 odd numbers?

[**F**] Find the sum of the first 9 odd numbers.

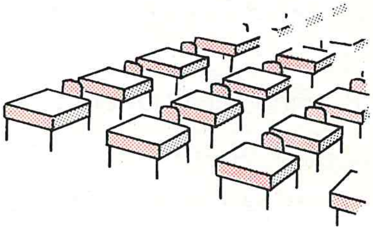
[**G**] Find the sum of the first 10 odd numbers.

6. No numbers are given in this exercise. Decide whether you would add, subtract, multiply, or divide if numbers were given. Answer **A**, **S**, **M**, or **D**.



[A] Joe had |||||| baseball cards.
Bill had |||||| baseball cards.
How many more baseball cards did Bill have than Joe?

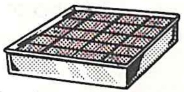
[B] We drove |||||| miles on Friday and |||||| miles on Saturday.
How far did we drive altogether?



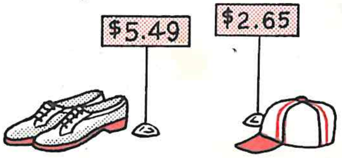
[C] In Jane's classroom there were |||||| rows
with |||||| children in each row.
How many children?

[D] Ted had |||||| hamsters. He gave |||||| away.
How many did he have left?

[E] Emily had |||||| stamps in her collection. She had |||||| stamps on each page. How many pages of stamps did she have?



[F] Sandy made |||||| pieces of fudge. She put the same number of pieces in each of |||||| boxes. How many pieces did she put in each box?



7. [A] John spent $5.49 for a pair of shoes.
He bought a cap for $2.65.
Find the total amount he spent.

[B] Nancy's tennis racket cost $9.99.
Ken spent $18.45 for his racket.
How much more did Ken spend than Nancy?

Chapter review

1. Find the products.

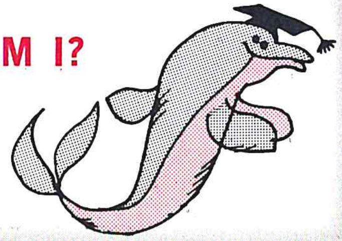
[A] 1×1
[B] 9×1
[C] 0×0
[D] 4×0
[E] 2×7
[F] 4×9
[G] 3×8
[H] 8×3
[I] 9×4
[J] 4×8
[K] 5×8
[L] 6×7
[M] 8×6
[N] 9×9
[O] 7×8
[P] 9×6
[Q] 9×8
[R] 7×9
[S] 9×10
[T] 10×10

Think

Just for multiplication
I'm called the identity,
Since you get the other factor
When you multiply by me.

×

WHO AM I?



2. Find the quotients.

[A] $24 \div 1$
[B] $0 \div 7$
[C] $9 \div 9$
[D] $27 \div 3$
[E] $36 \div 6$
[F] $48 \div 8$
[G] $48 \div 6$
[H] $45 \div 9$
[I] $42 \div 7$
[J] $49 \div 7$
[K] $54 \div 6$
[L] $63 \div 7$
[M] $63 \div 9$
[N] $56 \div 8$
[O] $64 \div 8$
[P] $72 \div 9$
[Q] $72 \div 8$
[R] $81 \div 9$
[S] $90 \div 9$
[T] $100 \div 10$

3. Solve the equations to find the products.

[A] $12 \times 7 = (6 \times 7) + (6 \times 7) = \boxed{n}$
[B] $13 \times 5 = (7 \times 5) + (6 \times 5) = \boxed{n}$
[C] $16 \times 7 = (8 \times 7) + (8 \times 7) = \boxed{n}$
[D] $18 \times 8 = (9 \times 8) + (9 \times 8) = \boxed{n}$

4. Find the quotients.

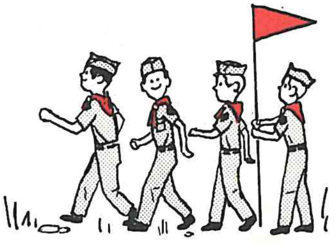
[A] $5\overline{)25}$ [B] $6\overline{)54}$ [C] $7\overline{)42}$ [D] $9\overline{)54}$ [E] $9\overline{)0}$ [F] $7\overline{)56}$

5. Give the missing numbers.

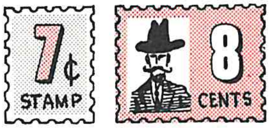
[A] Since $54 \times 398 = 21{,}492$, we know that $398 \times 54 = \boxed{n}$.
[B] Since $54 \times 398 = 21{,}492$, we know that $21{,}492 \div 398 = \boxed{n}$.
[C] Since $4672 + 4672 + 4672 = 14{,}016$, we know that $3 \times 4672 = \boxed{n}$.
[D] Since $144 \div 48 = 3$, we know that $3 \times 48 = \boxed{n}$.

6. [**A**] How many weeks are there in 49 days?

[**B**] 32 boy scouts were divided into 4 patrols. There were the same number of scouts in each patrol. How many scouts were in each patrol?

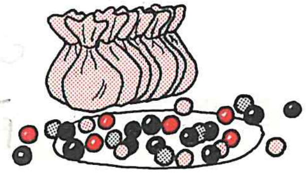


[**C**] How much does it cost to buy eight 7-cent stamps and one 8-cent stamp?

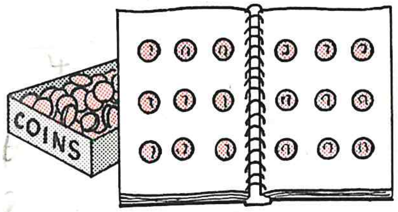


[**D**] How many pennies can you get for 7 nickels and 3 dimes?

[**E**] Tom had 8 bags with 8 marbles in each bag. He had 29 extra marbles on the ground. How many marbles did he have in all?



[**F**] Sally put 9 coins on each of 7 pages in her coin-collection book. She had 77 other coins in a box. How many coins did she have in all?



★[**G**] Ted had 35 rocks. Jim had 63 rocks. Each boy decided to store his rocks in cans, with 7 rocks in each can. Who used more cans? How many more?

★**7.** [**A**] Does $56 \div 8 = (56 \div 4) \div 2$?

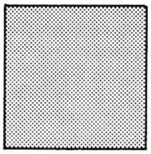
[**B**] Does $42 \div 6 = (42 \div 3) \div 2$?

CHAPTER 4 Measurement

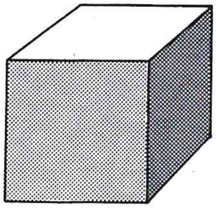
Length, area, and volume

We use segments, squares, and cubes for measuring.

Segment



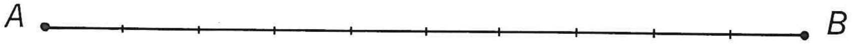
Square



Cube

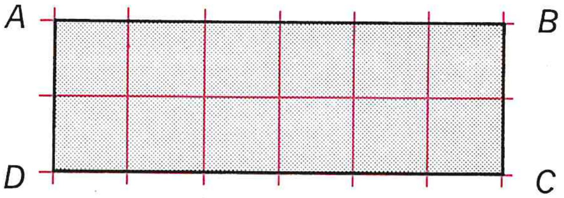
A We count **segment units** to find length.

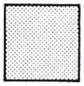
unit



What do we mean when we say that the length of segment **AB** is 10 units?

B We count **square units** to find area.

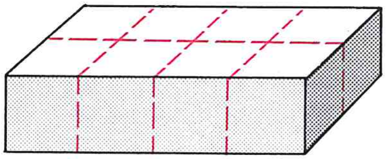


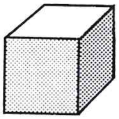


unit

What do we mean when we say that the area of rectangle **ABCD** is 12 units?

C We count **cube units** to find volume.





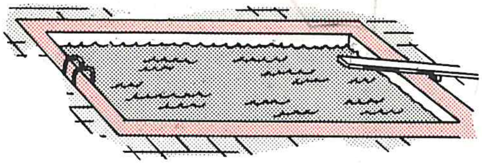
unit

What do we mean when we say that the volume of the figure is 8 units?

EXERCISES

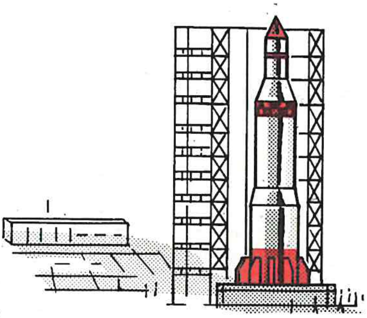
Do you need to find length, area, or volume to answer each question?

1. How deep is the pool?

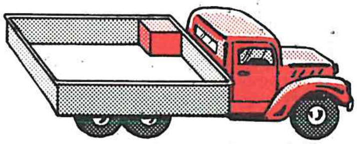


2. How much water does the pool hold?

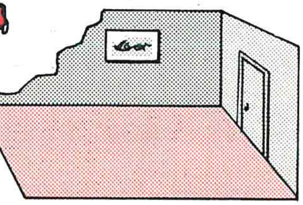
3. How large a piece of plastic must we buy to cover the pool?



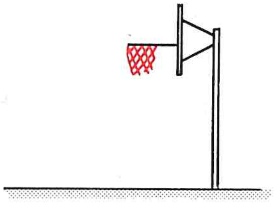
4. How tall is the rocket?



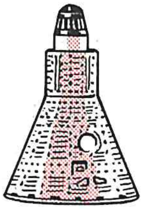
5. How many boxes will the truck hold?



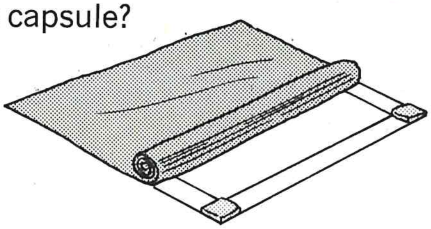
6. What size rug shall we buy to cover the floor?



7. How high is the basket?



8. How much room is inside the space capsule?

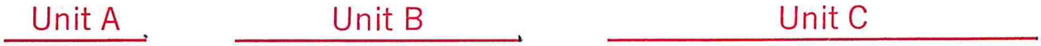


9. How large is the canvas that covers the baseball diamond?

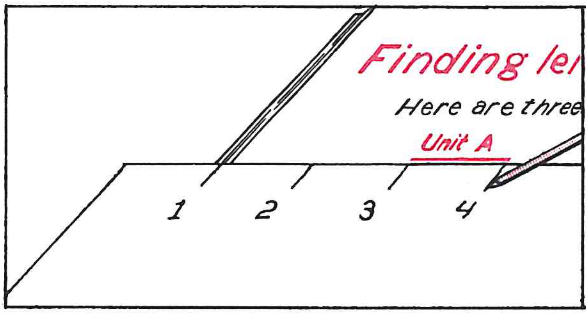
10. What size belt do you wear?

Finding length

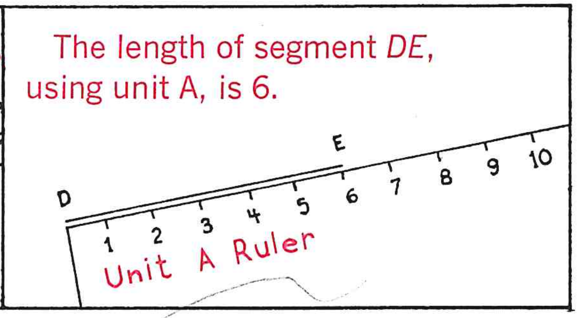
Here are three units we could use to find length.



On the edge of your paper, make a ruler, using unit A.



Making a unit A ruler.


The length of segment *DE*, using unit A, is 6.

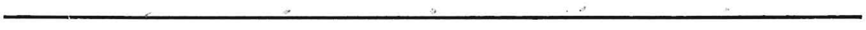
Using the unit A ruler.

Now make a unit B ruler and a unit C ruler on separate sheets of paper.

EXERCISES

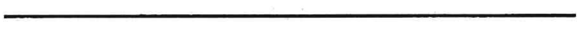
1. Use these units to give the length of this segment.

[A] unit A [B] unit B [C] unit C



2. Use these units to give the length of this segment.

[A] unit A [B] unit B



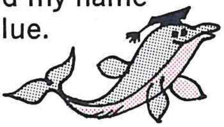
3. Use the unit A ruler to give the length of unit C.

4. Use the unit A ruler to give the length of unit B.

★**5.** Using unit B, you find that the length of a pencil is 5. What is the length of the pencil if you use unit A?

Think

You think about 10 hundreds.
100 tens will do.
That's enough to find my name
You need no other clue.



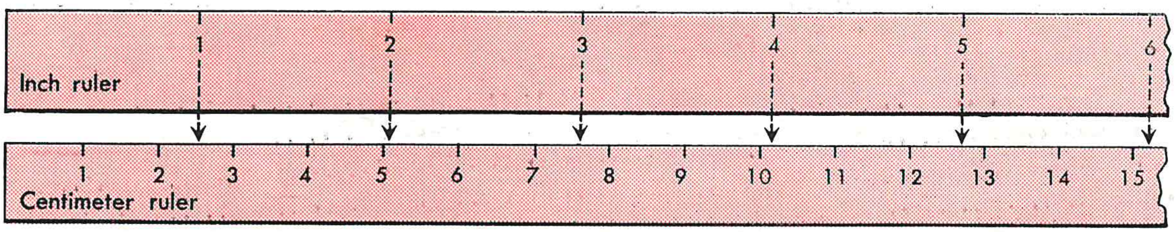
WHO AM I?

Inch and centimeter rulers

To find the length of an object, we must first choose a unit. We could choose any unit, but it is easier if everyone uses certain special units. Here are two units that are used in many countries.

inch (in.)   centimeter (cm)

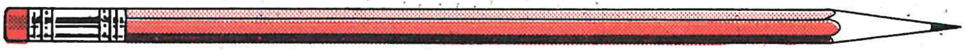
The inch and centimeter rulers will help you compare these units.



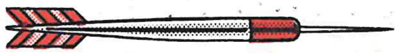
EXERCISES

1. Use your inch ruler to measure these objects.

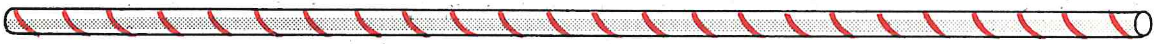
[A]



[B]

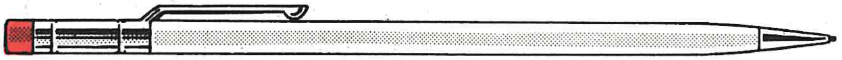


[C]

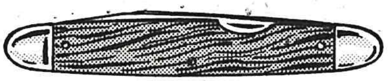


2. Use your centimeter ruler to measure these objects.

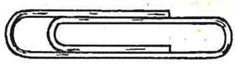
[A]



[B]



[C]



3. Draw segments of the following lengths.

[A] 10 in. **[B]** 10 cm **[C]** 2 in. **[D]** 5 cm **[E]** $\frac{1}{2}$ in. **[F]** 1 cm

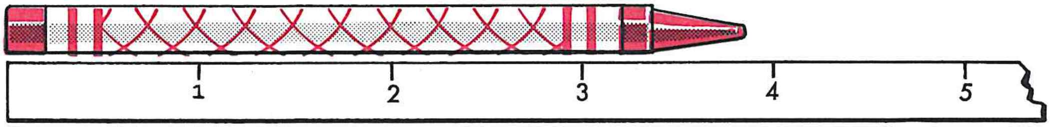
4. **[A]** Which is longer, an inch or a centimeter?

[B] A red wire is 7 centimeters long. A black wire is 7 inches long. Which wire is longer?

[C] A rope is 59 inches long. Will you get a number greater or less than 59 when you measure the rope with a centimeter ruler?

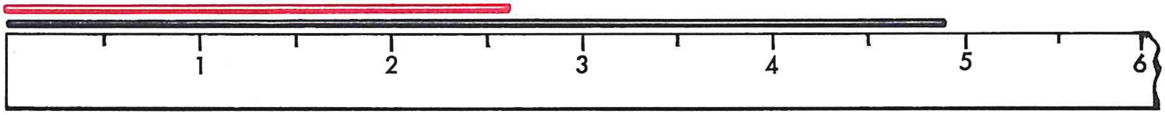
Estimating length

The ruler below has black marks one inch apart. The length of the crayon is more than 3 but less than 4. Is it closer to 3 or to 4?



The length of the crayon (to the nearest inch) is 4.

The ruler below has marks **one-half** inch apart. The length of the black wire is more than $4\frac{1}{2}$ but less than 5. Is it closer to $4\frac{1}{2}$ or to 5?

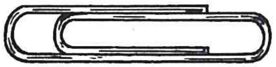


The length of the black wire (to the nearest half inch) is 5.
The length of the red wire (to the nearest half inch) is $2\frac{1}{2}$.

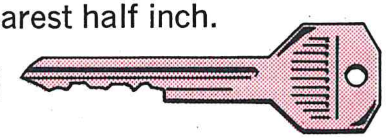
EXERCISES

1. Give the length of each object to the nearest half inch.

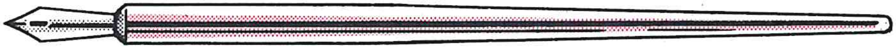
[A]



[B]



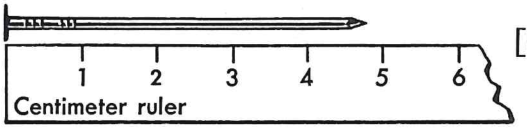
[C]



2. Give the length of each object in exercise 1 to the nearest inch.

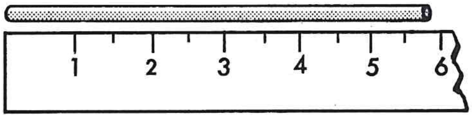
3. Give the missing numbers.

[A]



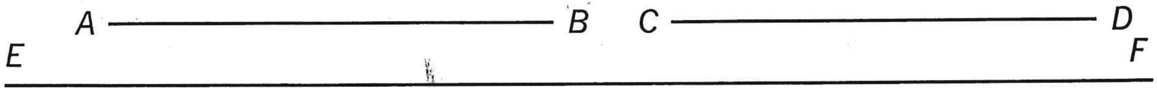
length (to the nearest cm) = ▯

[B]



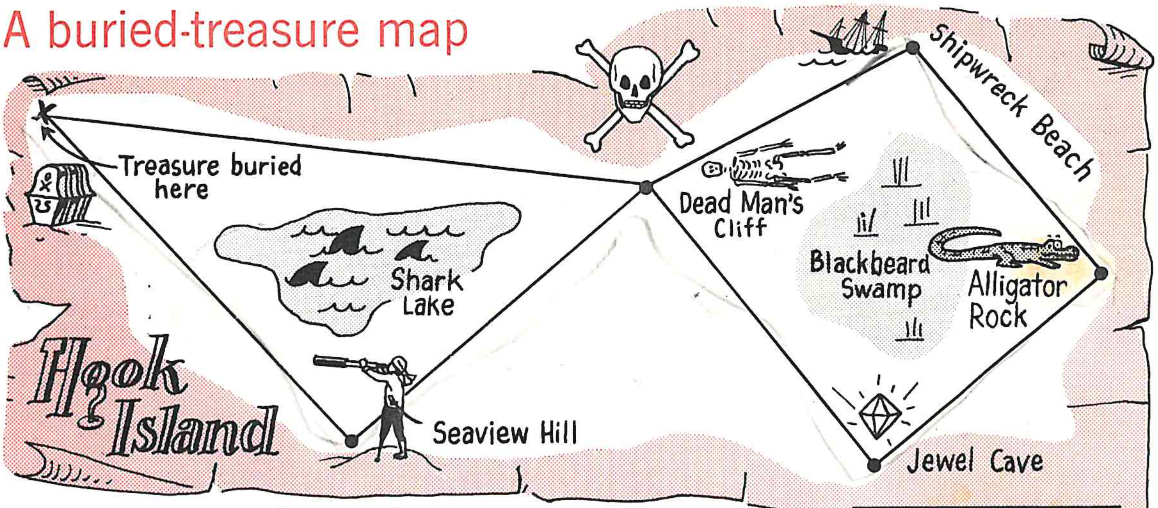
length (to the nearest $\frac{1}{2}$ cm) = ▯

4. Give the length of each segment to the nearest centimeter.



5. Give the length of each segment in exercise 4 to the nearest half centimeter.

A buried-treasure map



SCALE
Each centimeter on this map means 1 mile on the island.

When Dan was digging a hole to plant a tree, he found this map in a metal box. Answer the questions. Measure to the nearest centimeter.

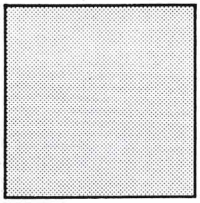
1. How many miles from
   - [A] Alligator Rock to Shipwreck Beach?
   - [B] the cliff to the treasure?
   - [C] the cave to Seaview Hill?
   - [D] Seaview Hill to the treasure?
   - [E] the cliff to the cave?
2. Which is shorter, to go from Dead Man's Cliff straight to the treasure or to go from Dead Man's Cliff past Seaview Hill to the treasure? How many miles shorter?
3. How far must a crow fly to go from Alligator Rock straight to the treasure?
4. Which is shorter, to go from the cliff past the beach to Alligator Rock or to go from the cliff past Jewel Cave to Alligator Rock? How many miles shorter?
5. Is it farther to walk on the paths around Shark Lake or around Blackbeard Swamp? How many miles farther?

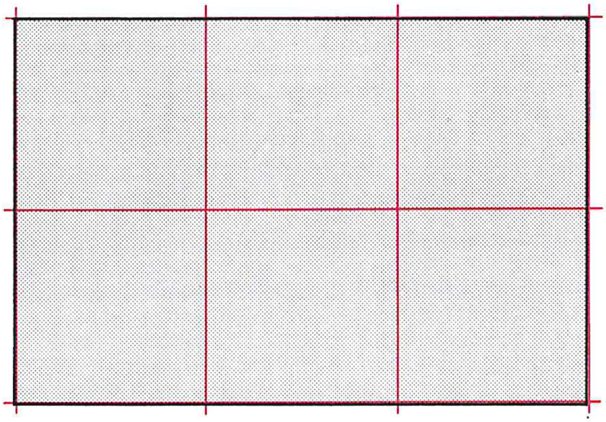
★ 6. If you start at Alligator Rock, which is the shortest path to the treasure? How many miles is it?

★ 7. What is the longest way from Alligator Rock to the treasure if you always use a path that takes you closer to the treasure?

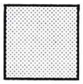
Finding area

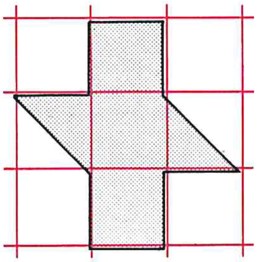
We can count square units to find area.



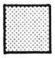


When the square inch is the unit, the area of this region is 6.

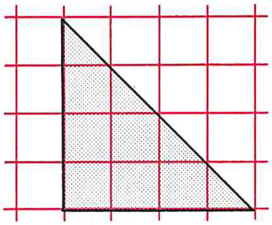




When the square centimeter is the unit, the area of this region is 4.



unit



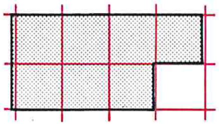
Using this unit, the area of this region is 8.

EXERCISES

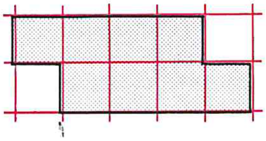
This ▨ is the unit for these exercises.

1. Find the number of square units (area) in each shaded region.

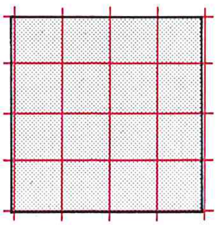
[A]



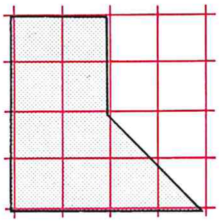
[B]



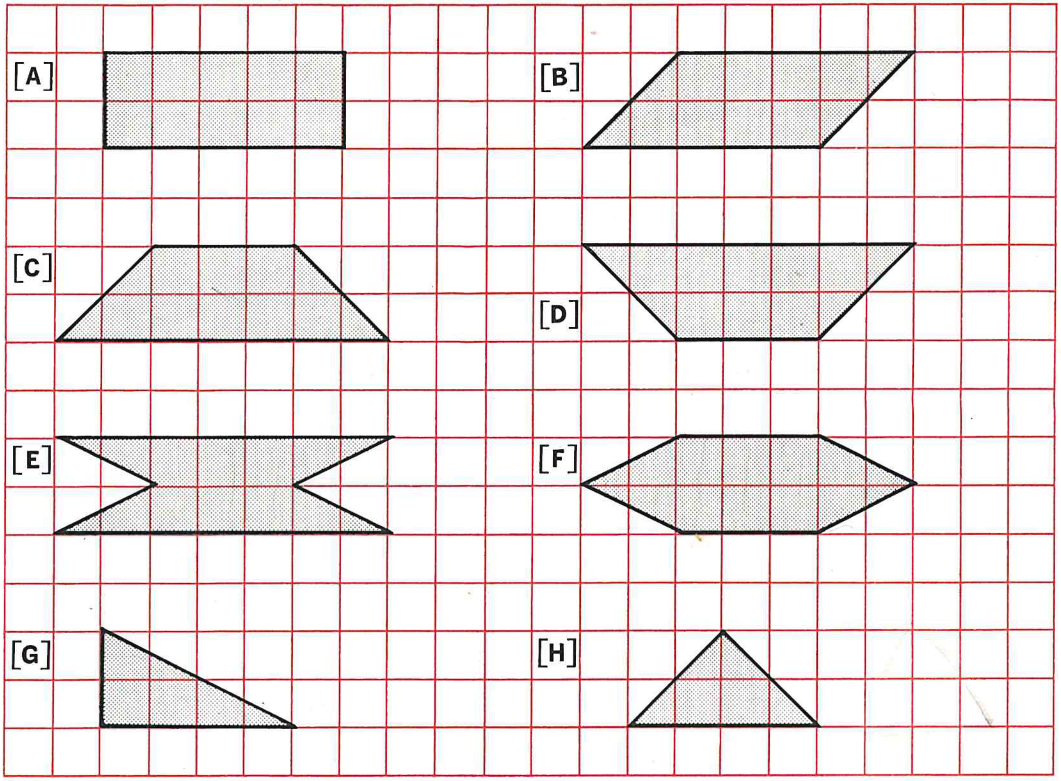
[C]



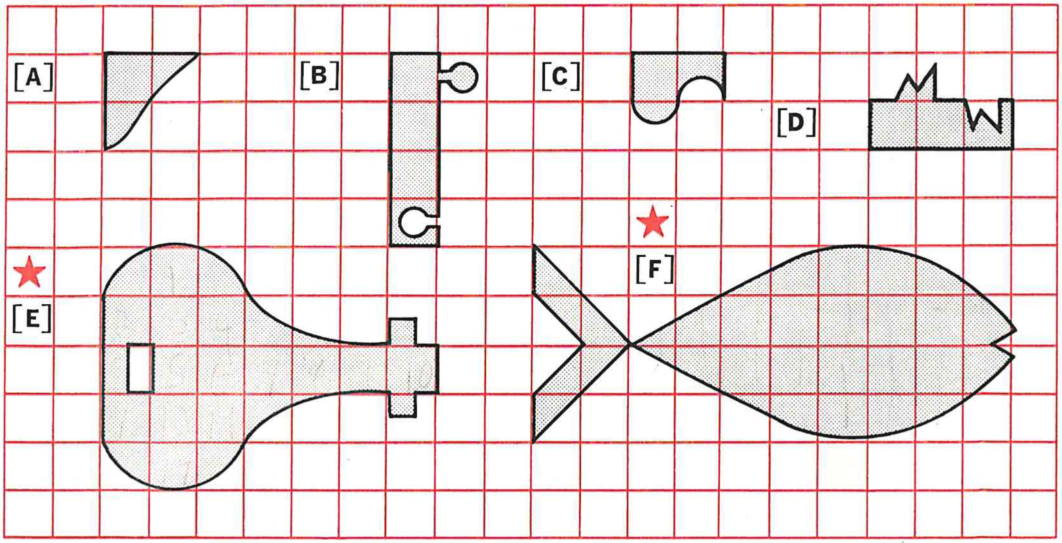
[D]



2. Find the number of square units (area) in each shaded region.



3. Estimate the area of each region.



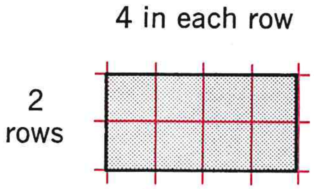
Area—rectangles

These exercises may help you discover a short cut for finding the area of a rectangle.

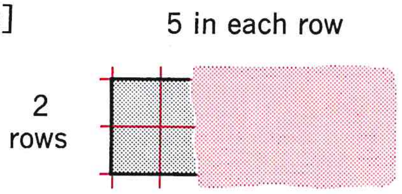
EXERCISES

1. This —— is the segment unit, and this ▨ is the square unit for the exercise. Give the area of each rectangle.

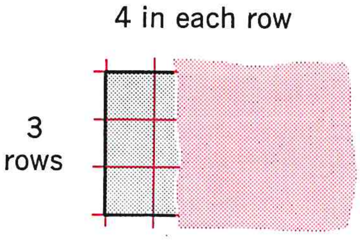
[A]



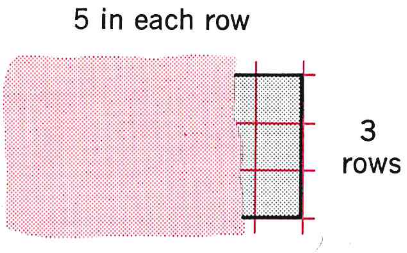
[B]



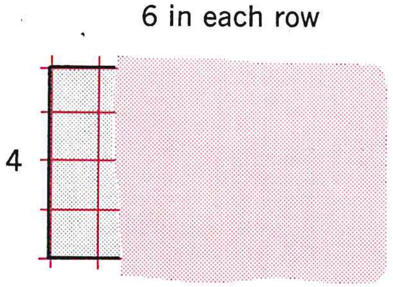
[C]



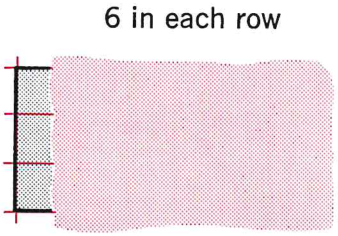
[D]



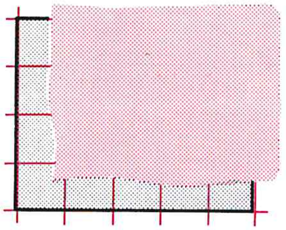
[E]



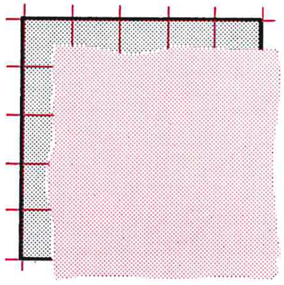
[F]



[G]

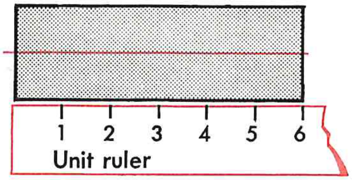


[H]

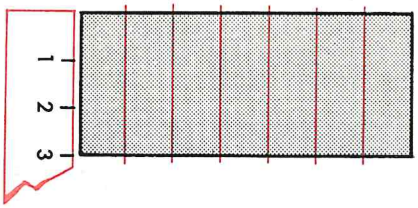


2. Give the area of each rectangle.

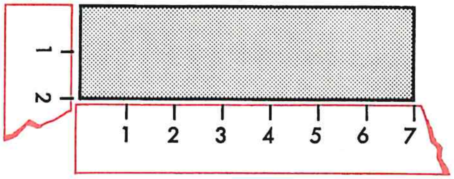
[A]



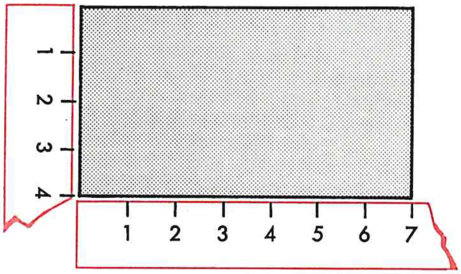
[B]



[C]

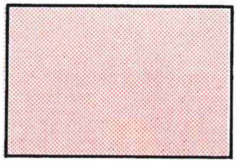


[D]

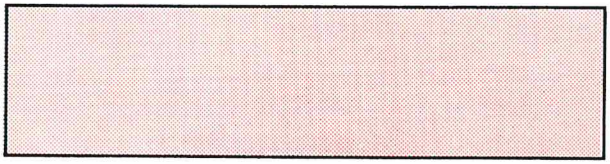


3. Use your centimeter ruler to find the area of each region.

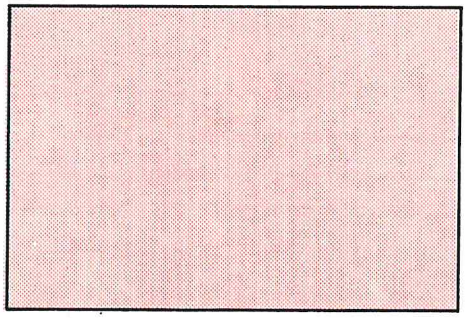
[A]



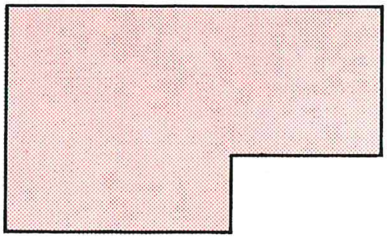
[B]



[C]

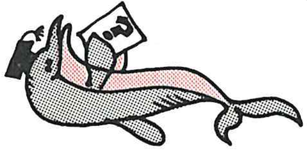


[E]



Think

I'm the smallest number that
Will get you less than fifty
If you want to take away
From seven more than sixty.

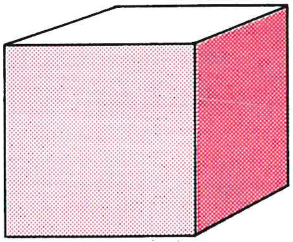


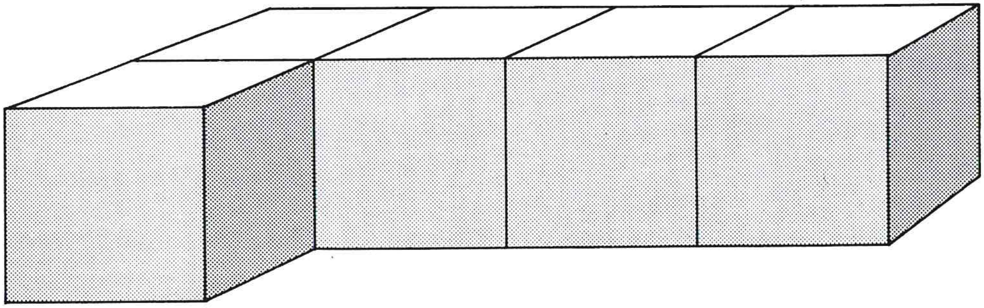
WHO AM I?

Finding volume

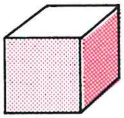
We can count cubic units to find volume.

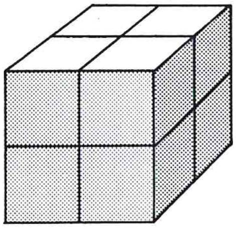
When the cubic inch → is the unit, the volume of this figure is 5.



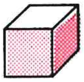


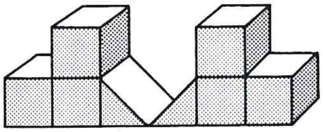
When the cubic centimeter → is the unit, the volume of this figure is 8.



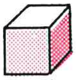


When this → is the unit, the volume of this figure is 7.



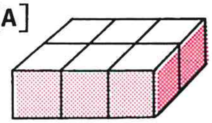


EXERCISES

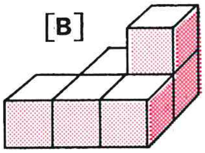
This  is the unit for these exercises.

1. Find the number of cubic units (volume) for each figure.

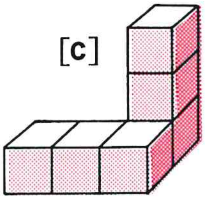
[A]



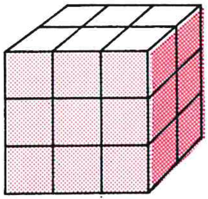
[B]



[C]

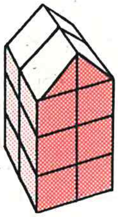


[D]

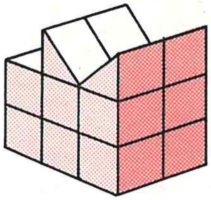


2. Find the number of cubic units (volume) for each figure.

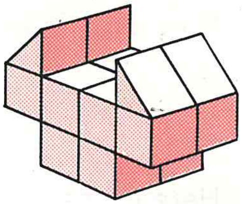
[A]



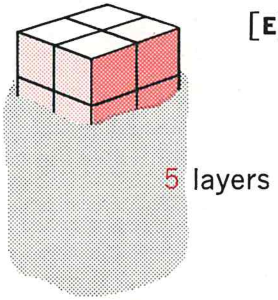
[B]



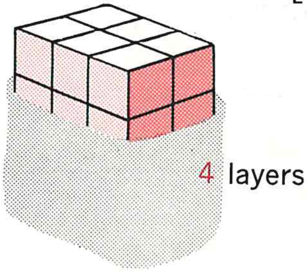
[C]



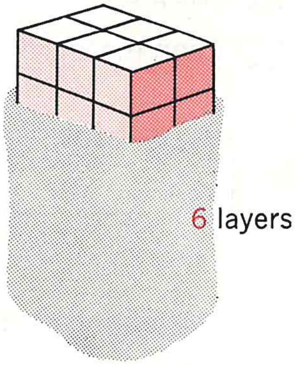
[D]



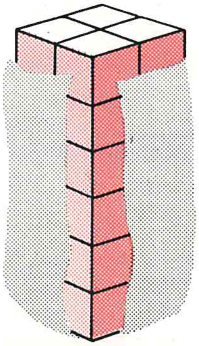
[E]



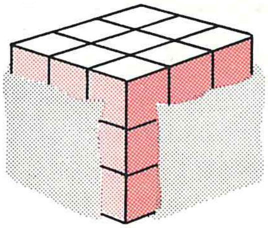
[F]



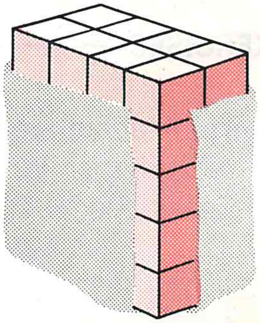
[G]



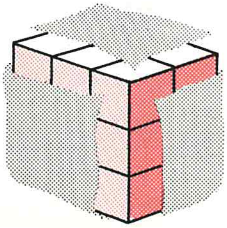
[H]



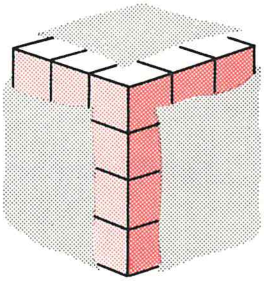
[I]



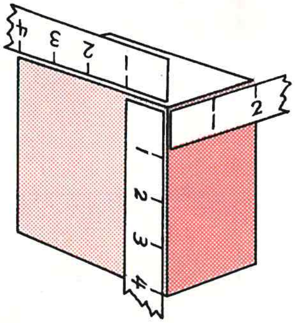
[J]



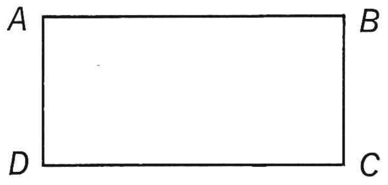
[K]



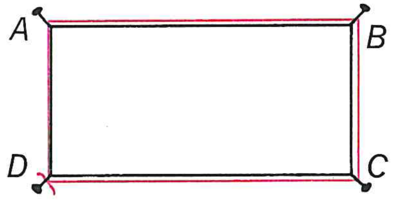
★ **[L]**



Estimating perimeter



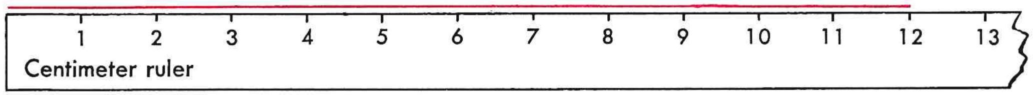
How far is it around rectangle *ABCD*?



Here is one way to estimate this distance.

① Wrap a string around the rectangle.

② Then measure the string.

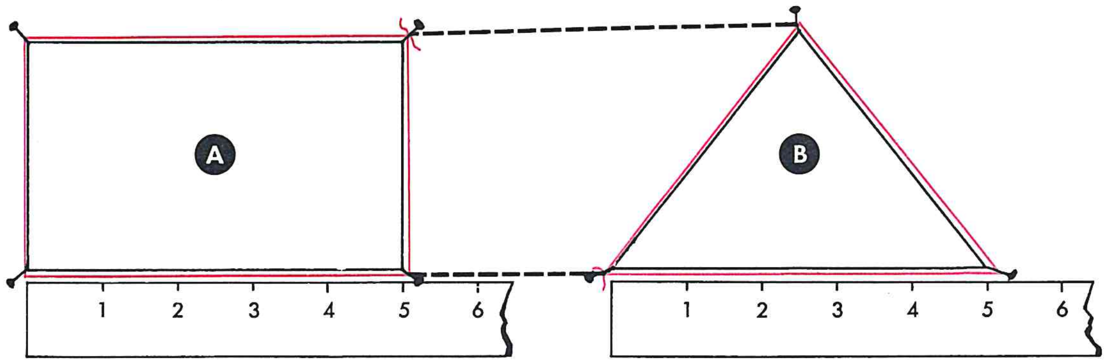


When the centimeter is the unit, we say:

The perimeter of the rectangle is about 12.

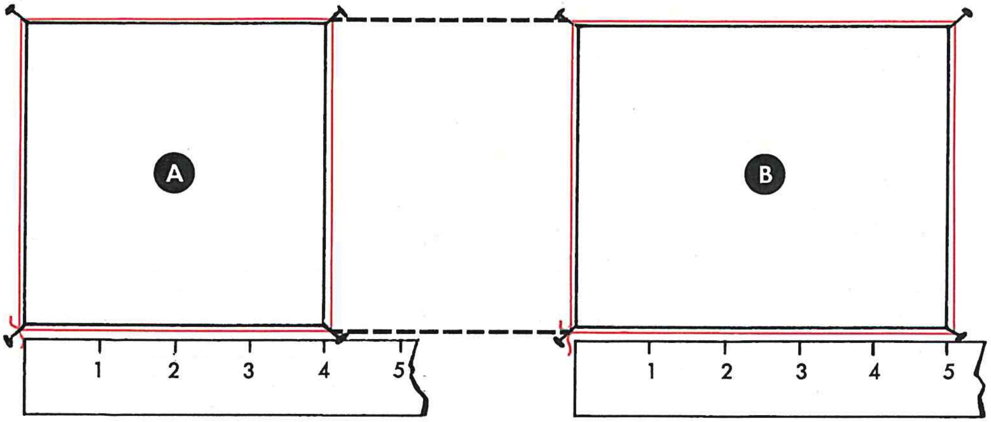
EXERCISES

1. Think about measuring the red string that is wrapped around each figure. Without measuring, tell which figure, A or B, has the greater perimeter.

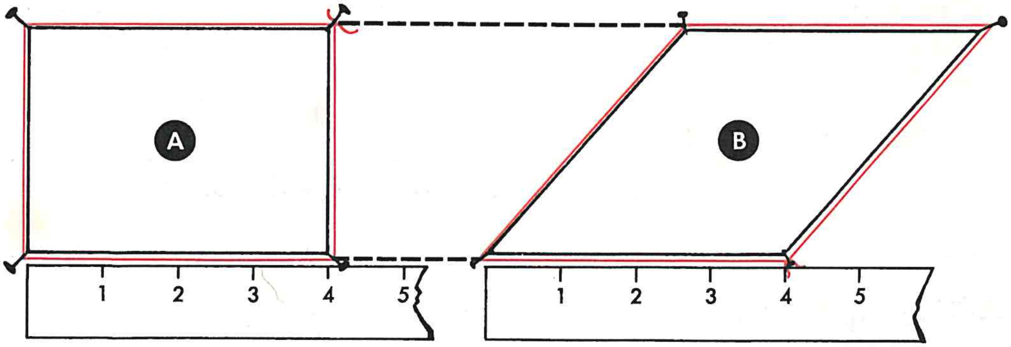


For exercises 2 and 3, without measuring, tell which figure, A or B, has the greater perimeter.

2.

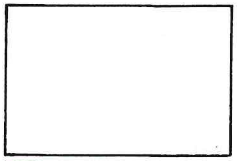


3.



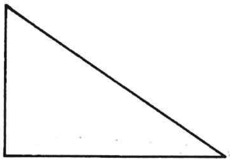
For exercises 4 through 7, use only the ruler shown to choose the best estimate for the perimeter of each figure.

4.

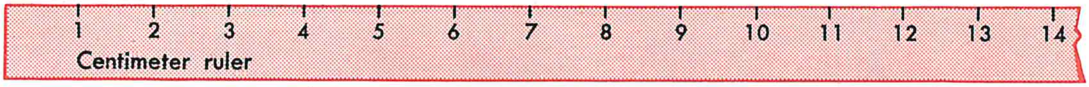


[A] 6
[B] 10
[C] 15

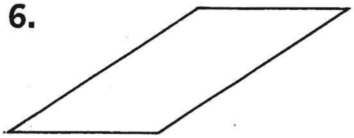
5.



[A] 5
[B] 6
[C] 9

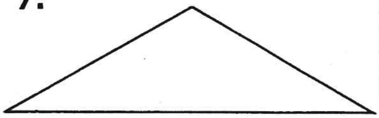


6.

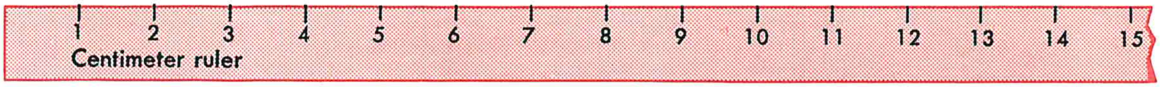


[A] 15
[B] 5
[C] 10

7.

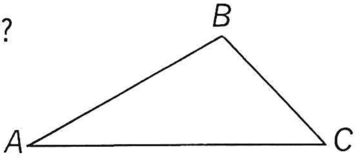


[A] 11
[B] 15
[C] 8

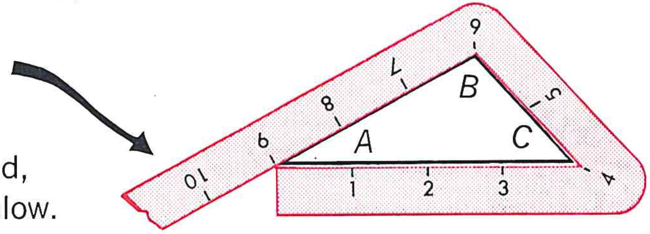


Measuring to find perimeter

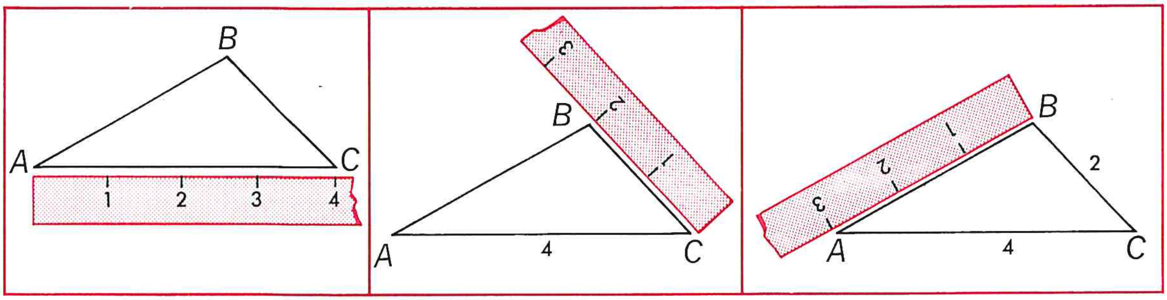
How far is it around triangle *ABC*?



Instead of string, rulers are usually used to find perimeter. We could find the perimeter of triangle *ABC* easily if we could bend our ruler like this.



Since most rulers do not bend, we find perimeter as shown below.

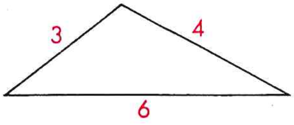


The perimeter of triangle *ABC* is 4 + 2 + 3, or 9.

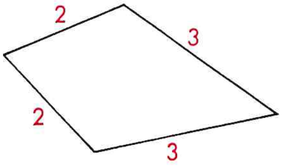
EXERCISES

1. The length of each side of the figure is given. Find the perimeter of each figure.

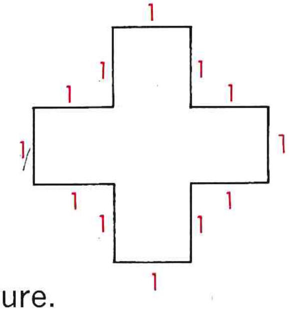
[A]



[B]

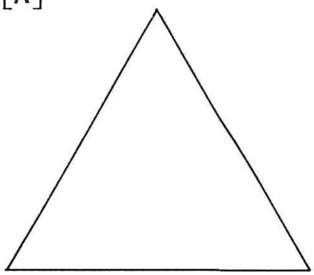


[C]

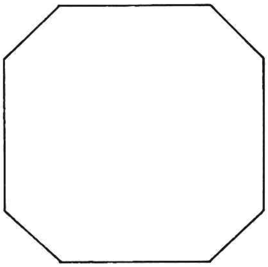


2. Use a centimeter ruler to find the perimeter of each figure.

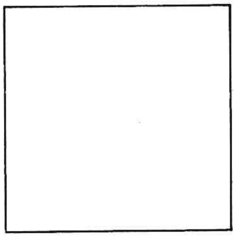
[A]



[B]

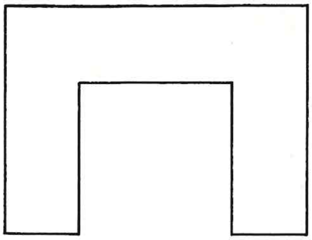


[C]

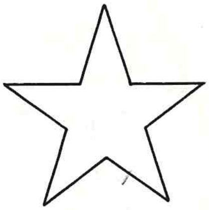


3. Use a centimeter ruler to find the perimeter of each figure.

[A]

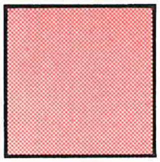


[B]

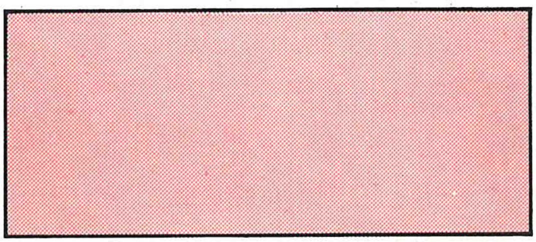


4. Use your centimeter ruler to find the perimeter and area of each rectangle. Tell which is the larger number, the number you found for the area or the number you found for the perimeter.

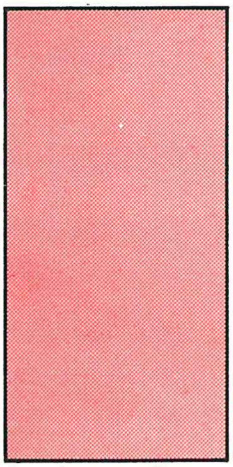
[A]



[B]

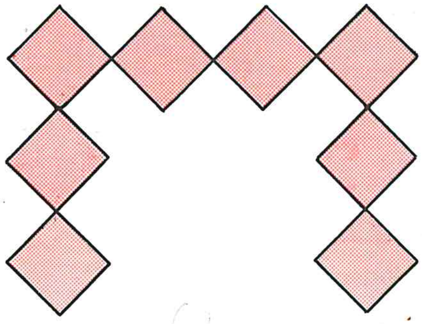


[C]



★**5.** Draw a square so that the number for the area is the same as the number for the perimeter.

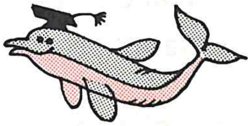
★**6.** [A] What is the perimeter of this figure formed with 8 squares?



[B] What is the smallest perimeter you can get by rearranging these 8 squares? Draw a figure to show your answer.

Think

Divide a number by itself
If zero's not its name.
I'm the quotient every time.
It always is the same.

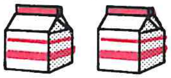


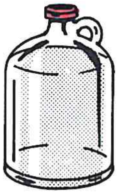
WHO AM I?

Liquid measure

It takes 8 ounces (16 tablespoons) to fill 1 cup .

It takes 2 cups to fill 1 pint .

It takes 2 pints  to fill 1 quart .

It takes 4 quarts to fill 1 gallon  .

EXERCISES

1. Give the word [ounce(s), cup(s), pint(s), quart(s), gallon(s)] that best completes each sentence.

   [A] A glass holds 1 _?_ of milk.
   [B] A tablespoon holds $\frac{1}{2}$ _?_ of soup.
   [C] A car's tank holds 18 _?_ of gasoline.
   [D] A _?_ of juice is more than a cup but less than a quart of juice.
   [E] A boy who drinks 4 cups of cocoa drinks a _?_ of cocoa.
   [F] A 16-ounce bottle holds 1 _?_ of soda.
   [G] An engine that uses 4 quarts of oil uses 1 _?_ of oil.
   [H] A pitcher that holds 32 ounces of tea holds 1 _?_ of tea.

2. Give the missing numbers.

   [A] It takes ▥ quarts to fill 2 gallons.
   [B] 1 quart of juice is enough for ▥ cups.
   [C] 1 pint of cough syrup fills ▥ tablespoons.
   [D] 1 gallon and 2 pints fill ▥ quart cartons.
   [E] 1 quart and 2 cups fill ▥ pint jars.

★3. The volume of a gallon container is 231 cubic inches. Estimate (in cubic inches) the volume of a 1-quart milk carton.

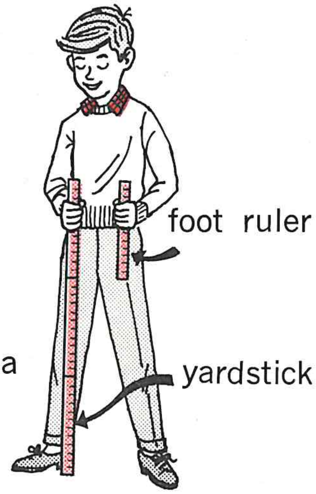
Inches, feet, yards, and miles

Ⓐ 1 inch

Ⓑ A foot is 12 inches long.

Ⓒ A yard is 3 feet long.

Ⓓ A mile is 1760 yards long.



You probably know the lengths of the inch, foot, and yard. These statements may give you a better idea about the length of a mile.

① A train with 120 cars is about a mile long.

② 10 city blocks are about a mile long.

③ 18 football fields placed end-to-end are about a mile long.

④ When you run around a school track 4 times, you have run 1 mile.

EXERCISES

1. Give the word [inch(es), foot(feet), yard(s), mile(s)] that best completes each sentence.
   - [A] A pencil is about 6 __?__ long.
   - [B] A man is about 6 __?__ tall.
   - [C] A basketball hoop is 10 __?__ high.
   - [D] A large airplane flying 2 __?__ high looks small from the ground.
   - [E] A football field is 100 __?__ long.
   - [F] You could walk a __?__ in 25 minutes.
   - [G] A dining table is about 30 __?__ high.
   - [H] A bicycle is about 2 __?__ long.

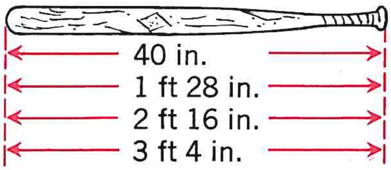
2. Give the missing numbers.
   - [A] 1 foot 6 inches is ▥ inches.
   - [B] 1 yard 2 feet is ▥ feet.
   - [C] 9 feet is ▥ yards.
   - [D] 2 feet 3 inches is ▥ inches.
   - ★[E] 1 yard is ▥ inches.
   - ★[F] 1 mile is ▥ feet.
   - ★[G] 1 square yard is ▥ square feet.
   - ★[H] 1 cubic yard is ▥ cubic feet.

Converting units

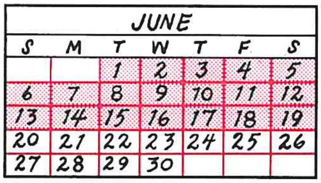
DISCUSSION EXERCISES

1. [A] Explain why both 1 ft 28 in. and 40 in. can be used to give the length of the bat.

   [B] Explain why both 2 ft 16 in. and 3 ft 4 in. can be used to give the length of the bat.



2. Explain these ways of measuring 19 days.



JUNE						
S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

19 days
1 week 12 days
2 weeks 5 days

★ 3. Explain the steps shown in combining two measures.

$$\begin{array}{r} 1\text{ ft}\ \ 9\text{ in.}\\ 1\text{ ft}\ \ 7\text{ in.}\\ \hline 2\text{ ft } 16\text{ in.}\end{array} \longrightarrow 3\text{ ft } 4\text{ in.}$$

★ 4. Explain the steps in finding the difference of two measures.

$$\begin{array}{r} 3\text{ ft}\ \ \ 4\text{ in.}\\ 1\text{ ft } 10\text{ in.}\\ \hline ?\end{array} \longrightarrow \begin{array}{r} 2\text{ ft } 16\text{ in.}\\ 1\text{ ft } 10\text{ in.}\\ \hline 1\text{ ft}\ \ \ 6\text{ in.}\end{array}$$

EXERCISES

1. Copy the exercises below on your paper, and give as many of the missing numbers as you can. Look in the Table of Measures at the back of the book to find the others.

[A] 1 yd = ▥ ft
[B] 1 ft = ▥ in.
[C] 1 day = ▥ hr
[D] 1 hr = ▥ min
[E] 1 min = ▥ sec

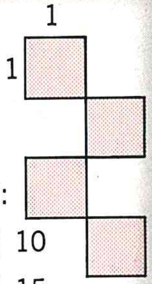
[F] 1 lb = ▥ oz
[G] 1 gal = ▥ qt
[H] 1 qt = ▥ pt
[I] 1 pt = ▥ cups
[J] 1 wk = ▥ days

[K] 1 bu = ▥ pk
[L] 1 pk = ▥ qt
[M] 1 mi = ▥ ft
[N] 1 mi = ▥ yd
[O] 1 ton = ▥ lb

2. Give the missing numbers.

[A] 1 qt 3 pt = 2 qt ▥ pt
[B] 3 ft 13 in. = ▥ ft 1 in.
[C] 2 hr 75 min = 3 hr ▥ min
[D] 15 min 90 sec = ▥ min 30 sec
[E] 4 gal 4 qt = ▥ gal 0 qt
[F] 0 lb 20 oz = 1 lb ▥ oz
[G] 6 pt ▥ cups = 7 pt 1 cup
[H] 3 days 30 hr = 4 days ▥ hr

Think



The perimeter of this figure is 16. Arrange the squares to get a perimeter of:

[A] 14 [B] 12 [C] 10
[D] 8 [E] 9 [F] 15

★3. Change each of these measures so that you have the greatest number of the larger unit. (Example: 3 weeks 10 days ⟶ 4 weeks 3 days)

[A] 2 pt 3 cups
[B] 5 gal 7 qt
[C] 3 lb 20 oz
[D] 6 yd 17 ft
[E] 3 lb 30 oz
[F] 8 min 80 sec
[G] 3 hr 62 min
[H] 0 qt 2 pt
[I] 3 bu 8 pk
[J] 9 ft 27 in.
[K] 5 qt 6 pt
[L] 8 pk 9 qt
[M] 7 wk 21 days
[N] 5 days 25 hr
[O] 6 pt 40 cups
[P] 7 qt 9 cups

★4. Find the totals. Change your answer as you did in exercise 3.

[A] 2 lb 7 oz
3 lb 14 oz

[B] 1 qt 1 pt
2 qt 1 pt

[C] 5 yd 2 ft
6 yd 2 ft

[D] 7 bu 3 pk
9 bu 7 pk

[E] 5 days 10 hr
2 days 14 hr

[F] 3 hr 50 min
4 hr 30 min

[G] 2 hr 30 min 40 sec
1 hr 40 min 35 sec

[H] 3 wk 6 days 12 hr
4 wk 5 days 18 hr

[I] 7 gal 1 qt 1 pt
1 gal 3 qt 1 pt

★5. Find the differences. You may have to change the top measure as in the example.

Example:

8 lb 7 oz
5 lb 9 oz
⟶
7 lb 23 oz
5 lb 9 oz
2 lb 14 oz

[A] 6 days 7 hr
1 day 13 hr

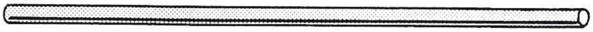
[B] 4 ft 3 in.
1 ft 10 in.

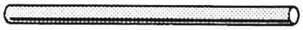
[C] 7 min 14 sec
6 min 52 sec

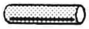
[D] 10 wk 2 days
3 wk 5 days

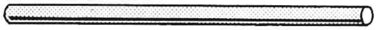
Chapter review

1. If rod **A** is 8 units long, how long are rods **B**, **C**, and **D**?

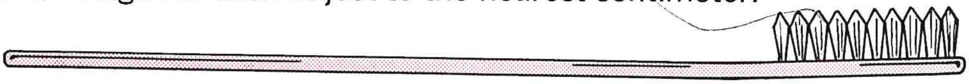
[A] 

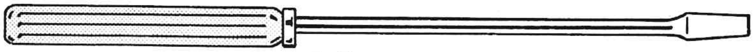
[B] 

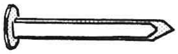
[C] 

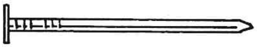
[D] 

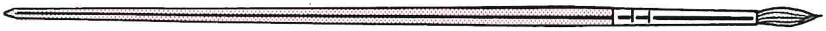
2. Give the length of each object to the nearest centimeter.

[A] 

[B] 

[C] 

[D] 

[E] 

3. Give the length of each object in exercise 2 to the nearest inch.

4. Dan chose an inch as his unit. Neil decided to use one fourth of an inch as his unit. Since it was a quarter inch, Neil called it a "quinch."

[A] The length of an eraser is 2 inches. How many "quinches" long is the eraser?

[B] Dan used inches, and Neil used "quinches" to measure the outer edge of this page. Without measuring, tell who got the larger number.

[C] How many inches long is the outer edge of this page?

[D] How many "quinches" would this be?

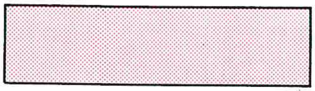
5. Copy each sentence and write inch(es) or centimeter(s) in the proper blanks.

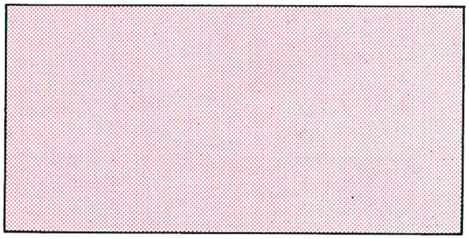
[A] A segment 4 _?_ long is greater than a segment 4 _?_ long.

[B] A segment 6 _?_ long is about the same length as a segment 15 _?_ long.

[C] It takes about $2\frac{1}{2}$ _?_ to make 1 _?_.

6. Find the area of each rectangle. Use the square centimeter as your unit.

[A] 

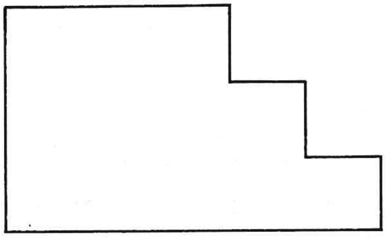
[B] 

7. Which is greater, 1860 yards or 1 mile? How much greater?

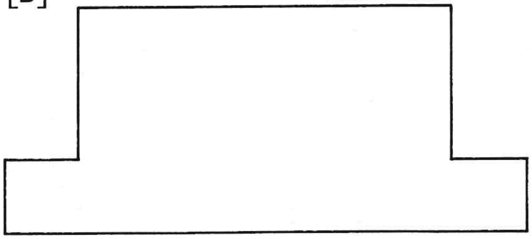
8. Would you find length, area, volume, or perimeter to find
   [A] how much space there is inside a refrigerator?
   [B] how far you go when you walk around one city block?

9. Find the perimeter of each figure. Use your centimeter ruler.

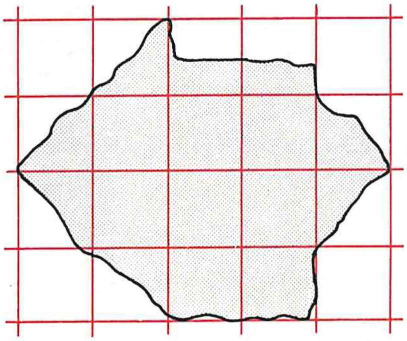
[A]



[B]

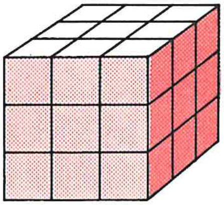


10. Estimate the area of this lake.
1 square centimeter on the map means
1 square mile of the lake.

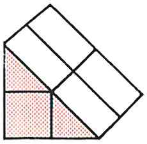


11. Give the volume of each figure.

[A]



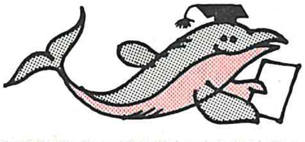
[B]



Think

1. Find the smallest 4-digit number that has 3 as one of its digits.

2. Find the largest 4-digit number that has the digits 1 and 7 with no two digits alike.

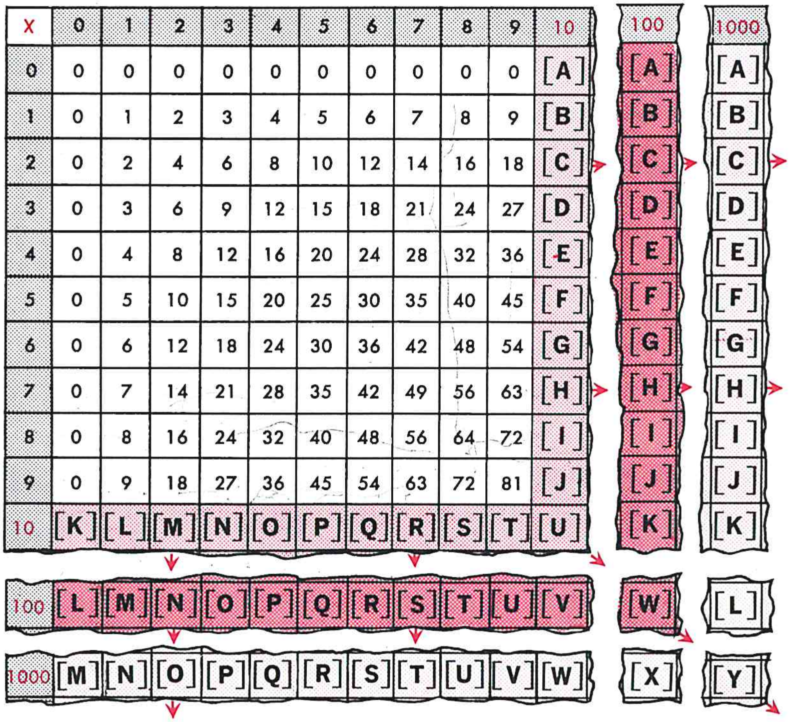


CHAPTER 5 Special products and quotients

Multiples of 10, 100, and 1000

You may think of the multiplication facts reviewed in Chapter 3 as a small part of a giant multiplication table that goes on and on.

The numbers in the 10 row and column of this table are called **multiples** of 10. The multiples of 100 and 1000 are in the 100 and 1000 rows and columns.



×	0	1	2	3	4	5	6	7	8	9	10	100	1000
0	0	0	0	0	0	0	0	0	0	0	[A]	[A]	[A]
1	0	1	2	3	4	5	6	7	8	9	[B]	[B]	[B]
2	0	2	4	6	8	10	12	14	16	18	[C]	[C]	[C]
3	0	3	6	9	12	15	18	21	24	27	[D]	[D]	[D]
4	0	4	8	12	16	20	24	28	32	36	[E]	[E]	[E]
5	0	5	10	15	20	25	30	35	40	45	[F]	[F]	[F]
6	0	6	12	18	24	30	36	42	48	54	[G]	[G]	[G]
7	0	7	14	21	28	35	42	49	56	63	[H]	[H]	[H]
8	0	8	16	24	32	40	48	56	64	72	[I]	[I]	[I]
9	0	9	18	27	36	45	54	63	72	81	[J]	[J]	[J]
10	[K]	[L]	[M]	[N]	[O]	[P]	[Q]	[R]	[S]	[T]	[U]	[K]	[K]
100	[L]	[M]	[N]	[O]	[P]	[Q]	[R]	[S]	[T]	[U]	[V]	[W]	[L]
1000	[M]	[N]	[O]	[P]	[Q]	[R]	[S]	[T]	[U]	[V]	[W]	[X]	[Y]

EXERCISES

1. Use the table to give the products in the

[A] 10 column and row. [B] 100 column and row. [C] 1000 column and row.

2. Other products in the 10 column are indicated below.

Solve the equations to find these products.

[A] 30 × 10 = 3 × 10 × 10 = n

[B] 40 × 10 = 4 × 10 × 10 = n

[C] 50 × 10 = 5 × 10 × 10 = n

[D] 60 × 10 = 6 × 10 × 10 = n

[E] 70 × 10 = 7 × 10 × 10 = n

[F] 80 × 10 = 8 × 10 × 10 = n

3. Solve the equations.

[A] 12 × 10 = (10 × 10) + (2 × 10) = n

[B] 29 × 10 = (20 × 10) + (9 × 10) = n

[C] 47 × 10 = (40 × 10) + (7 × 10) = n

[D] 93 × 10 = (90 × 10) + (3 × 10) = n

Jim made this poster to tell about a short cut he discovered for multiplying by 10 and 100.

To write the **product** of a number and 10, just write this number in the **tens place**.

$8 \times 10 = \overset{\text{TENS}}{8}0$

$30 \times 10 = \overset{\text{TENS}}{30}0$

$97 \times 10 = \overset{\text{TENS}}{97}0$

$365 \times 10 = \overset{\text{TENS}}{365}0$

To write the **product** of a number and 100 just write this number in the **hundreds place.**

$6 \times 100 = \overset{\text{HUNDREDS}}{6}00$

$87 \times 100 = \overset{\text{HUNDREDS}}{87}00$

DISCUSSION EXERCISE

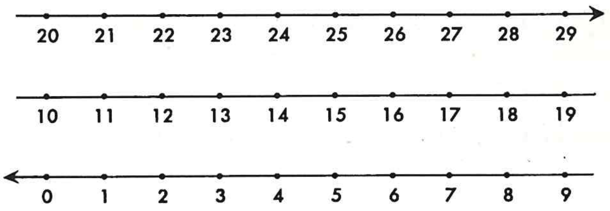
Study the short cut and describe a quick way to multiply a number by 1000.

EXERCISES

1. Find the products.

[A] 32 × 10
[B] 54 × 10
[C] 70 × 10
[D] 46 × 10
[E] 65 × 10
[F] 10 × 97
[G] 86 × 10
[H] 10 × 23
[I] 69 × 10
[J] 324 × 10
[K] 700 × 10
[L] 32 × 100
[M] 40 × 100
[N] 100 × 79
[O] 93 × 1000
[P] 68 × 1000

★**2.** Give the landing point for each jump described below. Imagine as many additional rows in the number-line picture as you need to work each exercise.



[A] 0 $\overset{\uparrow}{\textcircled{1}}$
[B] 0 $\overset{\uparrow}{\textcircled{3}}$
[C] 0 $\overset{\uparrow}{\textcircled{8}}$
[D] 0 $\overset{\uparrow}{\textcircled{9}}$
[E] 0 $\overset{\uparrow}{\textcircled{10}}$
[F] 0 $\overset{\uparrow}{\textcircled{30}}$
[G] 0 $\overset{\uparrow}{\textcircled{70}}$
[H] 0 $\overset{\uparrow}{\textcircled{83}}$
[I] 1 $\overset{\uparrow}{\textcircled{6}}$
[J] 5 $\overset{\uparrow}{\textcircled{7}}$
[K] 32 $\overset{\uparrow}{\textcircled{5}}$
[L] 412 $\overset{\uparrow}{\textcircled{8}}$
[M] 676 $\underset{\downarrow}{\textcircled{7}}$
[N] 71 $\underset{\downarrow}{\textcircled{7}}$
[O] 99 $\underset{\downarrow}{\textcircled{9}}$
[P] 25 $\underset{\downarrow}{\textcircled{2}}$ $\overset{\uparrow}{\textcircled{12}}$
[Q] 9 ${}^{\nwarrow}\textcircled{1}$
[R] 9 ${}^{\nwarrow}\textcircled{2}$
[S] 9 ${}^{\nwarrow}\textcircled{9}$
[T] 9 ${}^{\nwarrow}\textcircled{9}$ $\overset{\uparrow}{\textcircled{9}}$
[U] 0 $\textcircled{1}^{\nearrow}$
[V] 0 $\textcircled{2}^{\nearrow}$
[W] 0 $\textcircled{8}^{\nearrow}$
[X] 0 $\textcircled{8}^{\nearrow}$ $\overset{\uparrow}{\textcircled{8}}$
[Y] 9 ${}^{\nwarrow}\textcircled{10}$ $\overset{\uparrow}{\textcircled{10}}$

Using the grouping principle to find products

For three numbers in a certain order, there are two choices of which factors we could multiply first.

A We could multiply these first.

$$\overset{\searrow}{9} \times \overset{\searrow}{4} \times 10$$

B We could multiply these first.

$$9 \times \overset{\swarrow}{4} \times \overset{\swarrow}{10}$$

$$(9 \times 4) \times 10 = 9 \times (4 \times 10)$$

When we multiply, we can change the **grouping** and get the same product.
We call this principle the **grouping principle.**

DISCUSSION EXERCISES

1. Explain step 1.
2. Explain how the grouping principle is used to get from step 1 to step 2.
3. Explain steps 3 and 4.
4. Solve: $9 \times 40 = \boxed{n}$

1150

9×40

① $9 \times (4 \times 10)$

② $(9 \times 4) \times 10$

③ 36×10

④ 360

EXERCISES

1. Solve the equations.

[A] $3 \times 20 = 3 \times 2 \times 10 = \boxed{n}$
[B] $5 \times 30 = 5 \times 3 \times 10 = \boxed{n}$
[C] $8 \times 40 = 8 \times 4 \times 10 = \boxed{n}$
[D] $9 \times 50 = 9 \times 5 \times 10 = \boxed{n}$
[E] $7 \times 60 = 7 \times 6 \times 10 = \boxed{n}$
[F] $4 \times 70 = 4 \times 7 \times 10 = \boxed{n}$
[G] $7 \times 80 = 7 \times 8 \times 10 = \boxed{n}$
[H] $6 \times 90 = 6 \times 9 \times 10 = \boxed{n}$
[I] $5 \times 70 = 5 \times 7 \times 10 = \boxed{n}$
[J] $9 \times 70 = 9 \times 7 \times 10 = \boxed{n}$
[K] $6 \times 40 = 6 \times 4 \times 10 = \boxed{n}$
[L] $5 \times 60 = 5 \times 6 \times 10 = \boxed{n}$
[M] $9 \times 80 = 9 \times 8 \times 10 = \boxed{n}$
[N] $8 \times 90 = 8 \times 9 \times 10 = \boxed{n}$
[O] $6 \times 80 = 6 \times 8 \times 10 = \boxed{n}$
[P] $9 \times 60 = 9 \times 6 \times 10 = \boxed{n}$
[Q] $3 \times 400 = 3 \times 4 \times 100 = \boxed{n}$
[R] $8 \times 600 = 8 \times 6 \times 100 = \boxed{n}$
[S] $5 \times 700 = 5 \times 7 \times 100 = \boxed{n}$
[T] $9 \times 8000 = 9 \times 8 \times 1000 = \boxed{n}$

Special products

1. Find the products.

[A] 3×6
[B] 3×60
[C] 5×3
[D] 5×30
[E] 5×300
[F] 4×5
[G] 4×50
[H] 5×7
[I] 5×70
[J] 5×700
[K] 6×4
[L] 6×40
[M] 3×8
[N] 3×80
[O] 3×800
[P] 7×8
[Q] 7×80
[R] 9×3
[S] 9×30
[T] 9×300
[U] 8×5
[V] 8×50
[W] 9×7
[X] 9×70
[Y] 9×700

2. Find the products.

[A] 4×30
[B] 5×60
[C] 7×70
[D] 6×80
[E] 9×50
[F] 3×70
[G] 8×70
[H] 6×90
[I] 4×40
[J] 6×60
[K] 7×90
[L] 9×90
[M] 4×200
[N] 6×600
[O] 8×800
[P] 9×700
[Q] 7×600
[R] 5×500
[S] 3×900
[T] 8×500

3. Find the products.

[A] 7×50
[B] 3×900
[C] 6×300
[D] 4×900
[E] 7×600
[F] 9×800
[G] 8×200
[H] 2×900
[I] 9×700
[J] 400×5
[K] 700×7
[L] 600×9
[M] 7000×4
[N] 4000×8
[O] 5000×6
[P] 9000×7

★**4.** Solve the equations.

[A] $6 \times 90 = \boxed{n}$
[B] $\boxed{n} \times 80 = 320$
[C] $\boxed{n} \times 70 = 350$
[D] $\boxed{n} \times 9 = 630$
[E] $\boxed{n} \times 8 = 560$
[F] $\boxed{n} \times 5 = 400$
[G] $30 \times \boxed{n} = 180$
[H] $\boxed{n} = 7 \times 80$
[I] $6 \times \boxed{n} = 420$
[J] $\boxed{n} \times 40 = 360$
[K] $630 = \boxed{n} \times 70$
[L] $540 = 6 \times \boxed{n}$
[M] $\boxed{n} \times 800 = 3200$
[N] $\boxed{n} \times 900 = 6300$
[O] $9 \times \boxed{n} = 5400$
[P] $600 \times \boxed{n} = 4200$
[Q] $7200 = 8 \times \boxed{n}$
[R] $\boxed{n} \times 500 = 4500$
[S] $9 \times \boxed{n} = 8100$
[T] $\boxed{n} \times 8000 = 48{,}000$

Think

Finding nine times forty
Is simple as can be.
You first find nine times four.
Then multiply by me.

WHO AM I?

Missing factors

EXERCISES

1. Find the products.

[A] 8×10
[B] 13×10
[C] 60×9
[D] 10×10
[E] 50×8
[F] 60×10
[G] 9×80
[H] 100×10
[I] 47×10
[J] 10×65
[K] 10×93
[L] 132×10
[M] 10×96
[N] 0×10
[O] 831×10
[P] 10×729

2. Find the products.

[A] 20×9
[B] 5×30
[C] 4×70
[D] 10×35
[E] 7×90
[F] 21×10
[G] 3×700
[H] 6×500
[I] 9×400
[J] 80×4
[K] 34×10
[L] 8×700
[M] 456×100
[N] 100×37
[O] 100×42
[P] 54×100
[Q] 5000×6
[R] 9×4000

Think

To multiply by me
Is as easy as can be.
That it's hard, I'll not
  pretend.
Just put zero on the end.

WHO AM I?

3. Solve the equations.

[A] $n \times 3 = 21$
[B] $n \times 3 = 210$
[C] $n \times 7 = 28$
[D] $n \times 7 = 280$
[E] $n \times 5 = 30$
[F] $n \times 5 = 300$
[G] $n \times 4 = 24$
[H] $n \times 4 = 240$
[I] $n \times 4 = 2400$
[J] $n \times 6 = 42$
[K] $n \times 6 = 420$
[L] $n \times 6 = 4200$
[M] $n \times 6 = 54$
[N] $n \times 6 = 540$
[O] $n \times 6 = 5400$
[P] $n \times 9 = 18$
[Q] $n \times 9 = 180$
[R] $n \times 9 = 1800$

4. Solve the equations.

[A] $n \times 3 = 240$
[B] $n \times 6 = 360$
[C] $n \times 2 = 160$
[D] $n \times 5 = 200$
[E] $n \times 7 = 210$
[F] $n \times 4 = 320$
[G] $n \times 8 = 400$
[H] $n \times 2 = 60$
[I] $n \times 4 = 120$
[J] $n \times 9 = 270$
[K] $n \times 5 = 450$
[L] $n \times 7 = 420$
[M] $n \times 3 = 2400$
[N] $n \times 4 = 2800$
[O] $n \times 2 = 1600$
[P] $n \times 5 = 2000$
[Q] $n \times 7 = 4900$
[R] $n \times 6 = 3000$

5. Solve the equations.

[A] $\boxed{n} \times 3 = 24$
[B] $\boxed{n} \times 30 = 240$
[C] $\boxed{n} \times 5 = 20$
[D] $\boxed{n} \times 50 = 200$
[E] $\boxed{n} \times 4 = 32$
[F] $\boxed{n} \times 40 = 320$
[G] $\boxed{n} \times 2 = 16$
[H] $\boxed{n} \times 20 = 160$
[I] $\boxed{n} \times 200 = 1600$
[J] $\boxed{n} \times 4 = 28$
[K] $\boxed{n} \times 40 = 280$
[L] $\boxed{n} \times 40 = 2800$
[M] $\boxed{n} \times 6 = 24$
[N] $\boxed{n} \times 60 = 240$
[O] $\boxed{n} \times 600 = 2400$
[P] $\boxed{n} \times 8 = 48$
[Q] $\boxed{n} \times 80 = 480$
[R] $\boxed{n} \times 800 = 4800$

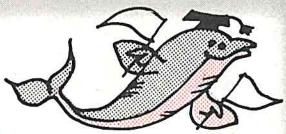
6. Solve the equations.

[A] $\boxed{n} \times 30 = 180$
[B] $\boxed{n} \times 20 = 100$
[C] $\boxed{n} \times 50 = 150$
[D] $\boxed{n} \times 40 = 280$
[E] $\boxed{n} \times 60 = 240$
[F] $\boxed{n} \times 80 = 640$
[G] $\boxed{n} \times 30 = 180$
[H] $\boxed{n} \times 40 = 160$
[I] $\boxed{n} \times 70 = 280$
[J] $\boxed{n} \times 60 = 420$
[K] $\boxed{n} \times 50 = 450$
[L] $\boxed{n} \times 90 = 720$
[M] $\boxed{n} \times 400 = 2800$
[N] $\boxed{n} \times 200 = 1200$
[O] $\boxed{n} \times 500 = 2500$
[P] $\boxed{n} \times 300 = 2100$
[Q] $\boxed{n} \times 600 = 3600$
[R] $\boxed{n} \times 800 = 2400$

7. Solve the equations.

[A] $\boxed{n} \times 4 = 320$
[B] $\boxed{n} \times 7 = 420$
[C] $\boxed{n} \times 20 = 140$
[D] $\boxed{n} \times 8 = 160$
[E] $\boxed{n} \times 400 = 800$
[F] $\boxed{n} \times 30 = 270$
[G] $\boxed{n} \times 600 = 3000$
[H] $\boxed{n} \times 50 = 350$
[I] $\boxed{n} \times 9 = 810$
[J] $\boxed{n} \times 700 = 4900$
[K] $\boxed{n} \times 800 = 5600$
[L] $\boxed{n} \times 3 = 150$
[M] $\boxed{n} \times 500 = 1000$
[N] $\boxed{n} \times 90 = 360$

Think



Let's agree that $\circledast$ between two numbers means:

Multiply the first number by itself and add the second to this product.

So, $2 \circledast 1 = 5$, $3 \circledast 2 = 11$, and $4 \circledast 2 = 18$.

1. Solve the equations.

[A] $2 \circledast 4 = \boxed{n}$
[B] $5 \circledast 1 = \boxed{n}$
[C] $6 \circledast 6 = \boxed{n}$
[D] $1 \circledast 1 = \boxed{n}$

2. Answer T (true) or F (false).

[A] $2 \circledast 5 = 5 \circledast 2$
[B] $4 \circledast 3 = 3 \circledast 4$
[C] $0 \circledast 4 = 4$
[D] $4 \circledast 0 = 0$
[E] $0 \circledast 1 = 1 \circledast 0$
[F] $0 \circledast 2 = 2 \circledast 0$

Special quotients

1. Solve the equations.

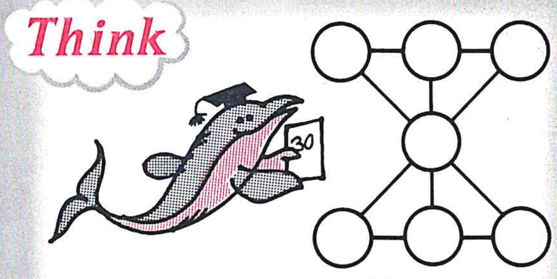
[A] $\boxed{n} \times 7 = 14$
[B] $\boxed{n} \times 7 = 140$
[C] $14 \div 7 = \boxed{n}$
[D] $140 \div 7 = \boxed{n}$
[E] $\boxed{n} \times 4 = 32$
[F] $\boxed{n} \times 4 = 320$
[G] $32 \div 4 = \boxed{n}$
[H] $320 \div 4 = \boxed{n}$
[I] $\boxed{n} \times 8 = 24$
[J] $\boxed{n} \times 8 = 240$
[K] $24 \div 8 = \boxed{n}$
[L] $240 \div 8 = \boxed{n}$
[M] $\boxed{n} \times 7 = 28$
[N] $\boxed{n} \times 7 = 280$
[O] $28 \div 7 = \boxed{n}$
[P] $280 \div 7 = \boxed{n}$
[Q] $\boxed{n} \times 9 = 27$
[R] $\boxed{n} \times 9 = 270$
[S] $27 \div 9 = \boxed{n}$
[T] $270 \div 9 = \boxed{n}$
[U] $\boxed{n} \times 6 = 24$
[V] $\boxed{n} \times 6 = 240$
[W] $24 \div 6 = \boxed{n}$
[X] $240 \div 6 = \boxed{n}$

2. Find the quotients.

[A] $21 \div 3 = \boxed{n}$
[B] $210 \div 3 = \boxed{n}$
[C] $14 \div 7 = \boxed{n}$
[D] $140 \div 7 = \boxed{n}$
[E] $24 \div 4 = \boxed{n}$
[F] $240 \div 4 = \boxed{n}$
[G] $40 \div 5 = \boxed{n}$
[H] $400 \div 5 = \boxed{n}$
[I] $48 \div 8 = \boxed{n}$
[J] $480 \div 8 = \boxed{n}$
[K] $36 \div 6 = \boxed{n}$
[L] $360 \div 6 = \boxed{n}$
[M] $27 \div 9 = \boxed{n}$
[N] $270 \div 9 = \boxed{n}$
[O] $49 \div 7 = \boxed{n}$
[P] $490 \div 7 = \boxed{n}$
[Q] $56 \div 8 = \boxed{n}$
[R] $560 \div 8 = \boxed{n}$

3. Find the quotients.

[A] $20 \div 5 = \boxed{n}$
[B] $200 \div 50 = \boxed{n}$
[C] $28 \div 4 = \boxed{n}$
[D] $280 \div 40 = \boxed{n}$
[E] $32 \div 8 = \boxed{n}$
[F] $320 \div 80 = \boxed{n}$
[G] $48 \div 6 = \boxed{n}$
[H] $480 \div 60 = \boxed{n}$
[I] $72 \div 9 = \boxed{n}$
[J] $720 \div 90 = \boxed{n}$



Draw a figure like this one. Put the numbers 7, 8, 9, 10, 11, 12, and 13 in the circles so that the sum of the numbers along any line is 30.

EXERCISES

1. Solve the equations.

[A] $\boxed{n} \times 7 = 140$

$140 \div 7 = \boxed{n}$

[B] $\boxed{n} \times 8 = 240$

$240 \div 8 = \boxed{n}$

[C] $\boxed{n} \times 4 = 320$

$320 \div 4 = \boxed{n}$

[D] $\boxed{n} \times 6 = 240$

$240 \div 6 = \boxed{n}$

[E] $\boxed{n} \times 9 = 540$

$540 \div 9 = \boxed{n}$

[F] $\boxed{n} \times 30 = 210$

$210 \div 30 = \boxed{n}$

[G] $\boxed{n} \times 70 = 280$

$280 \div 70 = \boxed{n}$

[H] $\boxed{n} \times 90 = 270$

$270 \div 90 = \boxed{n}$

2. Find the quotients. Think about the missing factors.

[A] $320 \div 8 = \boxed{n}$
[B] $270 \div 3 = \boxed{n}$
[C] $350 \div 5 = \boxed{n}$
[D] $240 \div 4 = \boxed{n}$
[E] $300 \div 6 = \boxed{n}$
[F] $420 \div 7 = \boxed{n}$
[G] $360 \div 9 = \boxed{n}$
[H] $180 \div 2 = \boxed{n}$
[I] $480 \div 8 = \boxed{n}$
[J] $420 \div 6 = \boxed{n}$
[K] $280 \div 4 = \boxed{n}$
[L] $450 \div 5 = \boxed{n}$
[M] $210 \div 3 = \boxed{n}$
[N] $560 \div 7 = \boxed{n}$
[O] $630 \div 9 = \boxed{n}$
[P] $720 \div 8 = \boxed{n}$
[Q] $490 \div 7 = \boxed{n}$
[R] $540 \div 6 = \boxed{n}$
[S] $120 \div 20 = \boxed{n}$
[T] $150 \div 30 = \boxed{n}$
[U] $360 \div 40 = \boxed{n}$
[V] $480 \div 60 = \boxed{n}$
[W] $560 \div 70 = \boxed{n}$
[X] $810 \div 90 = \boxed{n}$

3. Find the quotients. Think about the missing factors.

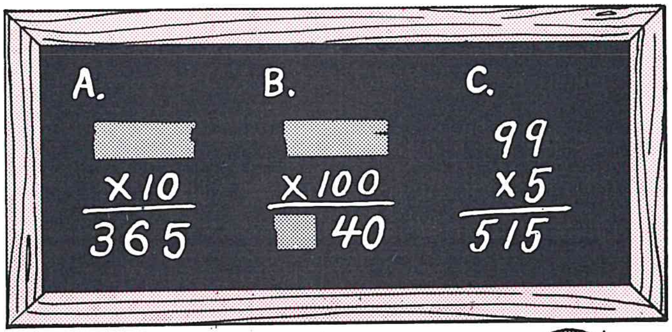
[A] $3\overline{)120}$
[B] $5\overline{)200}$
[C] $6\overline{)180}$
[D] $7\overline{)210}$
[E] $2\overline{)160}$
[F] $8\overline{)400}$
[G] $9\overline{)360}$
[H] $4\overline{)320}$
[I] $9\overline{)630}$
[J] $8\overline{)640}$
[K] $8\overline{)560}$
[L] $7\overline{)420}$
[M] $9\overline{)720}$
[N] $6\overline{)540}$
[O] $30\overline{)270}$
[P] $40\overline{)200}$
[Q] $50\overline{)300}$
[R] $60\overline{)480}$
[S] $70\overline{)490}$
[T] $90\overline{)540}$

4. Find the quotients.

[A] $1600 \div 4 = \boxed{n}$
[B] $2000 \div 5 = \boxed{n}$
[C] $1800 \div 9 = \boxed{n}$
[D] $2700 \div 3 = \boxed{n}$
[E] $5600 \div 7 = \boxed{n}$
[F] $4800 \div 8 = \boxed{n}$
[G] $5400 \div 9 = \boxed{n}$
[H] $4200 \div 7 = \boxed{n}$
[I] $3\overline{)2400}$
[J] $2\overline{)100}$
[K] $5\overline{)3500}$
[L] $4\overline{)2800}$
[M] $6\overline{)5400}$
[N] $7\overline{)2800}$
[O] $8\overline{)7200}$
[P] $9\overline{)8100}$

Reasoning

DISCUSSION EXERCISES



1. Ted wrote problem A on the chalkboard. He said he was thinking about a whole number that gave the product 365 when multiplied by 10. How could you show that Ted made a mistake?

2. One factor and part of the product have been covered in problem B. Show that the answer is not correct.

3. Without finding the correct product, show that 515 could not be the product in problem C. (Clue: What number is close to 99?)

EXERCISES

1. Copy each problem. Give the missing digits.

[A]	[B]	[C]	[D]	[E]	[F]
50	▒0	▒0	9▒	▒▒	▒▒
×1▒	×10	×9	×▒	×7	×9
500	700	36▒	540	5▒0	6▒0

★2. Give the pair of numbers that go in the gray spaces.

	Products		× +		Sums
[A]	50	▒	× +	▒	15
[B]	140	▒	× +	▒	27
[C]	140	▒	× +	▒	72
[D]	560	▒	× +	▒	87
[E]	4200	▒	× +	▒	706
[F]	3600	▒	× +	▒	904

	Products		× ÷		Quotients
[G]	100	▒	× ÷	▒	1
[H]	250	▒	× ÷	▒	10
[I]	160	▒	× ÷	▒	10
[J]	300	▒	× ÷	▒	3
[K]	1000	▒	× ÷	▒	10
[L]	2400	▒	× ÷	▒	24

Buying and selling clothes

1. Carol saw 12 skirts. Each skirt cost $10. How much would it cost to buy all the skirts?

2. A salesman sold these articles during one week. Find the amount of money received for →

8 suits,	$60 each
10 skirts,	$7 each
7 coats,	$60 each
6 suits,	$90 each

[A] the $60 suits. [C] the coats.

[B] the skirts. [D] the $90 suits. [E] all the clothes on the list.

3. The store manager bought 9 suits. At least one, but not all the suits cost $40. The others cost $80.

[A] The total cost of the suits could not have been $720. Why?

[B] The total cost of the suits could not have been $360. Why?

[C] What is the most he could have paid for the suits?

[D] What is the least he could have paid for the suits?

[E] If 4 suits cost $80 each and 5 cost $40 each, how much did he pay for all the suits?

★[F] If only 3 suits cost $80 each, what was the total cost?

★[G] Complete the table and answer the question.

If the total cost of the suits was $440, how many $40 suits did he buy?

Number of $40 suits	Number of $80 suits	Total cost of suits
1	8	$
2	7	$
3	6	$
4	5	$
5	4	$
6	3	$
7	2	$
8	1	$

Using the order and grouping principles

Change the order/grouping and the product is the same.

These examples help show how **order** and **grouping** changes affect the product, $2 \times 10 \times 3 \times 10$.

We could group these: ↘ ↙ $2 \times 10 \times 3 \times 10$ $20 \times 3 \times 10$ 60×10 600	We could group these: ↘ ↙ $2 \times 10 \times 3 \times 10$ $30 \times 2 \times 10$ 60×10 600	We could group these: ↘ ↙ $2 \times 10 \times 3 \times 10$ $30 \times 2 \times 10$ 60×10 600
We could group these: ↘ ↙ $2 \times 10 \times 3 \times 10$ $6 \times 10 \times 10$ 6×100 600	We could group these: ↘ ↙ $2 \times 10 \times 3 \times 10$ $100 \times 2 \times 3$ 100×6 600	We could group these: ↘ ↙ $2 \times 10 \times 3 \times 10$ $20 \times 10 \times 3$ 200×3 600

Because of the **order** and **grouping** principles, you can **multiply any two factors** first and get the same product. Study this example.

20×30
❶ $(2 \times 10) \times (3 \times 10)$
❷ $(2 \times 3) \times (10 \times 10)$
❸ 6×100
❹ 600

DISCUSSION EXERCISES

1. Explain step 1.
2. Which two factors were chosen to multiply first in step 2?
3. Explain steps 3 and 4.
4. Solve: $20 \times 30 = \boxed{n}$

EXERCISES

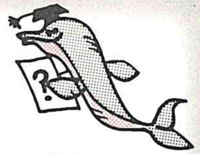
1. Solve the equations.

[A] $40 \times 60 = 4 \times 10 \times 6 \times 10 = \boxed{n}$
[B] $50 \times 60 = 5 \times 10 \times 6 \times 10 = \boxed{n}$
[C] $60 \times 70 = 6 \times 10 \times 7 \times 10 = \boxed{n}$
[D] $70 \times 80 = 7 \times 10 \times 8 \times 10 = \boxed{n}$
[E] $80 \times 90 = 8 \times 10 \times 9 \times 10 = \boxed{n}$
[F] $30 \times 50 = 3 \times 10 \times 5 \times 10 = \boxed{n}$
[G] $90 \times 600 = 9 \times 10 \times 6 \times 100 = \boxed{n}$
[H] $800 \times 80 = 8 \times 100 \times 8 \times 10 = \boxed{n}$

2. Find the products.

[A] $30 \times 60 = \boxed{n}$
[B] $80 \times 40 = \boxed{n}$
[C] $90 \times 30 = \boxed{n}$
[D] $70 \times 40 = \boxed{n}$
[E] $70 \times 90 = \boxed{n}$
[F] $40 \times 60 = \boxed{n}$
[G] $70 \times 30 = \boxed{n}$
[H] $40 \times 90 = \boxed{n}$
[I] $80 \times 70 = \boxed{n}$
[J] $60 \times 80 = \boxed{n}$
[K] $40 \times 40 = \boxed{n}$
[L] $80 \times 90 = \boxed{n}$
[M] $60 \times 90 = \boxed{n}$
[N] $90 \times 90 = \boxed{n}$
[O] $70 \times 70 = \boxed{n}$
[P] $60 \times 50 = \boxed{n}$

Think



To find 40 times 30
Much quicker than quick,
Just find me times 100.
There's really no trick.

WHO AM I?

3. Find the quotient by solving the multiplication equation.

[A] $\boxed{n} \times 40 = 800$
$800 \div 40 = \boxed{n}$

[B] $\boxed{n} \times 30 = 900$
$900 \div 30 = \boxed{n}$

[C] $\boxed{n} \times 50 = 1500$
$1500 \div 50 = \boxed{n}$

[D] $\boxed{n} \times 30 = 2700$
$2700 \div 30 = \boxed{n}$

[E] $\boxed{n} \times 70 = 3500$
$3500 \div 70 = \boxed{n}$

[F] $\boxed{n} \times 60 = 2400$
$2400 \div 60 = \boxed{n}$

[G] $\boxed{n} \times 90 = 6300$
$6300 \div 90 = \boxed{n}$

[H] $\boxed{n} \times 60 = 1800$
$1800 \div 60 = \boxed{n}$

[I] $\boxed{n} \times 80 = 4800$
$4800 \div 80 = \boxed{n}$

4. Find the quotients. Think about missing factors.

[A] $40\overline{)1200}$ [B] $20\overline{)1400}$ [C] $50\overline{)2500}$ [D] $10\overline{)900}$ [E] $30\overline{)2100}$

[F] $60\overline{)1800}$ [G] $70\overline{)4200}$ [H] $30\overline{)2700}$ [I] $10\overline{)800}$ [J] $80\overline{)800}$

[K] $20\overline{)1800}$ [L] $40\overline{)3200}$ [M] $50\overline{)4000}$ [N] $60\overline{)3000}$ [O] $40\overline{)3600}$

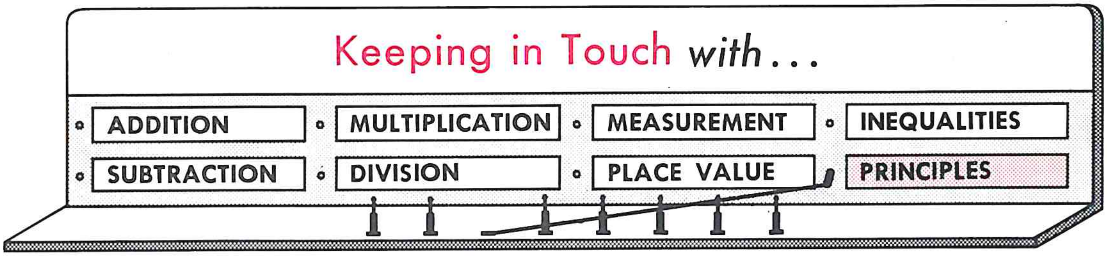
[P] $80\overline{)5600}$ [Q] $10\overline{)400}$ [R] $90\overline{)6300}$ [S] $10\overline{)500}$ [T] $90\overline{)2700}$

[U] $60\overline{)4800}$ [V] $80\overline{)7200}$ [W] $90\overline{)5400}$ [X] $70\overline{)4900}$ [Y] $80\overline{)6400}$

5. Find the products.

[A] $20 \times 700 = \boxed{n}$
[B] $40 \times 400 = \boxed{n}$
[C] $30 \times 800 = \boxed{n}$
[D] $60 \times 300 = \boxed{n}$
[E] $50 \times 900 = \boxed{n}$
[F] $70 \times 900 = \boxed{n}$
[G] $70 \times 800 = \boxed{n}$
[H] $80 \times 800 = \boxed{n}$

Keeping in Touch *with . . .*



The **basic principles** for addition and multiplication are very useful. The principles you have used to help you add and multiply are listed below.

Study this list carefully.

BASIC PRINCIPLES	
ADDITION	MULTIPLICATION
Order principle Grouping principle 0 principle	Order principle Grouping principle 1 principle
Multiplication-addition principle	

DISCUSSION EXERCISE

The statement $(3 + 9) + 7 = 3 + (9 + 7)$ illustrates the **grouping principle for addition.** Give statements that illustrate the other principles.

EXERCISES

The statement in each exercise illustrates one of the principles in the table above. Give the principle for each exercise.

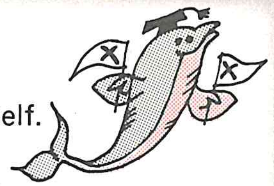
1. $30 \times (2 \times 10) = (30 \times 2) \times 10$
2. $54 + 37 = 37 + 54$
3. $93 + 0 = 93$
4. $87 \times 0 = 0 \times 87$
5. $63 \times 1 = 63$
6. $23 \times 10 = (20 \times 10) + (3 \times 10)$
7. $30 + (10 + 2) = (30 + 10) + 2$
★ **8.** $(3 \times 2) \times (5 \times 4) = (5 \times 4) \times (3 \times 2)$
★ **9.** $(6 \times 7) + (4 \times 7) = (4 \times 7) + (6 \times 7)$
★ **10.** $(20 \times 8) + (3 \times 8) = 23 \times 8$
★ **11.** $(25 + 7) + 3 = 3 + (25 + 7)$
★ **12.** $(30 \times 5) + (4 \times 5) = 34 \times 5$
★ **13.** $(8 + 5) + (4 + 7) = (4 + 7) + (8 + 5)$

Chapter review

1. Find the products.

[A] 10×7
[B] 18×10
[C] 10×23
[D] 50×10
[E] 75×10
[F] 6×100
[G] 100×18
[H] 57×100
[I] 70×100
[J] 100×32

Think



Multiply me by myself.
Then multiply by 2.
5000 is the answer.
You need no other clue.

WHO AM I?

2. Give the missing numbers.

[A] Since $3 \times 2 = 6$, we know that $3 \times 20 = \boxed{n}$.
[B] Since $7 \times 4 = 28$, we know that $7 \times 40 = \boxed{n}$.
[C] Since $9 \times 6 = 54$, we know that $9 \times 600 = \boxed{n}$.
[D] Since $3 \times 10 \times 4 \times 10 = 12 \times 100$, we know that $30 \times 40 = \boxed{n}$.
[E] Since $8 \times 10 \times 9 \times 10 = 72 \times 100$, we know that $80 \times 90 = \boxed{n}$.

3. Find the products.

[A] 4×30
[B] 5×70
[C] 6×80
[D] 40×4
[E] 50×6
[F] 7×200
[G] 3×500
[H] 600×4
[I] 8×300
[J] 400×8
[K] 70×20
[L] 30×40
[M] 50×60
[N] 80×40
[O] 30×90
[P] 7×90
[Q] 800×2
[R] 70×50
[S] 6×700
[T] 90×90

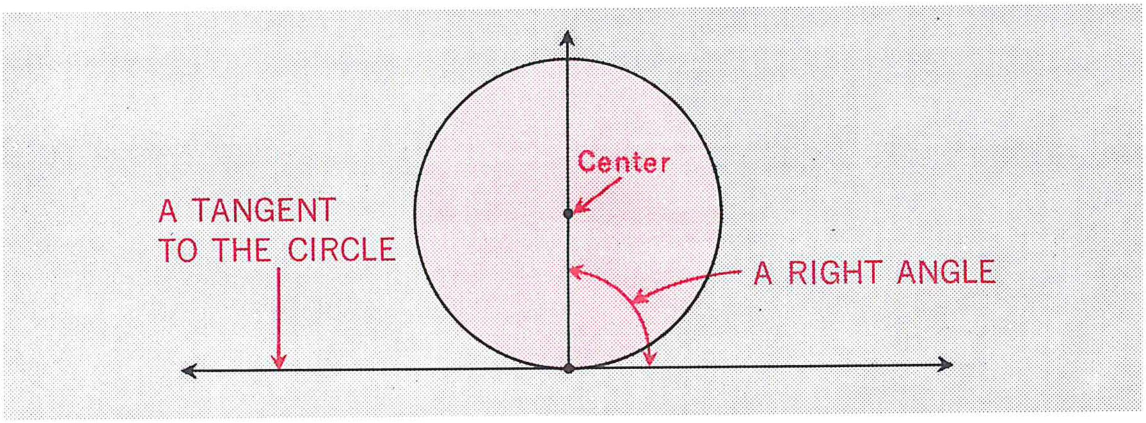
4. Find the quotients.

[A] $120 \div 4$
[B] $240 \div 6$
[C] $180 \div 2$
[D] $2500 \div 5$
[E] $2100 \div 7$
[F] $2400 \div 8$
[G] $2400 \div 30$
[H] $1600 \div 20$
[I] $2000 \div 40$
[J] $3500 \div 70$
[K] $4200 \div 60$
[L] $4500 \div 50$

5. Find the quotients.

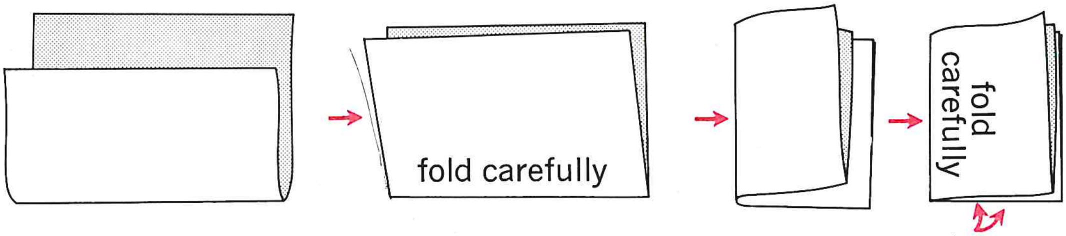
[A] $4\overline{)\overset{20}{280}}$
[B] $5\overline{)100}$
[C] $9\overline{)270}$
[D] $6\overline{)540}$
[E] $8\overline{)1600}$
[F] $40\overline{)280}$
[G] $60\overline{)600}$
[H] $20\overline{)140}$
[I] $5\overline{)4000}$
[J] $70\overline{)630}$

More about circles



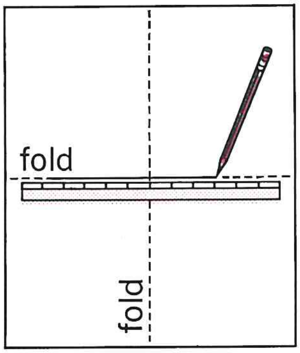
A line that touches a circle in exactly one point is called a **tangent** to the circle.

1. [A] Follow the steps below to obtain lines that cross at right angles.

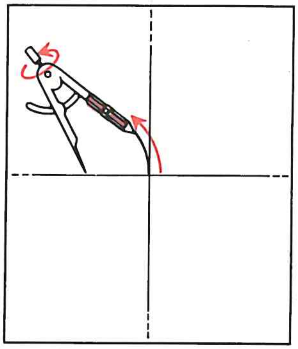


Make sure these two edges are together.

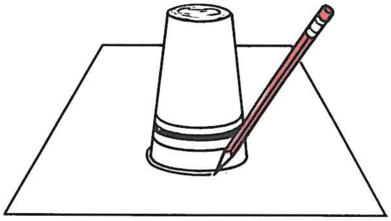
[B] Unfold your paper and use your ruler to trace over the lines that cross at right angles.



2. Use your paper from exercise 1 to draw 5 circles that are tangent to one of the two lines. Draw 3 on one side and 2 on the other.

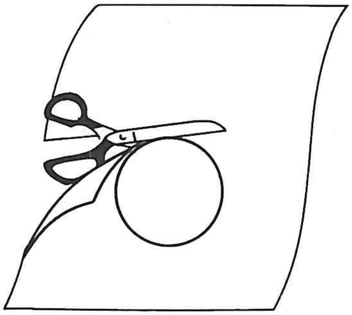


3. Draw a circle using a round object such as a cup, glass, or pop bottle. The object of this activity is to find the center of the circle.



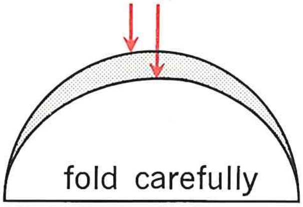
[A] Cut out the circle.

[B] Fold it carefully in half. Does this fold pass through the center of the circle?

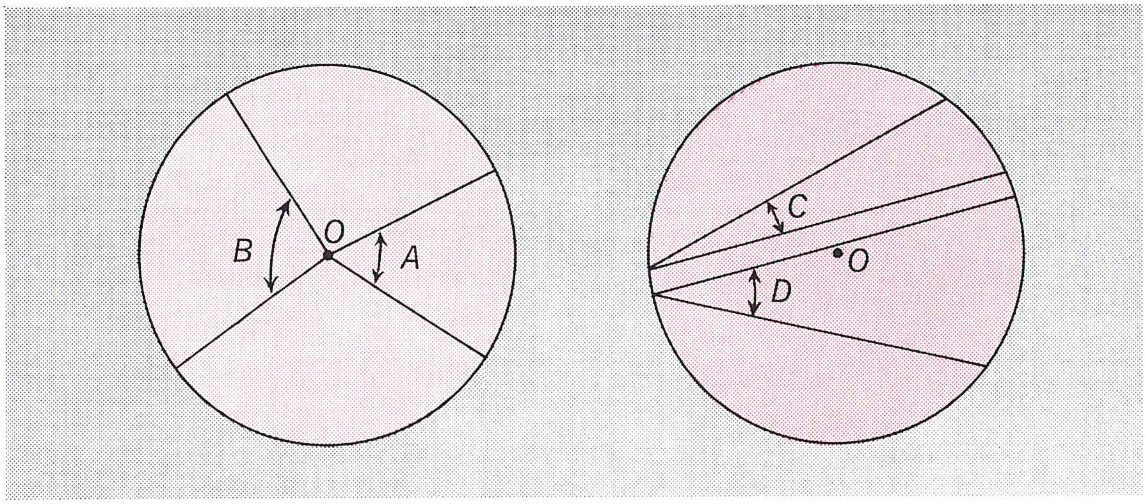


[C] Make another such fold and place a dot where you think the center is.

[D] Paste this circle on another sheet of paper and use your compass to check the accuracy of your work.


These edges should come together.

Central and inscribed angles



Angles A and B are **central angles** of the circle.

Angles C and D are **inscribed angles** of the circle.

1. Draw a circle with a $2\frac{1}{2}$-inch or 3-inch radius.

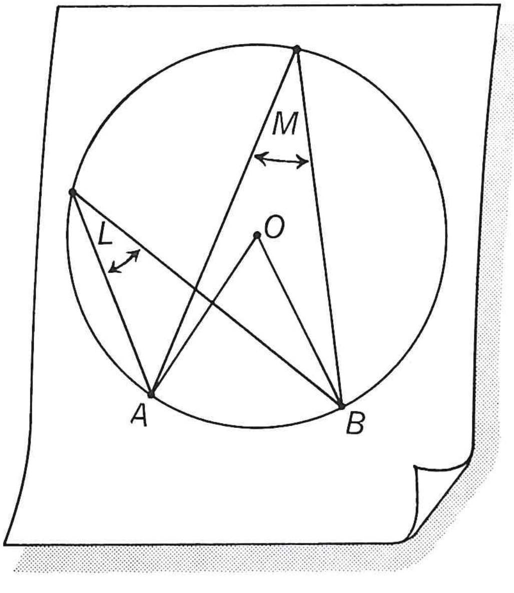
[**A**] Mark the center of your circle and label it O.

[**B**] Mark two points on your circle about 2 or 3 inches apart. Label these points A and B.

[**C**] Draw the central angle AOB.

[**D**] Mark two other points on your circle.

[**E**] Draw the inscribed angles to A and B and label them L and M as in the picture.



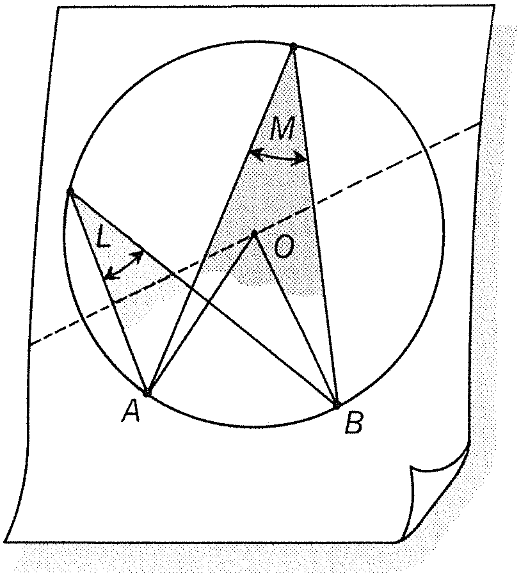
2. Color the inside of angle *L* one color and the inside of angle *M* another color.

[**A**] Cut your paper about where the dotted line shows in the picture.

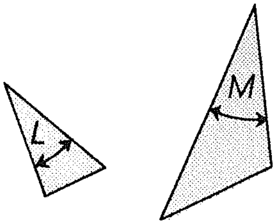
Do not cut any part of angle *AOB*.

Cut as much of angle *L* and *M* as you can.

[**B**] Cut out angles *L* and *M*.

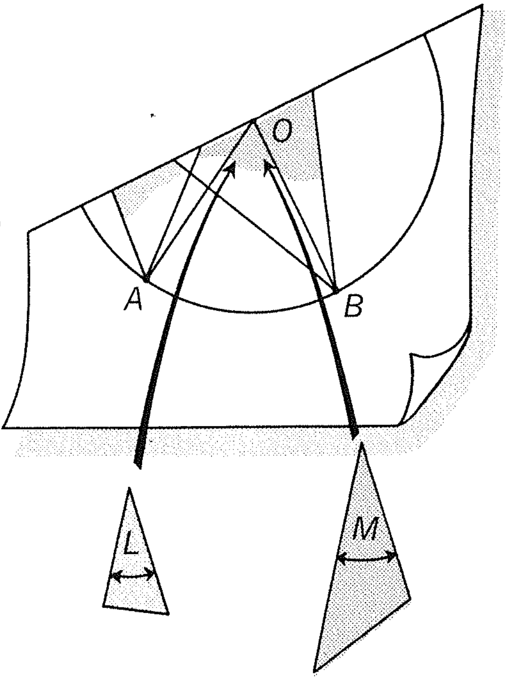


3. Place angles *L* and *M* on each other. How do they compare?

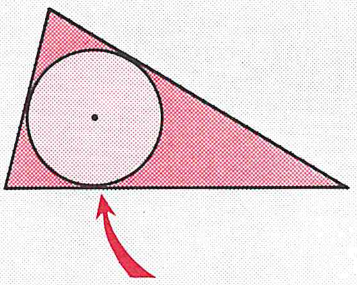


4. Paste angles *L* and *M* on central angle *AOB*. How do they fit?

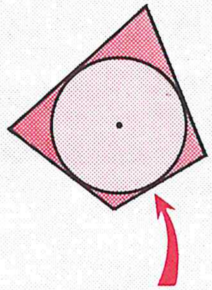
5. Do you think this would work for other angles *L* and *M* you might choose?



Inscribed circles

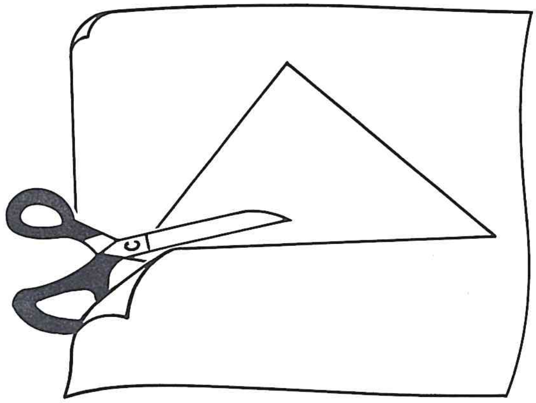


This circle is **inscribed** in the triangle.



This circle is **inscribed** in the quadrilateral.

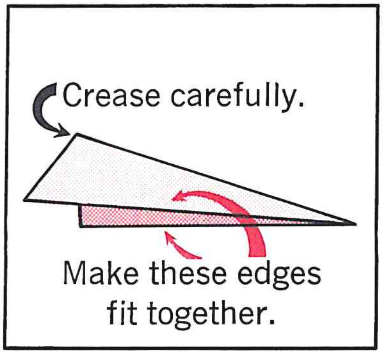
A circle is **inscribed** in figures such as those above when each side of the figure is tangent to the circle.



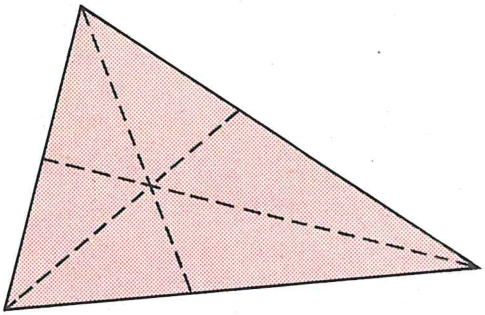
1. You can find the circle inscribed in a triangle in the following way.

[A] Draw and cut out a large triangle. The sides of your triangle should be at least 12 centimeters long.

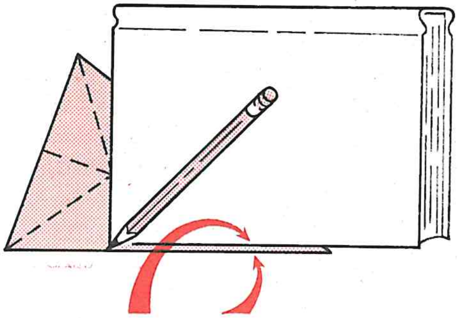
[B] Fold one corner of your triangle as shown in the figure.



[c] Now fold each of the other 2 corners as you did the first. If you did your work carefully, your triangle should look something like this. (The three creases should pass through one point.)



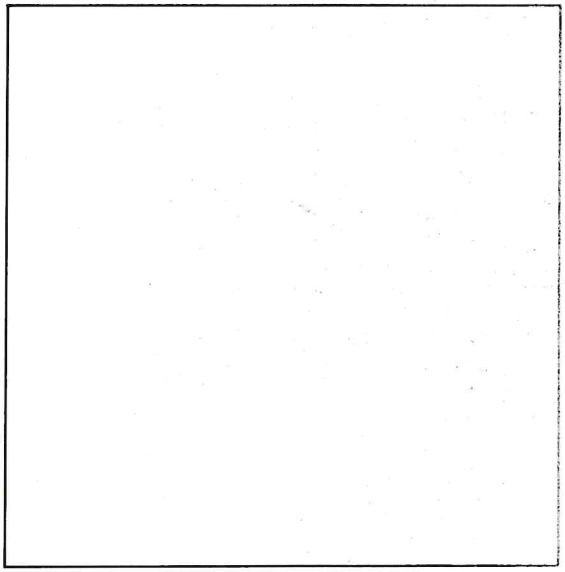
[d] Paste your triangle on another sheet of paper. Then use your book and mark the point right below the point where the folds intersect.



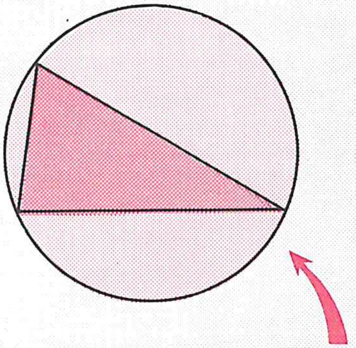
Be sure these edges fit upon each other.

[e] Decide where to place your compass, and draw the circle inscribed in the triangle.

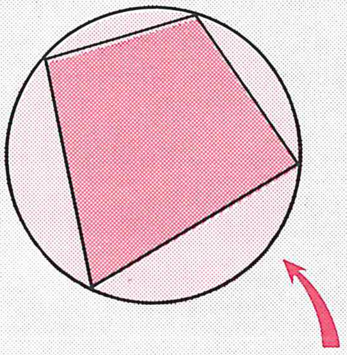
2. Trace this square on your paper. Figure out a way to draw a circle that is inscribed in the square. You should begin by searching for the center of the circle.



Circumscribed circles

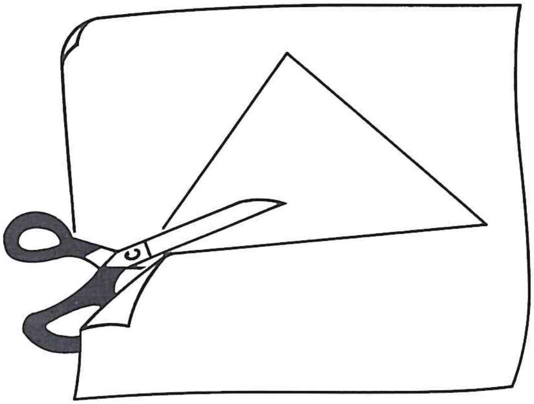


This circle is **circumscribed** about the triangle.



This circle is **circumscribed** about the quadrilateral.

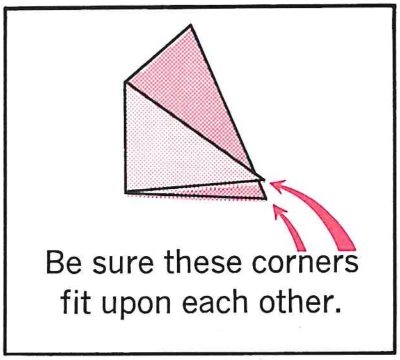
A circle is **circumscribed** about figures such as those above when each corner (vertex) of the figure is on the circle.



1. You can find the circle circumscribed about a triangle in the following way.

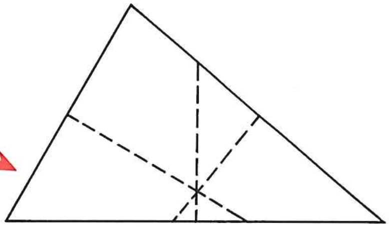
[A] Draw and cut out a triangle: Make the sides of your triangle between 5 and 12 centimeters.

[B] Find the middle point of each side of your triangle by folding as in the figure.

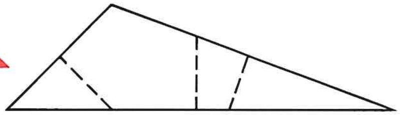


Be sure these corners fit upon each other.

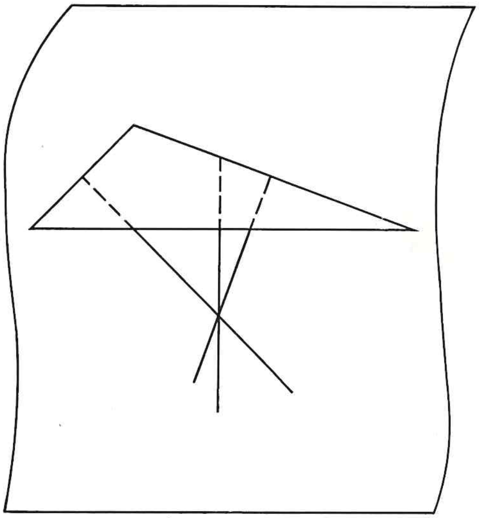
[C] When you complete the folding in B, your triangle should look something like this if the creases cross inside your triangle.



If the creases do not cross inside, your triangle should look something like this.



[D] Paste your triangle on another sheet of paper. Draw lines along the three creases. Make them long enough so they intersect. They should all pass through one point. This point is the center of the circumscribed circle. Draw this circle.



2. Trace the square on page 153 and draw the circle that is circumscribed about the square.

★**3.** Draw a right triangle on your paper.

[A] Draw the circle inscribed in the triangle.

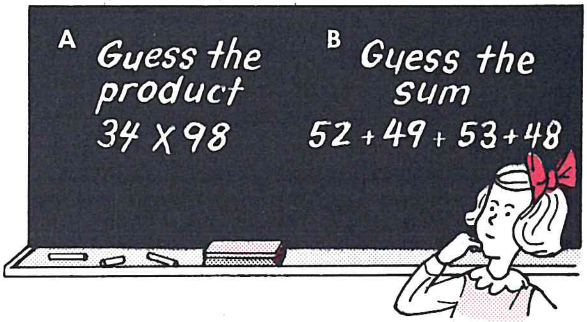
[B] Where is the center of the circle which circumscribes the triangle?

[C] Draw the circle which circumscribes the triangle.

CHAPTER 6 Estimation

Estimating

Miss Brown wrote these directions on the chalkboard. Cindy was surprised. She could **find** the sum and the product, but Miss Brown's directions said **''guess''!**



Here are the guesses made by Cindy and some of her classmates.

	Cindy	Lee	Nancy
Product	400	10,500	3400
Sum	300	400	200

DISCUSSION EXERCISES

1. Which guess do you think is best for the product?

2. How do you think Nancy decided to guess 3400?

3. Which guess do you think is best for the sum?

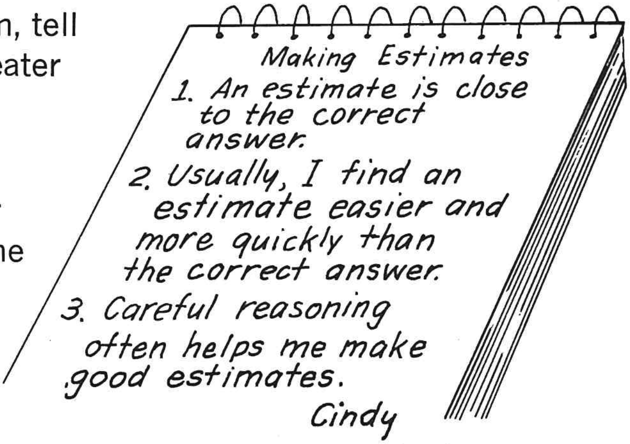
4. How do you think Nancy decided to guess 200?

5. Without finding the correct sum, explain why it cannot be 400.

6. Without finding the correct product, tell whether Nancy's guess was greater or less than the correct product.

7. Without finding the correct sum, tell whether Nancy's guess was greater or less than the correct sum.

After Miss Brown and the children had talked about their guesses, they decided to use the word **estimate** instead of **guess.** This is what Cindy wrote in her notebook.



Making Estimates

1. An estimate is close to the correct answer.

2. Usually, I find an estimate easier and more quickly than the correct answer.

3. Careful reasoning often helps me make good estimates.

Cindy

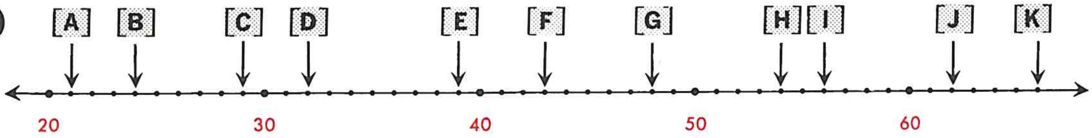
To estimate 29 × 43, we can find the product 30 × 40.
To estimate 48 + 73, we can find the sum 50 + 70.
To estimate 312 × 67, we can find the product 300 × 70.
To estimate 687 + 218, we can find the sum 700 + 200.

Multiples of 10 and 100 are helpful in making estimates.

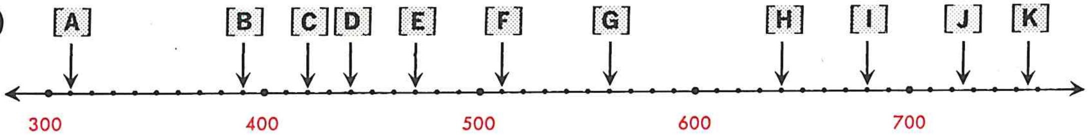
The number-line pictures and the exercises will help you choose multiples of 10 and 100 to use in estimating.

EXERCISES

A



B



1. Give the numbers (A through K) that go with the points on number-line picture A.

2. Give the multiple of 10 that is closest to each number in exercise 1. Number-line picture A should help you.

3. Give the multiple of 10 that is closest to each number.
   [A] 31 [B] 42 [C] 53 [D] 64 [E] 26 [F] 37 [G] 48 [H] 59

4. Here are the first two numbers for the gray spaces in picture B: [A] 310, [B] 390.
   Give the numbers for **C** through **K.**

5. Give the multiple of 100 that is closest to each number in exercise 4.

6. Give the multiple of 100 that is closest to each number.
   [A] 320 [B] 590 [C] 499 [D] 619 [E] 735 [F] 549 [G] 551

7. Give a number that is as close to 50 as it is to 60.

Estimating sums and differences

EXERCISES

1. Give the multiples of 10 that are closest to the addends. Then give an estimate for the sum.

Example:
$$\begin{array}{r}58 \rightarrow 60\\ +96 \rightarrow 100\\ \hline \text{Estimate} \rightarrow 160\end{array}$$

[A]
$$\begin{array}{r}19 \rightarrow ▥\\ +48 \rightarrow ▥\\ \hline ▥\end{array}$$

[B]
$$\begin{array}{r}23 \rightarrow ▥\\ +78 \rightarrow ▥\\ \hline ▥\end{array}$$

[C]
$$\begin{array}{r}27 \rightarrow ▥\\ +69 \rightarrow ▥\\ \hline ▥\end{array}$$

[D]
$$\begin{array}{r}18 \rightarrow ▥\\ +47 \rightarrow ▥\\ \hline ▥\end{array}$$

[E]
$$\begin{array}{r}68 \rightarrow ▥\\ +96 \rightarrow ▥\\ \hline ▥\end{array}$$

[F]
$$\begin{array}{r}76 \rightarrow ▥\\ +94 \rightarrow ▥\\ \hline ▥\end{array}$$

[G]
$$\begin{array}{r}44 \rightarrow ▥\\ +88 \rightarrow ▥\\ \hline ▥\end{array}$$

2. Give the number you think should go in each ▥.

[A] To estimate 98 + 49, we can find the sum ▥ + 50.

[B] To estimate 79 + 19, we can find the sum 80 + ▥.

[C] To estimate 92 − 47, we can find the difference ▥ − 50.

[D] To estimate 12 + 87, we can find the sum ▥ + 90.

[E] To estimate 67 + 34, we can find the sum 70 + ▥.

[F] To estimate 53 − 38, we can find the difference 50 − ▥.

[G] To estimate 29 + 74, we can find the sum ▥ + 70.

[H] To estimate 86 − 38, we can find the difference ▥ − 40.

[I] To estimate 74 + 65, we can find the sum ▥ + 70.

[J] To estimate 54 − 26, we can find the difference ▥ − 30.

[K] To estimate 92 + 88, we can find the sum 90 + ▥.

3. Give an estimate for each sum and difference.

[A] 59 + 19
[B] 92 − 48
[C] 43 + 69
[D] 67 − 27
[E] 69 + 69
[F] 93 − 27
[G] 86 + 59
[H] 87 − 28
[I] 44 + 86
[J] 97 − 46
[K] 97 + 98
[L] 77 − 38
[M] 12 + 23 + 29
[N] 58 + 63 + 47
[O] 69 + 46 + 44
[P] 86 + 84 + 98

4. Give the multiples of 100 that are closest to the addends. Then give an estimate for the sum.

[A] 898 → ▥
407 → ▥
296 → ▥
+513 → ▥
▥

[B] 690 → ▥
418 → ▥
387 → ▥
+209 → ▥
▥

[C] 457 → ▥
541 → ▥
327 → ▥
+273 → ▥
▥

[D] 651 → ▥
849 → ▥
378 → ▥
+764 → ▥
▥

~~5.~~ Find the correct sums in exercise 4. Which sum is closest to your estimate?

6. Give the number you think should go in each ▥.

[A] To estimate $298+67$, we can find the sum ▥ $+70$.
[B] To estimate $702-397$, we can find the difference ▥ -400.
[C] To estimate $417+686$, we can find the sum $400+$ ▥.
[D] To estimate $641+856$, we can find the sum ▥ $+900$.
[E] To estimate $847-568$, we can find the difference ▥ -600.

7. Which sum will give the best estimate for $85+85$?

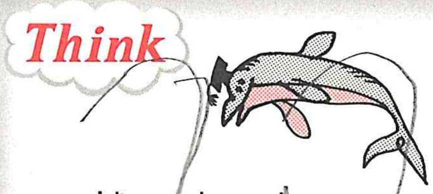
[A] $80+80$
[B] $90+90$
[C] $80+90$

8. Give an estimate for each sum.

[A] $85+65$
[B] $35+45$
[C] $65+65$
[D] $73+45$

9. Give an estimate for each sum and difference.

[A] $395+407$
[B] $402-198$
[C] $787+218$
[D] $479-324$
[E] $963+439$
[F] $804-387$
[G] $497+69$
[H] $886-39$
[I] $88+317$
[J] $761-458$
[K] $869+468$
[L] $729-346$
[M] $301+649$
[N] $649-301$
[O] $489-92$
[P] $86+215$



I have always been
A two-digit multiple of 10.
I am closer to 94
Than to 80 and 5 more.

WHO AM I?

Estimating products

EXERCISES

1. For each exercise, write the equation from the table which could help you best estimate the product.

[A] 7 × 61
[B] 9 × 72
[C] 4 × 98
[D] 7 × 79
[E] 4 × 81
[F] 9 × 19
[G] 7 × 23
[H] 7 × 48
[I] 4 × 18
[J] 4 × 44
[K] 9 × 47
[L] 7 × 41

$4 \times 100 = 400$
$7 \times 40 = 280$
$4 \times 40 = 160$
$7 \times 80 = 560$
$4 \times 80 = 320$
$9 \times 20 = 180$
$7 \times 20 = 140$
$9 \times 50 = 450$
$7 \times 60 = 420$
$4 \times 20 = 80$
$9 \times 70 = 630$
$7 \times 50 = 350$

2. Give the number you think should go in each ▥.

[A] To estimate 3 × 51, we can find the product 3 × ▥.
[B] To estimate 8 × 42, we can find the product 8 × ▥.
[C] To estimate 6 × 39, we can find the product 6 × ▥.
[D] To estimate 9 × 78, we can find the product 9 × ▥.
[E] To estimate 7 × 63, we can find the product 7 × ▥.
[F] To estimate 9 × 37, we can find the product ▥ × 37.
[G] To estimate 4 × 96, we can find the product 4 × ▥.
[H] To estimate 6 × 84, we can find the product 6 × ▥.
[I] To estimate 87 × 7, we can find the product ▥ × 7.
★[J] To estimate 8 × 45, we can find the product 8 × ▥.

3. **Estimate** these products.

[A] 6 × 21
[B] 5 × 32
[C] 7 × 81
[D] 8 × 39
[E] 9 × 68
[F] 3 × 47
[G] 6 × 73
[H] 8 × 18
[I] 7 × 99
[J] 37 × 9
[K] 17 × 6
[L] 7 × 19
[M] 98 × 3
[N] 6 × 56
[O] 4 × 74
★[P] 8 × 65

4. Give the number you think should go in each gray space.

[A] To estimate 32 × 29, we can find the product ▨ ×30. 20 × 70

[B] To estimate 98 × 18, we can find the product ▨ × 20.

[C] To estimate 45 × 53, we can find the product ▨ × 50.

[D] To estimate 9 × 207, we can find the product 9 × ▨.

[E] To estimate 7 × 496, we can find the product 7 × ▨.

[F] To estimate 32 × 387, we can find the product 30 × ▨.

[G] To estimate 46 × 51, we can find the product ▨.

[H] To estimate 83 × 19, we can find the product ▨.

[I] To estimate 6 × 396, we can find the product ▨.

[J] To estimate 52 × 417, we can find the product ▨.

5. Give an estimate for each product.

[A] 31 × 29
[B] 98 × 21
[C] 69 × 19
25 **[D]** 41 × 51
60 **[E]** 82 × 89

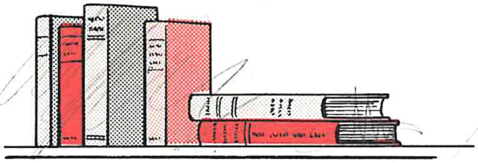
[F] 64 × 56
[G] 38 × 88
[H] 65 × 11
[I] 45 × 45
[J] 93 × 18

[K] 6 × 198
[L] 3 × 696
[M] 7 × 516
[N] 8 × 999
[O] 9 × 356

★**[P]** 51 × 207
★**[Q]** 39 × 289
★**[R]** 44 × 318
★**[S]** 469 × 34
★**[T]** 538 × 651

6. Estimate the answer.

[A] What did Joe have to pay for 6 books at 49¢ each? 294



[B] A cup holds about 8 ounces. How many ounces are in 98 cups? 800

[C] 24 hours make one day. How many hours are in 62 days? 784

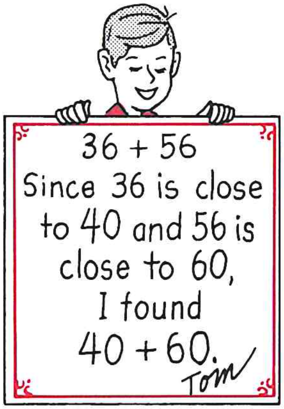
Think

Count my digits; you'll find two.
They're like as like can be.
When you find my sum with 1.
Three digits you will see.

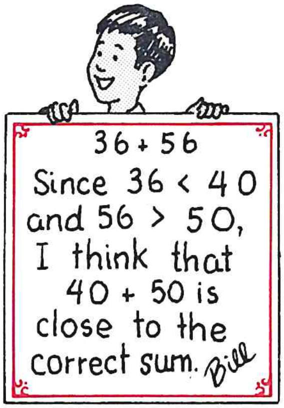
WHO AM I?

Which estimate is best?

Tom and Bill were asked to give an estimate for the sum 36 + 56. Here is what they wrote to show how they found their estimates.



36 + 56
Since 36 is close
to 40 and 56 is
close to 60,
I found
40 + 60.
Tom



36 + 56
Since 36 < 40
and 56 > 50,
I think that
40 + 50 is
close to the
correct sum.
Bill

DISCUSSION EXERCISES

1. Without finding the correct sum, tell which estimate you think is better.

2. Find the correct sum. Which estimate is better?

3. Which estimate, 70 + 50 or 70 + 60, is closer to the correct sum for 74 + 54?

EXERCISES

1. Different ways are given to estimate each sum. Choose the one you think will give the best estimate and write it on your paper.

[A] 45 + 45 40 + 40, 50 + 50, 50 + 40
[B] 24 + 36 20 + 40, 20 + 30, 30 + 40
[C] 56 + 87 60 + 90, 50 + 80, 50 + 90
[D] 64 + 74 60 + 80, 60 + 70, 70 + 80
[E] 41 + 69 40 + 60, 40 + 70, 50 + 70

★ 2. Different ways are given to estimate each product. Choose the one you think will give the best estimate.

[A] 35 × 35 30 × 30, 40 × 40, 30 × 40
[B] 25 × 34 20 × 30, 30 × 30, 30 × 40
[C] 65 × 46 60 × 40, 70 × 50, 60 × 50
[D] 36 × 56 40 × 60, 40 × 50, 30 × 50

Estimating quotients

EXERCISES

Three numbers are given for each problem. Choose the number that is closest to the correct answer.

1. 224 apples

How many packages of 7?

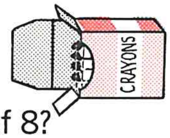
[A] 100 [B] 30 [C] 80

2. 386 erasers

How many boxes of 9?

[A] 400 [B] 4 [C] 40

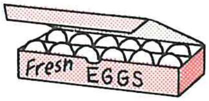
3. 504 crayons



How many boxes of 8?

[A] 60 [B] 100 [C] 7

4. 276 eggs



How many cartons of 12?

[A] 20 [B] 10 [C] 100

5. 87 ÷ 3 How many threes in 87? [A] 3 [B] 30 [C] 20

6. 128 ÷ 4 How many fours in 128? [A] 10 [B] 300 [C] 30

7. 252 ÷ 6 How many sixes in 252? [A] 40 [B] 50 [C] 400

8. 686 ÷ 7 How many sevens in 686? [A] 10 [B] 1000 [C] 100

9. 5760 ÷ 8 How many eights in 5760? [A] 70 [B] 700 [C] 800

10. 217 ÷ 31 How many thirty-ones in 217? [A] 5 [B] 6 [C] 7

11. 152 ÷ 27 How many twenty-sevens in 152? [A] 10 [B] 5 [C] 8

12. $3\overline{)279}$ How many threes in 279? [A] 9 [B] 900 [C] 90

13. $5\overline{)525}$ How many fives in 525? [A] 90 [B] 100 [C] 200

14. $7\overline{)343}$ How many sevens in 343? [A] 5 [B] 40 [C] 50

15. $7\overline{)4760}$ How many sevens in 4760? [A] 600 [B] 70 [C] 700

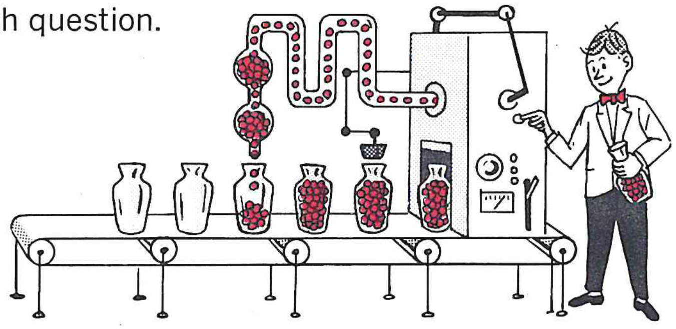
16. $9\overline{)4680}$ How many nines in 4680? [A] 50 [B] 5000 [C] 500

17. $53\overline{)212}$ How many fifty-threes in 212? [A] 8 [B] 5 [C] 4

★ 18. $25\overline{)224}$ How many twenty-fives in 224? [A] 8 [B] 10 [C] 100

Short stories—estimation

Estimate the answer to each question.



1. 48 pills in each bottle.
   7 bottles. How many pills?

2. 6 tomatoes in each bag.
   32 bags. How many tomatoes?

3. Travel 434 miles in 7 hours.
   About how many miles each hour?

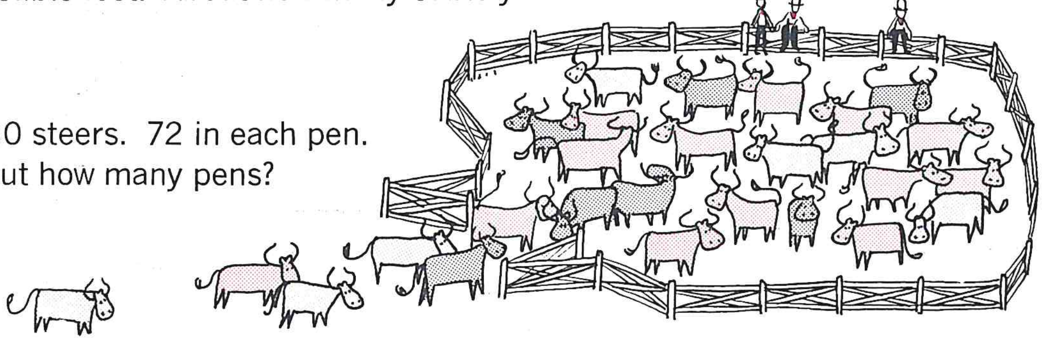
4. About 31 children in each class.
   213 children. How many classes?

5. Empire State Building. 1250 feet high.
   102 stories. How high is one story?

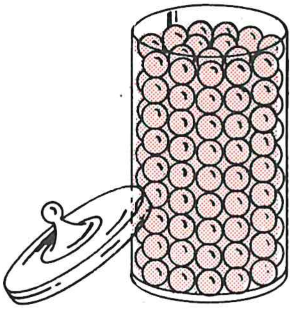
6. About 365 days in a year. 7 days in a week.
   About how many weeks in a year?

7. 27 cubic feet make a cubic yard.
   89 cubic feet. About how many cubic yards?

★ 8. 2780 steers. 72 in each pen.
   About how many pens?

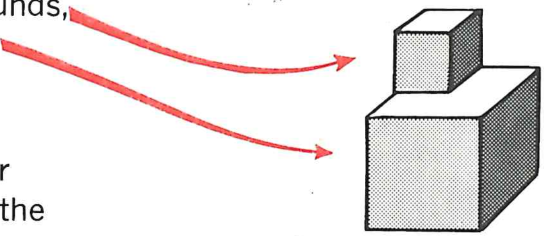


Estimation for fun



1. Estimate the number of marbles in this plastic jar.

2. If this block of lead weighs 43 pounds, estimate the weight of this block.



3. Estimate the number of words in exercise 5 by counting the number of words in **one line** and counting the number of lines.

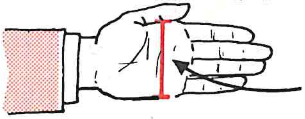
4. Count the number of breaths you take in 1 minute. Estimate the number of breaths you take

   [A] in an hour, ★ [B] in a 24-hour day.

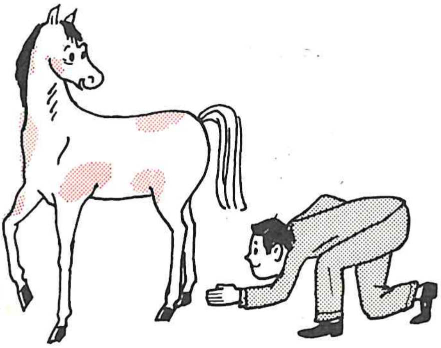
5. Sally took 4 drinks from the fountain on Monday. Tim took 6 drinks. Bill took 5 drinks. There are 33 children in Sally's class. Estimate how many drinks Sally's class takes from the fountain in [A] one day, [B] one week.

6. The tallest horse on record is 21 hands tall. Estimate this horse's height in

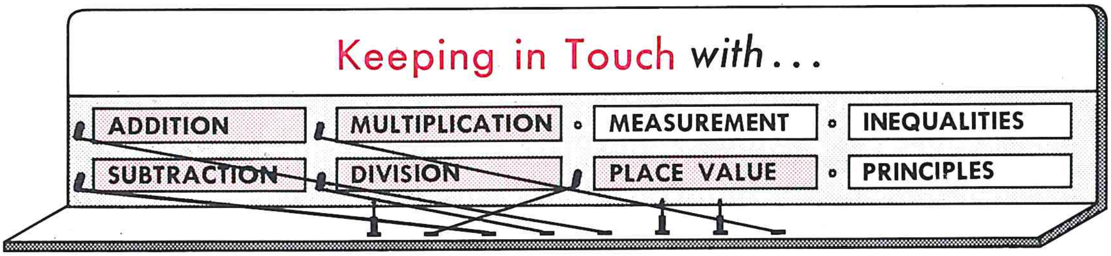
   [A] inches, ★ [B] feet.



On a 6-foot man this length is about one hand.



Keeping in Touch with . . .



1. Find the missing numbers.

[A] $356 = 300 + 50 + \boxed{n}$ [C] $639 = \boxed{n} + 30 + 9$ [E] $6947 = 6047 + \boxed{n}$

[B] $872 = 800 + \boxed{n} + 2$ [D] $5436 = 5430 + \boxed{n}$ [F] $4673 = 4603 + \boxed{n}$

2. Find the sums.

[A] $\begin{array}{r}39\\68\\+57\\\hline\end{array}$ [B] $\begin{array}{r}56\\39\\+87\\\hline\end{array}$ [C] $\begin{array}{r}856\\729\\+347\\\hline\end{array}$ [D] $\begin{array}{r}638\\463\\+806\\\hline\end{array}$ [E] $\begin{array}{r}5947\\6879\\+9435\\\hline\end{array}$

3. Find the differences.

[A] $\begin{array}{r}234\\-39\\\hline\end{array}$ [B] $\begin{array}{r}195\\-39\\\hline\end{array}$ [C] $\begin{array}{r}156\\-39\\\hline\end{array}$ [D] $\begin{array}{r}117\\-39\\\hline\end{array}$ [E] $\begin{array}{r}78\\-39\\\hline\end{array}$ [F] $\begin{array}{r}39\\-39\\\hline\end{array}$

4. Use exercise 3 to give the number of thirty-nines in 234.

5. List the problems that have no whole-number answers. Then find the differences for the other exercises.

[A] $\begin{array}{r}147\\-108\\\hline\end{array}$ [B] $\begin{array}{r}281\\-315\\\hline\end{array}$ [C] $\begin{array}{r}370\\-106\\\hline\end{array}$ [D] $\begin{array}{r}304\\-137\\\hline\end{array}$ [E] $\begin{array}{r}874\\-396\\\hline\end{array}$ [F] $\begin{array}{r}387\\-431\\\hline\end{array}$

[G] $\begin{array}{r}500\\-287\\\hline\end{array}$ [H] $\begin{array}{r}1008\\-369\\\hline\end{array}$ [I] $\begin{array}{r}1870\\-1798\\\hline\end{array}$ [J] $\begin{array}{r}9007\\-8438\\\hline\end{array}$ [K] $\begin{array}{r}4976\\-1098\\\hline\end{array}$

6. Solve the equations.

[A] $54 + 38 + 7 = \boxed{n}$ [C] $83 + 19 + 7 = \boxed{n}$ [E] $405 - 398 = \boxed{n}$

[B] $457 + 39 + 4 = \boxed{n}$ [D] $567 + 348 + 9 = \boxed{n}$ [F] $697 - 688 = \boxed{n}$

7. Find the products and quotients.

[A] 9 × 40 [D] 200 × 8 [G] 20 × 50 [J] 30 × 400

[B] 30 × 7 [E] 360 ÷ 4 [H] 60 × 30 [K] 3600 ÷ 4

[C] 70 × 6 [F] 720 ÷ 8 [I] 800 ÷ 40 [L] 500 × 300

Chapter review

1. Give what you think should go in each gray space.

[A] To estimate 59 + 83, we can find the sum ▨ + 80.
[B] To estimate 396 + 507, we can find the sum 400 + ▨.
[C] To estimate 7 × 68, we can find the product 7 × ▨.
[D] To estimate 8 × 497, we can find the product 8 × ▨.
[E] To estimate 53 × 47, we can find the product ▨.
[F] To estimate 413 × 89, we can find the product ▨.
[G] To estimate 426 + 568, we can find the sum ▨.
[H] To estimate 65 × 9, we can find the product ▨.
[I] To estimate 403 − 196, we can find the difference ▨.

2. Estimate the sums, products, and differences.

[A] 69 + 23
[B] 82 − 49
[C] 8 × 37
[D] 33 × 58
[E] 517 + 89
[F] 95 + 95
[G] 503 − 293
[H] 407 × 22
[I] 659 + 435
[J] 398 × 21
[K] 49 × 313
[L] 301 × 297
[M] 205 + 298
[N] 39 × 604

Think

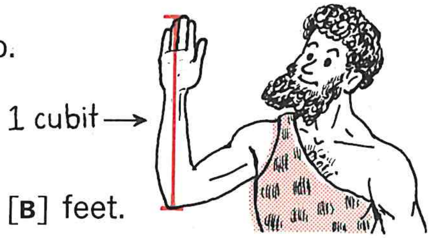
I'm quite close to one hundred three.
Closer still to ninety-eight.
Yet for both numbers I am used
When you want to estimate.

WHO AM I?

3. Choose the best estimate for each exercise.

[A] 88 ÷ 4	How many fours in 88?	[A] 20	[B] 30	[C] 10
[B] 434 ÷ 7	How many sevens in 434?	[A] 100	[B] 70	[C] 60
[C] 558 ÷ 6	How many sixes in 558?	[A] 80	[B] 90	[C] 100
[D] 1728 ÷ 32	How many thirty-twos in 1728?	[A] 100	[B] 50	[C] 6

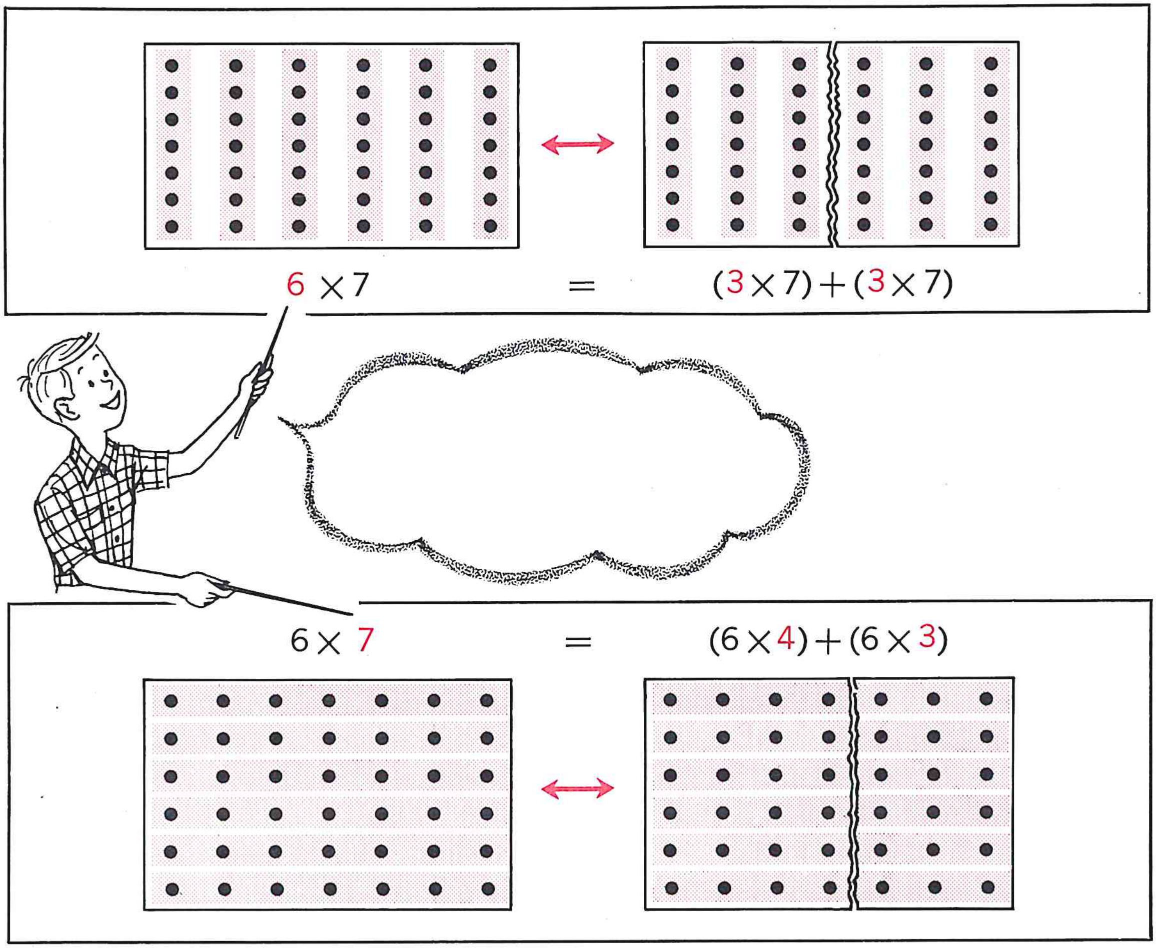
4. A cubit is a unit of length first used long ago. It is the distance from a man's elbow to the tip of his fingers. Goliath was reported to be over 6 cubits tall. Estimate Goliath's height in [A] inches, ★ [B] feet.



CHAPTER 7 Multiplying

The multiplication-addition principle

Here is how Steve explained **two ways** to think about the **multiplication-addition principle.**



DISCUSSION EXERCISES

1. What do you think Steve means when he says "break apart a factor"?
2. Show that both methods above give the same product.
3. Give some other ways to use the multiplication-addition principle to find the product 6 × 7.

Using the multiplication-addition principle

EXERCISES

1. Solve the equations.

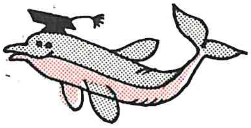
[A] $4 \times 25 = (4 \times 20) + (4 \times n)$
[B] $6 \times 32 = (6 \times 30) + (6 \times n)$
[C] $5 \times 18 = (5 \times 10) + (5 \times n)$
[D] $3 \times 56 = (3 \times 50) + (3 \times n)$
[E] $8 \times 21 = (8 \times 20) + (8 \times n)$
[F] $7 \times 92 = (7 \times n) + (7 \times 2)$
[G] $3 \times 76 = (3 \times n) + (3 \times 6)$
[H] $9 \times 58 = (9 \times n) + (9 \times 8)$
[I] $4 \times 35 = (4 \times n) + (4 \times 5)$
[J] $8 \times 47 = (8 \times n) + (8 \times 7)$

2. Solve the equations.

[A] $3 \times 16 = (3 \times 10) + (3 \times 6) = n$
[B] $5 \times 23 = (5 \times 20) + (5 \times 3) = n$
[C] $4 \times 38 = (4 \times 30) + (4 \times 8) = n$
[D] $9 \times 42 = (9 \times 40) + (9 \times 2) = n$
[E] $7 \times 57 = (7 \times 50) + (7 \times 7) = n$
[F] $8 \times 64 = (8 \times 60) + (8 \times 4) = n$
[G] $6 \times 79 = (6 \times 70) + (6 \times 9) = n$
[H] $5 \times 86 = (5 \times 80) + (5 \times 6) = n$
[I] $9 \times 95 = (9 \times 90) + (9 \times 5) = n$

Think

To multiply by me,
Here's what you can do:
First multiply by 10;
Then multiply by 2.



WHO AM I?

3. Give the missing numbers.

[A] Since $4 \times 80 = 320$, we know that $4 \times 81 = n$.
[B] Since $7 \times 50 = 350$, we know that $7 \times 51 = n$.
[C] Since $6 \times 80 = 480$, we know that $6 \times 82 = n$.
[D] Since $4 \times 90 = 360$, we know that $4 \times 94 = n$.
[E] Since $8 \times 70 = 560$, we know that $8 \times 73 = n$.

★ **4.** Solve the equations.

[A] $20 \times 43 = (20 \times 40) + (20 \times 3) = n$
[B] $60 \times 74 = (60 \times 70) + (60 \times 4) = n$
[C] $30 \times 86 = (30 \times 80) + (30 \times 6) = n$
[D] $50 \times 29 = (50 \times 20) + (50 \times 9) = n$
[E] $90 \times 98 = (90 \times 90) + (90 \times 8) = n$
[F] $23 \times 34 = (23 \times 30) + (23 \times 4) = n$
[G] $34 \times 45 = (34 \times 40) + (34 \times 5) = n$

Multiplying with one 2-digit factor

DISCUSSION EXERCISES

Solve the equations in parts **A** and **B**. Then explain how to find the product in part **C**.

1. [A] $3 \times 12 = (3 \times 10) + (3 \times \boxed{n})$
[B] $3 \times 12 = 30 + \boxed{n}$
[C] $3 \times 12 = \boxed{n}$

2. [A] $4 \times 21 = (4 \times \boxed{n}) + (4 \times 1)$
[B] $4 \times 21 = \boxed{n} + 4$
[C] $4 \times 21 = \boxed{n}$

3. [A] $3 \times 25 = (3 \times \boxed{n}) + (3 \times 5)$
[B] $3 \times 25 = \boxed{n} + 15$
[C] $3 \times 25 = \boxed{n}$

4. [A] $6 \times 14 = (6 \times 10) + (6 \times \boxed{n})$
[B] $6 \times 14 = 60 + \boxed{n}$
[C] $6 \times 14 = \boxed{n}$

5. [A] $5 \times 21 = (5 \times 20) + (5 \times \boxed{n})$
[B] $5 \times 21 = 100 + \boxed{n}$
[C] $5 \times 21 = \boxed{n}$

6. [A] $7 \times 32 = (7 \times \boxed{n}) + (7 \times 2)$
[B] $7 \times 32 = \boxed{n} + 14$
[C] $7 \times 32 = \boxed{n}$

7. Explain each step in the example below and give the missing numbers.

Step 1	Step 2	Step 3
$\begin{array}{r} 37 \\ \times 4 \\ \hline \square \end{array}$	$\begin{array}{r} 37 \\ \times 4 \\ \hline 28 \\ \square \end{array}$	$\begin{array}{r} 37 \\ \times 4 \\ \hline 28 \\ 120 \\ \hline \square \end{array}$
$4 \times 7 = \boxed{n}$	$4 \times 30 = \boxed{n}$	$28 + 120 = \boxed{n}$

EXERCISES

Find the products.

1. $\begin{array}{r} 46 \\ \times 3 \\ \hline \end{array}$ **2.** $\begin{array}{r} 64 \\ \times 7 \\ \hline \end{array}$ **3.** $\begin{array}{r} 18 \\ \times 4 \\ \hline \end{array}$ **4.** $\begin{array}{r} 39 \\ \times 8 \\ \hline \end{array}$ **5.** $\begin{array}{r} 43 \\ \times 5 \\ \hline \end{array}$ **6.** $\begin{array}{r} 84 \\ \times 6 \\ \hline \end{array}$ **7.** $\begin{array}{r} 65 \\ \times 9 \\ \hline \end{array}$

DISCUSSION EXERCISES

1. Explain each step in the short cuts below.

A

$$\begin{array}{r} 26\\ \times 3\\ \hline 18\\ 60\\ \hline 78 \end{array} \Rightarrow \begin{array}{r} \overset{1}{2}6\\ \times 3\\ \hline 8 \end{array} \Rightarrow \begin{array}{r} \overset{1}{2}6\\ \times 3\\ \hline 78 \end{array}$$

B

$$\begin{array}{r} 57\\ \times 5\\ \hline 35\\ 250\\ \hline 285 \end{array} \Rightarrow \begin{array}{r} \overset{3}{5}7\\ \times 5\\ \hline 285 \end{array}$$

2. Explain each example.

[A] $\begin{array}{r} \overset{4}{3}8\\ \times 6\\ \hline 228 \end{array}$ [B] $\begin{array}{r} \overset{2}{5}4\\ \times 7\\ \hline 378 \end{array}$ [C] $\begin{array}{r} \overset{2}{6}5\\ \times 4\\ \hline 260 \end{array}$ [D] $\begin{array}{r} \overset{1}{8}2\\ \times 9\\ \hline 738 \end{array}$

EXERCISES

1. Find the products.

[A] $\begin{array}{r} 48\\ \times 2\\ \hline \end{array}$ [B] $\begin{array}{r} 32\\ \times 6\\ \hline \end{array}$ [C] $\begin{array}{r} 16\\ \times 4\\ \hline \end{array}$ [D] $\begin{array}{r} 14\\ \times 6\\ \hline \end{array}$

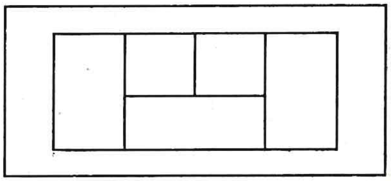
[E] $\begin{array}{r} 35\\ \times 9\\ \hline \end{array}$ [F] $\begin{array}{r} 62\\ \times 4\\ \hline \end{array}$ [G] $\begin{array}{r} 53\\ \times 8\\ \hline \end{array}$ [H] $\begin{array}{r} 71\\ \times 3\\ \hline \end{array}$

[I] $\begin{array}{r} 55\\ \times 8\\ \hline \end{array}$ [J] $\begin{array}{r} 46\\ \times 7\\ \hline \end{array}$ [K] $\begin{array}{r} 39\\ \times 2\\ \hline \end{array}$ [L] $\begin{array}{r} 18\\ \times 5\\ \hline \end{array}$

[M] $\begin{array}{r} 91\\ \times 5\\ \hline \end{array}$ [N] $\begin{array}{r} 83\\ \times 8\\ \hline \end{array}$ [O] $\begin{array}{r} 46\\ \times 5\\ \hline \end{array}$ [P] $\begin{array}{r} 55\\ \times 6\\ \hline \end{array}$

Think

Draw a figure like the one below, only larger. Now try to color this "map" with just 4 colors. All bordering regions should be different colors.



2. Solve the equations.

[A] $4 \times 6 \times 3 = \boxed{n}$

[B] $5 \times 7 \times 4 = \boxed{n}$

[C] $3 \times 6 \times 2 = \boxed{n}$

[D] $9 \times 4 \times 5 = \boxed{n}$

[E] $6 \times 7 \times 6 = \boxed{n}$

[F] $8 \times 7 \times 4 = \boxed{n}$

[G] $7 \times 2 \times 7 = \boxed{n}$

[H] $9 \times 8 \times 7 = \boxed{n}$

[I] $6 \times 9 \times 8 = \boxed{n}$

Factors with 3 and 4 digits

DISCUSSION EXERCISES

Find the products. Then solve the equations.

1. [A] 2×400 [D] $2 \times 423 = (2 \times 400) + (2 \times 20) + (2 \times n)$
[B] 2×20 [E] $2 \times 423 = 800 + n + 6$
[C] 2×3 [F] $2 \times 423 = n$

2. [A] 3×300 [D] $3 \times 326 = (3 \times n) + (3 \times 20) + (3 \times 6)$
[B] 3×20 [E] $3 \times 326 = 900 + 60 + n$
[C] 3×6 [F] $3 \times 326 = n$

3. [A] 4×500 [D] $4 \times 536 = (4 \times 500) + (4 \times n) + (4 \times 6)$
[B] 4×30 [E] $4 \times 536 = n + 120 + 24$
[C] 4×6 [F] $4 \times 536 = n$

4. Explain each step and give the missing numbers.

Step 1	Step 2	Step 3	Step 4
$\begin{array}{r} 347 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 347 \\ \times 5 \\ \hline 35 \\ \end{array}$	$\begin{array}{r} 347 \\ \times 5 \\ \hline 35 \\ 200 \\ \end{array}$	$\begin{array}{r} 347 \\ \times 5 \\ \hline 35 \\ 200 \\ 1500 \\ \hline \end{array}$
$5 \times 7 = n$	$5 \times 40 = n$	$5 \times 300 = n$	Add

EXERCISES

Find the products.

1. $\begin{array}{r} 213 \\ \times 3 \\ \hline \end{array}$ **2.** $\begin{array}{r} 416 \\ \times 2 \\ \hline \end{array}$ **3.** $\begin{array}{r} 532 \\ \times 4 \\ \hline \end{array}$ **4.** $\begin{array}{r} 618 \\ \times 3 \\ \hline \end{array}$ **5.** $\begin{array}{r} 729 \\ \times 5 \\ \hline \end{array}$

6. $\begin{array}{r} 843 \\ \times 6 \\ \hline \end{array}$ **7.** $\begin{array}{r} 2531 \\ \times 2 \\ \hline \end{array}$ **8.** $\begin{array}{r} 3618 \\ \times 3 \\ \hline \end{array}$ **9.** $\begin{array}{r} 7654 \\ \times 8 \\ \hline \end{array}$ **10.** $\begin{array}{r} 9315 \\ \times 7 \\ \hline \end{array}$

DISCUSSION EXERCISES

1. Explain the short cuts in the examples below.

A

$$\begin{array}{r} 436\\ \times 4\\ \hline 24\\ 120\\ 1600\\ \hline 1744 \end{array} \Rightarrow \begin{array}{r} {\scriptstyle 1\,2\;\,}\\ 436\\ \times 4\\ \hline 1744 \end{array}$$

B

$$\begin{array}{r} 5{,}486\\ \times 3\\ \hline 18\\ 240\\ 1{,}200\\ 15{,}000\\ \hline 16{,}458 \end{array} \Rightarrow \begin{array}{r} {\scriptstyle 1\;\;2\,1\;\,}\\ 5{,}486\\ \times 3\\ \hline 16{,}458 \end{array}$$

2. Explain each example.

[**A**] $\begin{array}{r} {\scriptstyle 1\,2\,4\;\,}\\ 1247\\ \times 6\\ \hline 7482 \end{array}$ [**B**] $\begin{array}{r} {\scriptstyle 1\,4\;\,}\\ 329\\ \times 5\\ \hline 1645 \end{array}$ [**C**] $\begin{array}{r} {\scriptstyle 1\,3\,2\;\,}\\ 1285\\ \times 4\\ \hline 5140 \end{array}$ [**D**] $\begin{array}{r} {\scriptstyle 5\;\;4\,1\;\,}\\ 3{,}652\\ \times 8\\ \hline 29{,}216 \end{array}$

EXERCISES

Find the products.

1. $\begin{array}{r} 348\\ \times 4\\ \hline \end{array}$ **2.** $\begin{array}{r} 217\\ \times 5\\ \hline \end{array}$ **3.** $\begin{array}{r} 618\\ \times 2\\ \hline \end{array}$ **4.** $\begin{array}{r} 521\\ \times 4\\ \hline \end{array}$ **5.** $\begin{array}{r} 603\\ \times 9\\ \hline \end{array}$ **6.** $\begin{array}{r} 735\\ \times 2\\ \hline \end{array}$

7. $\begin{array}{r} 642\\ \times 3\\ \hline \end{array}$ **8.** $\begin{array}{r} 815\\ \times 6\\ \hline \end{array}$ **9.** $\begin{array}{r} 921\\ \times 3\\ \hline \end{array}$

10. $\begin{array}{r} 926\\ \times 2\\ \hline \end{array}$ **11.** $\begin{array}{r} 138\\ \times 6\\ \hline \end{array}$ **12.** $\begin{array}{r} 507\\ \times 8\\ \hline \end{array}$

13. $\begin{array}{r} 7651\\ \times 7\\ \hline \end{array}$ **14.** $\begin{array}{r} 3716\\ \times 4\\ \hline \end{array}$ **15.** $\begin{array}{r} 5481\\ \times 7\\ \hline \end{array}$

16. $\begin{array}{r} 6567\\ \times 6\\ \hline \end{array}$ **17.** $\begin{array}{r} 8035\\ \times 5\\ \hline \end{array}$ **18.** $\begin{array}{r} 7604\\ \times 4\\ \hline \end{array}$

Think

Study the pattern. Then solve the equations.

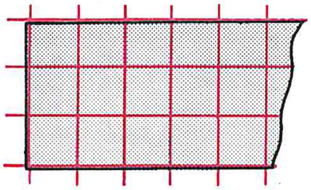
$$\begin{aligned} (1 \times 9) + 2 &= 11\\ (12 \times 9) + 3 &= 111\\ (123 \times 9) + 4 &= 1111\\ (1234 \times 9) + 5 &= 11{,}111\\ (12{,}345 \times 9) + 6 &= \boxed{n}\\ (123{,}456 \times 9) + 7 &= \boxed{n} \end{aligned}$$

Now check your answers.

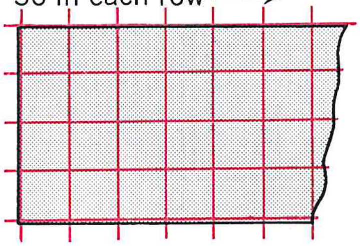
Area exercises

Find the area for each exercise.

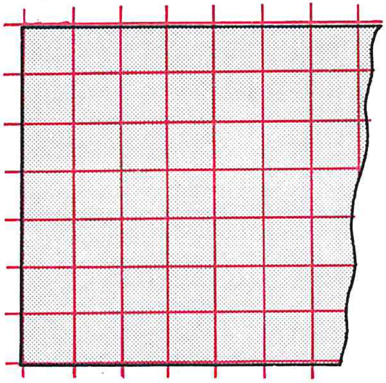
1. 16 in each row →



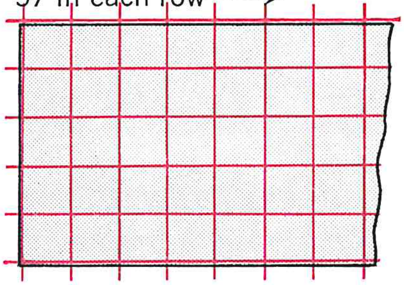
2. 36 in each row →



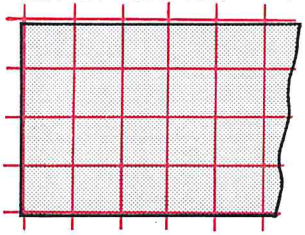
3. 28 in each row →



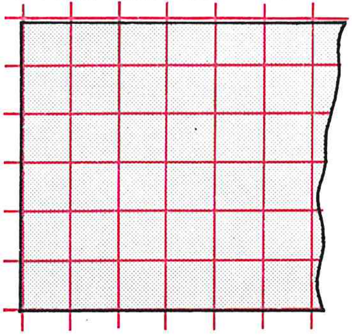
4. 57 in each row →



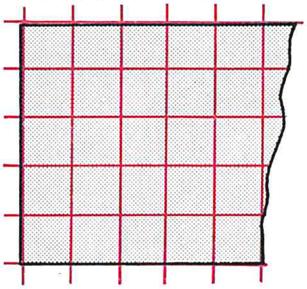
5. 167 in each row →



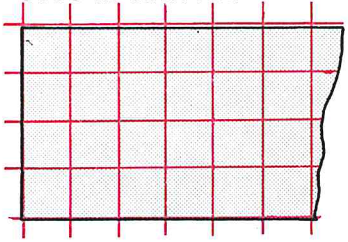
6. 349 in each row →



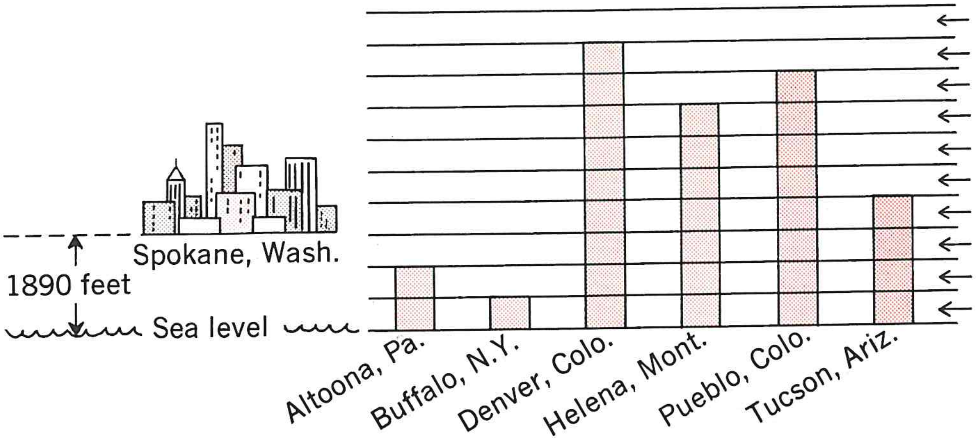
7. 2643 in each row →



8. 5368 in each row →



City altitudes



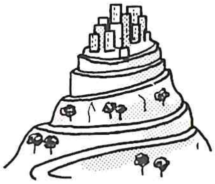
Each space shows about 585 feet above sea level.

City	Altitude
Altoona, Pa.	1170
Buffalo, N.Y.	585
Denver, Colo.	▨
Helena, Mont.	▨
Pueblo, Colo.	▨
Tucson, Ariz.	▨

1. Study the bar graph. Then make a table like the one on the right. Give the missing altitudes.

2. How much higher is Denver than Tucson?

3. The highest large city in the United States is Santa Fe, New Mexico. It is 6950 feet above sea level. How much higher is Santa Fe than Altoona?



4. Akron, Ohio, is about 875 feet above sea level. Salt Lake City, Utah, is at an altitude about 5 times as high as Akron. What is the altitude of Salt Lake City?

5. Phoenix, Arizona, is about 6 times as high as New Britain, Connecticut. If New Britain is 179 feet above sea level, what is the altitude of Phoenix?

★ **6.** Death Valley, California, is about 282 feet **below** sea level. How much higher is Buffalo than Death Valley?

2-digit factors

DISCUSSION EXERCISES

1. Explain step 1 in the diagram.
2. What principle is used for step 2?
3. Explain steps 3 and 4.
4. Solve: $43 \times 20 = n$
5. Explain this statement:

   Since $43 \times 2 = 86$, we know that $43 \times 20 = 860$.

43×20
① $43 \times (2 \times 10)$
② $(43 \times 2) \times 10$
③ 86×10
④ 860

EXERCISES

1. Find the products.

[A] Since $34 \times 2 = 68$, we know that $34 \times 20 = n$.
[B] Since $17 \times 3 = 51$, we know that $17 \times 30 = n$.
[C] Since $23 \times 4 = 92$, we know that $23 \times 40 = n$.
[D] Since $41 \times 5 = 205$, we know that $41 \times 50 = n$.
[E] Since $36 \times 7 = 252$, we know that $36 \times 70 = n$.
[F] Since $58 \times 4 = 232$, we know that $58 \times 40 = n$.
[G] Since $16 \times 8 = 128$, we know that $16 \times 80 = n$.
[H] Since $79 \times 3 = 237$, we know that $79 \times 30 = n$.
[I] Since $45 \times 6 = 270$, we know that $45 \times 60 = n$.
[J] Since $34 \times 9 = 306$, we know that $34 \times 90 = n$.

2. Find the products.

[A] $\begin{array}{r} 36 \\ \times 4 \\ \hline \end{array}$ $\begin{array}{r} 36 \\ \times 40 \\ \hline \end{array}$ [B] $\begin{array}{r} 53 \\ \times 2 \\ \hline \end{array}$ $\begin{array}{r} 53 \\ \times 20 \\ \hline \end{array}$ [C] $\begin{array}{r} 78 \\ \times 3 \\ \hline \end{array}$ $\begin{array}{r} 78 \\ \times 30 \\ \hline \end{array}$ [D] $\begin{array}{r} 93 \\ \times 5 \\ \hline \end{array}$ $\begin{array}{r} 93 \\ \times 50 \\ \hline \end{array}$

[E] $\begin{array}{r} 86 \\ \times 4 \\ \hline \end{array}$ $\begin{array}{r} 86 \\ \times 40 \\ \hline \end{array}$ [F] $\begin{array}{r} 75 \\ \times 6 \\ \hline \end{array}$ $\begin{array}{r} 75 \\ \times 60 \\ \hline \end{array}$ [G] $\begin{array}{r} 47 \\ \times 8 \\ \hline \end{array}$ $\begin{array}{r} 47 \\ \times 80 \\ \hline \end{array}$ [H] $\begin{array}{r} 56 \\ \times 4 \\ \hline \end{array}$ $\begin{array}{r} 56 \\ \times 40 \\ \hline \end{array}$

[I] $\begin{array}{r} 67 \\ \times 8 \\ \hline \end{array}$ $\begin{array}{r} 67 \\ \times 80 \\ \hline \end{array}$ [J] $\begin{array}{r} 65 \\ \times 4 \\ \hline \end{array}$ $\begin{array}{r} 65 \\ \times 40 \\ \hline \end{array}$ [K] $\begin{array}{r} 78 \\ \times 9 \\ \hline \end{array}$ $\begin{array}{r} 78 \\ \times 90 \\ \hline \end{array}$ [L] $\begin{array}{r} 37 \\ \times 6 \\ \hline \end{array}$ $\begin{array}{r} 37 \\ \times 60 \\ \hline \end{array}$

Study this example.

$$\begin{array}{ccccc} \text{Step 1} & & \text{Step 2} & & \text{Step 3} \\ \begin{array}{r} 47 \\ \times 50 \\ \hline 0 \end{array} & \longrightarrow & \begin{array}{r} \overset{3}{4}7 \\ \times 50 \\ \hline 50 \end{array} & \longrightarrow & \begin{array}{r} \overset{3}{4}7 \\ \times 50 \\ \hline 2350 \end{array} \end{array}$$

EXERCISES

1. Find the products.

[A] $\begin{array}{r} 63 \\ \times 20 \\ \hline \end{array}$ [B] $\begin{array}{r} 75 \\ \times 30 \\ \hline \end{array}$ [C] $\begin{array}{r} 81 \\ \times 40 \\ \hline \end{array}$

[D] $\begin{array}{r} 67 \\ \times 30 \\ \hline \end{array}$ [E] $\begin{array}{r} 45 \\ \times 60 \\ \hline \end{array}$ [F] $\begin{array}{r} 54 \\ \times 80 \\ \hline \end{array}$

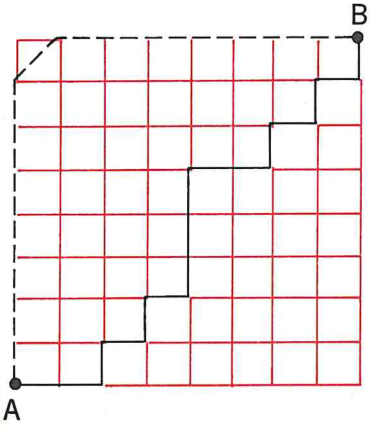
[G] $\begin{array}{r} 76 \\ \times 20 \\ \hline \end{array}$ [H] $\begin{array}{r} 49 \\ \times 60 \\ \hline \end{array}$ [I] $\begin{array}{r} 38 \\ \times 50 \\ \hline \end{array}$

[J] $\begin{array}{r} 67 \\ \times 90 \\ \hline \end{array}$ [K] $\begin{array}{r} 83 \\ \times 40 \\ \hline \end{array}$ [L] $\begin{array}{r} 92 \\ \times 50 \\ \hline \end{array}$

Think

In the figure below, which path, the solid or the dashed, is shorter between A and B?

Explain your answer.



2. Give the missing numbers.

[A] $25 \times 34 = (20 \times 34) + (n \times 34)$

[B] $36 \times 47 = (n \times 47) + (6 \times 47)$

[C] $57 \times 16 = (50 \times 16) + (n \times 16)$

[D] $15 \times 98 = (n \times 98) + (5 \times 98)$

[E] $49 \times 34 = (40 \times 34) + (9 \times n)$

[F] $61 \times 25 = (60 \times 25) + (n \times 25)$

3. Give the missing numbers.

[A] Since $30 \times 42 = 1260$, we know that $31 \times 42 = 1260 + n$.

[B] Since $20 \times 41 = 820$, we know that $21 \times 41 = 820 + n$.

[C] Since $30 \times 54 = 1620$, we know that $31 \times 54 = 1620 + n$.

[D] Since $40 \times 37 = 1480$, we know that $41 \times 37 = 1480 + n$.

[E] Since $60 \times 16 = 960$, we know that $61 \times 16 = 960 + n$.

[F] Since $30 \times 24 = 720$, we know that $32 \times 24 = 720 + n$.

More about 2-digit factors

DISCUSSION EXERCISES

$23 \times 46 = (20 \times 46) + (3 \times 46)$

Step 1	Step 2	Step 3
$\begin{array}{r} 46 \\ \times 23 \\ \hline 138 \end{array}$	$\begin{array}{r} 46 \\ \times 23 \\ \hline 138 \\ 920 \end{array}$	$\begin{array}{r} 46 \\ \times 23 \\ \hline 138 \\ 920 \\ \hline 1058 \end{array}$
$3 \times 46 = 138$	$20 \times 46 = 920$	$138 + 920 = 1058$

1. Explain step 1 in the example above.

2. Which part of the equation is completed in step 1?

3. Explain step 2.

4. Which part of the equation is completed in step 2?

5. Explain step 3.

6. Which part of the equation is completed in step 3?

EXERCISES

Find the products.

1. $\begin{array}{r} 46 \\ \times 23 \\ \hline \end{array}$ **2.** $\begin{array}{r} 54 \\ \times 12 \\ \hline \end{array}$ **3.** $\begin{array}{r} 71 \\ \times 24 \\ \hline \end{array}$ **4.** $\begin{array}{r} 32 \\ \times 14 \\ \hline \end{array}$ **5.** $\begin{array}{r} 65 \\ \times 24 \\ \hline \end{array}$ **6.** $\begin{array}{r} 56 \\ \times 57 \\ \hline \end{array}$ **7.** $\begin{array}{r} 28 \\ \times 75 \\ \hline \end{array}$

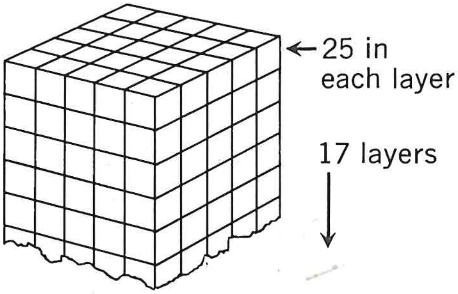
8. $\begin{array}{r} 61 \\ \times 78 \\ \hline \end{array}$ **9.** $\begin{array}{r} 47 \\ \times 36 \\ \hline \end{array}$ **10.** $\begin{array}{r} 55 \\ \times 26 \\ \hline \end{array}$ **11.** $\begin{array}{r} 34 \\ \times 96 \\ \hline \end{array}$ **12.** $\begin{array}{r} 48 \\ \times 48 \\ \hline \end{array}$ **13.** $\begin{array}{r} 39 \\ \times 93 \\ \hline \end{array}$ **14.** $\begin{array}{r} 57 \\ \times 61 \\ \hline \end{array}$

15. $\begin{array}{r} 99 \\ \times 22 \\ \hline \end{array}$ **16.** $\begin{array}{r} 68 \\ \times 45 \\ \hline \end{array}$ **17.** $\begin{array}{r} 15 \\ \times 48 \\ \hline \end{array}$ **18.** $\begin{array}{r} 49 \\ \times 60 \\ \hline \end{array}$ **19.** $\begin{array}{r} 69 \\ \times 96 \\ \hline \end{array}$ **20.** $\begin{array}{r} 25 \\ \times 68 \\ \hline \end{array}$ **21.** $\begin{array}{r} 76 \\ \times 67 \\ \hline \end{array}$

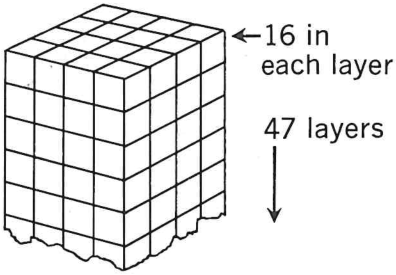
Volume exercises

Give the volume for each exercise.

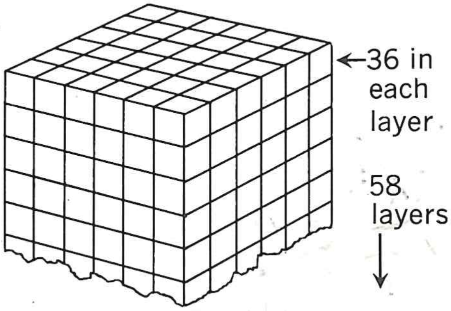
1.



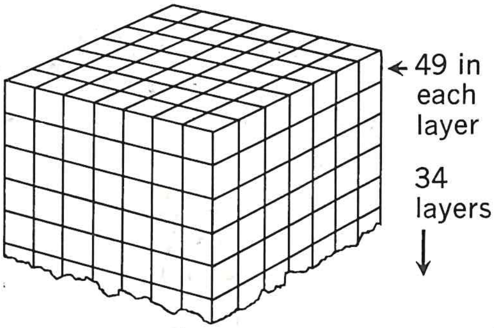
2.



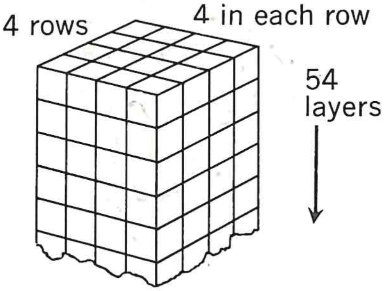
3.



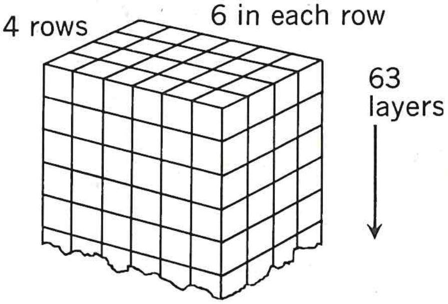
4.



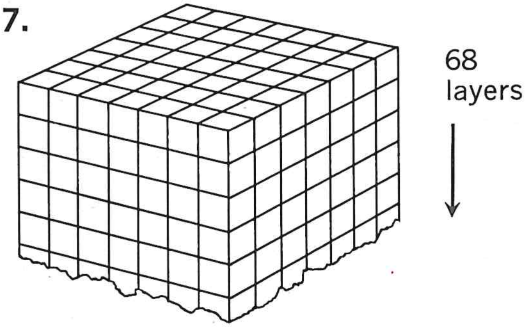
5.



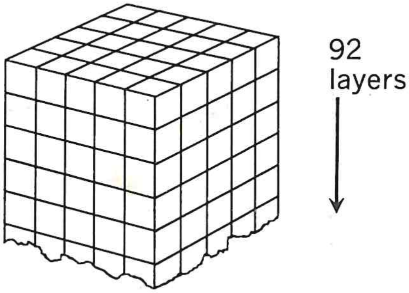
6.



7.

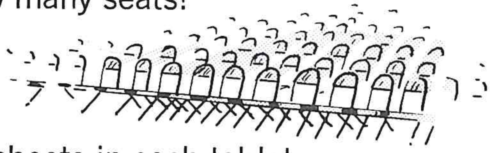


8.

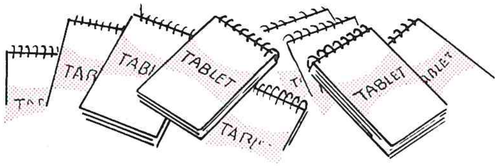


Short stories

1. Auditorium. 32 rows of seats.
25 seats in each row.
How many seats?



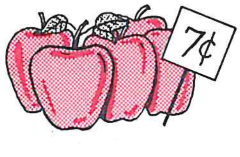
2. 48 sheets in each tablet.
36 tablets. How many sheets?



3. 17 teams. 12 on each team.
How many in all?

4. 74 children.
38 girls.
How many boys?

5. Apples. 7 cents each. How many apples can you buy for 63 cents?



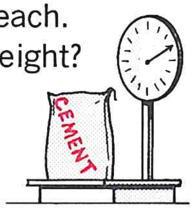
6. 15 shelves.
38 books on each shelf.
How many books?

7. 15,003 people at a ball game.
26 from Alaska.
How many not from Alaska?

8. 468 girls. 397 boys.
39 teachers. How many people in all?

9. Driving. 47 miles each hour.
25 hours. How far?

10. 34 sacks of cement.
Weigh 75 pounds each.
What is the total weight?



★ 11. Had 176 marbles.
Gave away 38.
Then gave away 66 more.
How many are left?

★ 12. Had 47 cents.
Earned 17 cents.
Spent 36 cents.
How many 5-cent candy bars can now be bought?

Records

1. Some records are made to play best when they go around 78 times each minute (78 revolutions per minute). How many times does a "78" record go around in 3 minutes?

2. The proper speed for some records is 45 revolutions per minute. If it takes 7 minutes to play a "45" record, how many times does it go around?

3. Some records play best when they go around about 33 times each minute. These are called long-playing (L.P.) records. If an L.P. takes 27 minutes to play, how many times does it go around?

4. Jan played 12 records one Saturday. Each one lasted 18 minutes. How long did Jan use her record player that day?

★ 5. Betty played 6 records, and each one lasted 3 minutes. If each was a 78 record, how many times did the turntable go around while the records were playing?

Multiplication exercises

1. Find the products.

[A] $\begin{array}{r}357\\ \times 3\\ \hline\end{array} \rightarrow \begin{array}{r}357\\ \times 30\\ \hline\end{array}$ [B] $\begin{array}{r}672\\ \times 4\\ \hline\end{array} \rightarrow \begin{array}{r}672\\ \times 40\\ \hline\end{array}$ [C] $\begin{array}{r}719\\ \times 5\\ \hline\end{array} \rightarrow \begin{array}{r}719\\ \times 50\\ \hline\end{array}$

[D] $\begin{array}{r}396\\ \times 8\\ \hline\end{array} \rightarrow \begin{array}{r}396\\ \times 80\\ \hline\end{array}$ [E] $\begin{array}{r}843\\ \times 4\\ \hline\end{array} \rightarrow \begin{array}{r}843\\ \times 40\\ \hline\end{array}$ [F] $\begin{array}{r}607\\ \times 7\\ \hline\end{array} \rightarrow \begin{array}{r}607\\ \times 70\\ \hline\end{array}$

2. Find the products.

[A] $\begin{array}{r}654\\ \times 2\\ \hline\end{array}$ $\begin{array}{r}654\\ \times 30\\ \hline\end{array} \longrightarrow \begin{array}{r}654\\ \times 32\\ \hline\end{array}$ [B] $\begin{array}{r}312\\ \times 6\\ \hline\end{array}$ $\begin{array}{r}312\\ \times 40\\ \hline\end{array} \longrightarrow \begin{array}{r}312\\ \times 46\\ \hline\end{array}$

[C] $\begin{array}{r}467\\ \times 5\\ \hline\end{array}$ $\begin{array}{r}467\\ \times 20\\ \hline\end{array} \longrightarrow \begin{array}{r}467\\ \times 25\\ \hline\end{array}$ [D] $\begin{array}{r}392\\ \times 8\\ \hline\end{array}$ $\begin{array}{r}392\\ \times 40\\ \hline\end{array} \longrightarrow \begin{array}{r}392\\ \times 48\\ \hline\end{array}$

[E] $\begin{array}{r}721\\ \times 3\\ \hline\end{array}$ $\begin{array}{r}721\\ \times 40\\ \hline\end{array} \longrightarrow \begin{array}{r}721\\ \times 43\\ \hline\end{array}$ [F] $\begin{array}{r}340\\ \times 5\\ \hline\end{array}$ $\begin{array}{r}340\\ \times 60\\ \hline\end{array} \longrightarrow \begin{array}{r}340\\ \times 65\\ \hline\end{array}$

[G] $\begin{array}{r}264\\ \times 4\\ \hline\end{array}$ $\begin{array}{r}264\\ \times 40\\ \hline\end{array} \longrightarrow \begin{array}{r}264\\ \times 44\\ \hline\end{array}$ [H] $\begin{array}{r}976\\ \times 2\\ \hline\end{array}$ $\begin{array}{r}976\\ \times 10\\ \hline\end{array} \longrightarrow \begin{array}{r}976\\ \times 12\\ \hline\end{array}$

3. Find the products.

[A] $\begin{array}{r}391\\ \times 12\\ \hline\end{array}$ [B] $\begin{array}{r}283\\ \times 41\\ \hline\end{array}$ [C] $\begin{array}{r}465\\ \times 26\\ \hline\end{array}$

[D] $\begin{array}{r}802\\ \times 81\\ \hline\end{array}$ [E] $\begin{array}{r}643\\ \times 56\\ \hline\end{array}$ [F] $\begin{array}{r}839\\ \times 83\\ \hline\end{array}$

[G] $\begin{array}{r}745\\ \times 28\\ \hline\end{array}$ [H] $\begin{array}{r}216\\ \times 35\\ \hline\end{array}$ [I] $\begin{array}{r}280\\ \times 62\\ \hline\end{array}$

[J] $\begin{array}{r}567\\ \times 14\\ \hline\end{array}$ [K] $\begin{array}{r}158\\ \times 10\\ \hline\end{array}$ [L] $\begin{array}{r}607\\ \times 68\\ \hline\end{array}$

Think

If you study this product carefully,

$$\begin{array}{r}12{,}345{,}679\\ \times 9\\ \hline 111{,}111{,}111\end{array}$$

you should be able to solve these equations quickly.

1. $12{,}345{,}679 \times 18 = \boxed{n}$
2. $12{,}345{,}679 \times 27 = \boxed{n}$
3. $12{,}345{,}679 \times 63 = \boxed{n}$

4. Find the products.

[A] Since $34 \times 2 = 68$, we know that $34 \times 20 = \boxed{n}$.

[B] Since $34 \times 2 = 68$, we know that $34 \times 200 = \boxed{n}$.

[C] Since $123 \times 4 = 492$, we know that $123 \times 40 = \boxed{n}$.

[D] Since $123 \times 4 = 492$, we know that $123 \times 400 = \boxed{n}$.

[E] Since $215 \times 5 = 1075$, we know that $215 \times 50 = \boxed{n}$.

[F] Since $215 \times 5 = 1075$, we know that $215 \times 500 = \boxed{n}$.

[G] Since $317 \times 8 = 2536$, we know that $317 \times 800 = \boxed{n}$.

[H] Since $691 \times 3 = 2073$, we know that $691 \times 300 = \boxed{n}$.

[I] Since $704 \times 6 = 4224$, we know that $704 \times 600 = \boxed{n}$.

5. Find the products.

[A] $\begin{array}{r}45\\ \times 3\\ \hline\end{array}$ $\quad\begin{array}{r}45\\ \times 30\\ \hline\end{array}$ $\quad\begin{array}{r}245\\ \times 3\\ \hline\end{array}$ $\quad\begin{array}{r}245\\ \times 30\\ \hline\end{array}$ $\quad\begin{array}{r}245\\ \times 300\\ \hline\end{array}$

[B] $\begin{array}{r}62\\ \times 4\\ \hline\end{array}$ $\quad\begin{array}{r}62\\ \times 40\\ \hline\end{array}$ $\quad\begin{array}{r}162\\ \times 4\\ \hline\end{array}$ $\quad\begin{array}{r}162\\ \times 40\\ \hline\end{array}$ $\quad\begin{array}{r}162\\ \times 400\\ \hline\end{array}$

[C] $\begin{array}{r}57\\ \times 6\\ \hline\end{array}$ $\quad\begin{array}{r}57\\ \times 60\\ \hline\end{array}$ $\quad\begin{array}{r}57\\ \times 600\\ \hline\end{array}$ $\quad\begin{array}{r}357\\ \times 60\\ \hline\end{array}$ $\quad\begin{array}{r}357\\ \times 600\\ \hline\end{array}$

[D] $\begin{array}{r}83\\ \times 5\\ \hline\end{array}$ $\quad\begin{array}{r}83\\ \times 50\\ \hline\end{array}$ $\quad\begin{array}{r}83\\ \times 500\\ \hline\end{array}$ $\quad\begin{array}{r}683\\ \times 50\\ \hline\end{array}$ $\quad\begin{array}{r}683\\ \times 500\\ \hline\end{array}$

6. Find the products.

[A] $\begin{array}{r}347\\ \times 600\\ \hline\end{array}$ $\quad$ [B] $\begin{array}{r}482\\ \times 300\\ \hline\end{array}$ $\quad$ [C] $\begin{array}{r}578\\ \times 200\\ \hline\end{array}$ $\quad$ [D] $\begin{array}{r}491\\ \times 800\\ \hline\end{array}$ $\quad$ [E] $\begin{array}{r}981\\ \times 700\\ \hline\end{array}$

7. Find the products.

[A] $\begin{array}{r}643\\ \times 2\\ \hline\end{array}$ $\quad\begin{array}{r}643\\ \times 50\\ \hline\end{array}$ $\quad\begin{array}{r}643\\ \times 300\\ \hline\end{array}$ $\quad\begin{array}{r}643\\ \times 352\\ \hline\end{array}$

[B] $\begin{array}{r}725\\ \times 3\\ \hline\end{array}$ $\quad\begin{array}{r}725\\ \times 10\\ \hline\end{array}$ $\quad\begin{array}{r}725\\ \times 500\\ \hline\end{array}$ $\quad\begin{array}{r}725\\ \times 513\\ \hline\end{array}$

[C] $\begin{array}{r}281\\ \times 4\\ \hline\end{array}$ $\quad\begin{array}{r}281\\ \times 20\\ \hline\end{array}$ $\quad\begin{array}{r}281\\ \times 100\\ \hline\end{array}$ $\quad\begin{array}{r}281\\ \times 124\\ \hline\end{array}$

[D] $\begin{array}{r}368\\ \times 3\\ \hline\end{array}$ $\quad\begin{array}{r}368\\ \times 70\\ \hline\end{array}$ $\quad\begin{array}{r}368\\ \times 500\\ \hline\end{array}$ $\quad\begin{array}{r}368\\ \times 573\\ \hline\end{array}$

Two 3-digit factors

263 × 514 = (200 × 514) + (60 × 514) + (3 × 514)

Step 1	Step 2	Step 3	Step 4
514 × 263 1542	514 × 263 1,542 30,840	514 × 263 1,542 30,840 102,800	514 × 263 1,542 30,840 102,800 135,182

DISCUSSION EXERCISES

1. Which part of the equation at the top is completed in step 1?
2. In step 2, is 514 being multiplied by 6 or by 60?
3. Which part of the equation is being completed in step 3?
4. In step 3, is 514 being multiplied by 2, 20, or 200?
5. Explain step 4.
6. Find the product 348 × 724 on the chalkboard using the four steps shown above.

EXERCISES

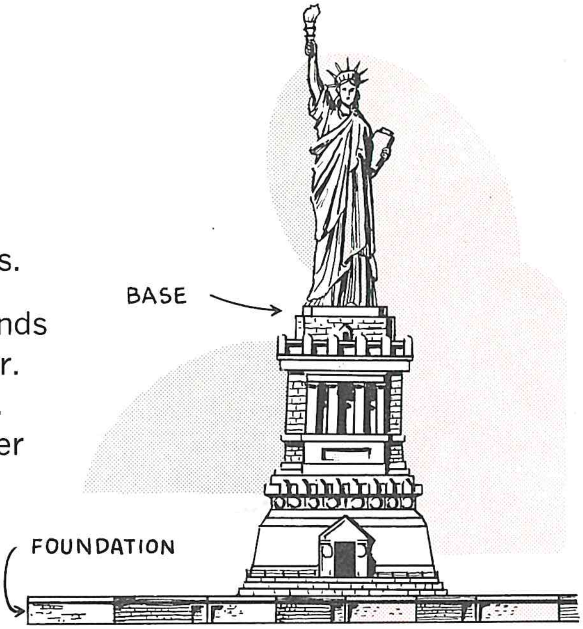
Find the products.

1. 621 ×243
2. 517 ×444
3. 823 ×615
4. 264 ×222
5. 575 ×314
6. 605 ×333
7. 316 ×634
8. 824 ×111
9. 967 ×121
10. 542 ×596
11. 568 ×666
12. 397 ×260
13. 908 ×307
14. 967 ×384
15. 659 ×555

The Statue of Liberty

The Statue of Liberty was a gift to the United States from France. It symbolizes freedom as well as friendship between the two countries.

The statue is 305 feet high. It stands on Liberty Island in New York Harbor. Visitors may go up inside the statue. As many as 30 or 40 people can enter the head of the statue at one time.



1. You can see from the table that the index finger is 8 feet long. What is the length of the finger in inches?

2. How many inches is it from the foundation to the torch?

Approximate Dimensions in **Feet**	
Base to torch	151
Foundation to torch	305
Length of hand	16
Index finger	8
Length of nose	$4\frac{1}{2}$
Right arm	42

3. There are 167 steps from the land level to the top of the pedestal. Inside the statue there are 168 steps from the base to the head. How many steps must you climb to get from the land level to the head?

4. In one month a guide made 264 trips from the land level to the head of the statue. How many steps did he climb (going up) that month?

5. The statue weighs about 225 tons. A ton is 2000 pounds. About how many pounds does the statue weigh?

Multiplying practice

Give an estimate for the first 20 products. Find all the products.

1. 37 ×6

2. 48 ×7

3. 56 ×4

4. 32 ×9

5. 68 ×5

6. 93 ×8

7. 64 ×28

8. 36 ×75

9. 52 ×43

10. 61 ×27

11. 50 ×36

12. 48 ×90

13. 327 ×5

14. 619 ×7

15. 823 ×9

16. 695 ×28

17. 824 ×69

18. 907 ×50

19. 762 ×348

20. 929 ×888

21. 654 ×289

22. 802 ×643

23. 765 ×567

24. 920 ×307

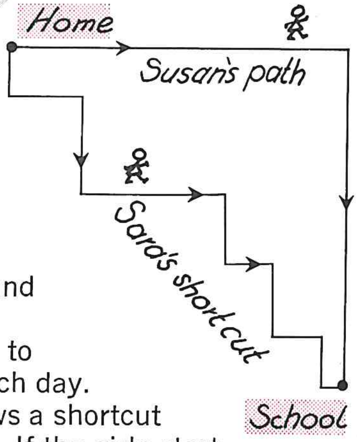
25. 700 ×64

26. 930 ×607

27. 807 ×444

28. 526 ×710

Think



Susan and her sister Sara walk to school each day. Sara knows a shortcut to school. If the girls start at the same time and walk at the same speed, who will get to school first? Explain your answer.

29. 362 × 46

30. 803 × 327

31. 9 × 7 × 4 × 6

32. 6 × 7 × 8 × 5

33. 4 × 9 × 8 × 6

34. 8 × 8 × 8 × 8

35. 8 × 7 × 326

36. 52 × 4 × 632

37. 8 × 125 × 356

38. 921 × 3 × 47

39. 25 × 31 × 4

40. 327 × 6 × 7

41. 4 × 389 × 0

42. 68 × 1 × 23

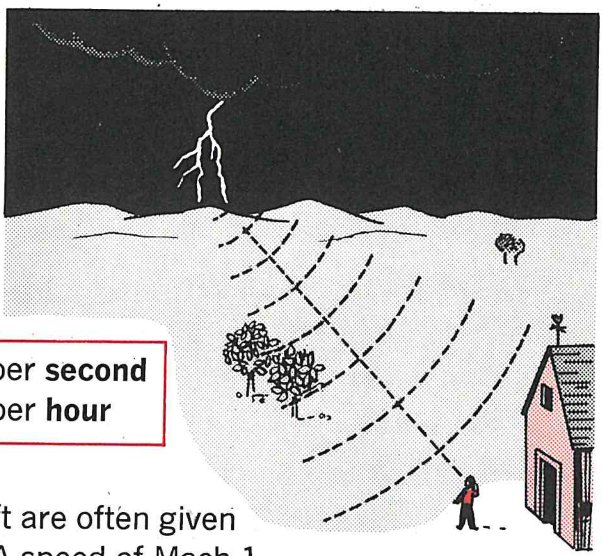
43. 17 × 4 × 62

44. 2 × 12 × 13

45. 20 × 21 × 22

46. 83 × 65 × 7

Speed of sound



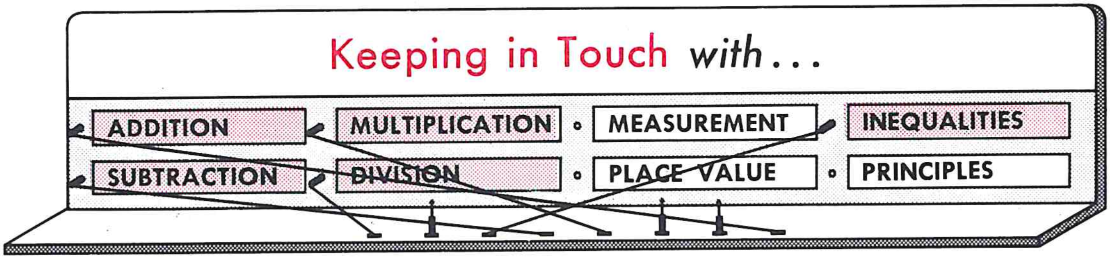
Have you ever seen lightning before you heard its thunder? We often **see** something happen before we **hear** it. This is because light travels much faster than sound.

Speed of light ..... 186,000 miles per **second**
Speed of sound .... 742 miles per **hour**

The speeds of supersonic aircraft are often given according to the speed of sound. A speed of Mach 1 is the speed of sound, Mach 2 is two times the speed of sound, Mach 3 is three times the speed of sound, and so on. Mach is the name of the scientist who made some important discoveries about sound.

1. How fast is [A] Mach 2, [B] Mach 3, [C] Mach 4?
2. The X–15 rocket plane can fly at speeds greater than Mach 5. How fast is Mach 5?
3. Some satellites travel at a speed that is about 23 times the speed of sound. How fast do they travel?
4. The earth travels in its orbit about the sun at a speed about 90 times the speed of sound. How fast does it travel?
5. The planet Mercury has an orbital speed that is about 146 times the speed of sound. How fast does Mercury travel in its orbit around the sun?

★ 6. Light travels at 669,600,000 miles per hour. How many miles per hour faster is the speed of light than the speed of sound?



1. Answer **T** (true) or **F** (false).

[A] $374{,}652 > 374{,}562$ [C] $921{,}354 = 912{,}354$ [E] $208{,}302 < 208{,}203$

[B] $483{,}007 < 483{,}010$ [D] $676{,}767 > 767{,}676$ [F] $543{,}210 > 543{,}120$

2. Match these.

[A] 6 less than 243 (Answer: d)	(a) $6 + 243$
[B] 243 divided by 6	(b) $6 - 243$
[C] 243 more than 6	(c) $243 \div 6$
[D] the product of 6 and 243	(d) $243 - 6$
[E] the difference of 243 and 6	(e) 6×243

3. Find the sums and differences.

[A] $\begin{array}{r}364\\287\\+521\\\hline\end{array}$ [B] $\begin{array}{r}614\\301\\+295\\\hline\end{array}$ [C] $\begin{array}{r}852\\361\\+725\\\hline\end{array}$ [D] $\begin{array}{r}916\\743\\+120\\\hline\end{array}$ [E] $\begin{array}{r}650\\843\\+921\\\hline\end{array}$

[F] $\begin{array}{r}7642\\-385\\\hline\end{array}$ [G] $\begin{array}{r}8603\\-105\\\hline\end{array}$ [H] $\begin{array}{r}7625\\-3468\\\hline\end{array}$ [I] $\begin{array}{r}9003\\-27\\\hline\end{array}$ [J] $\begin{array}{r}8080\\-808\\\hline\end{array}$

4. Find the products.

[A] Since $6 \times 9 = 54$, we know that $6 \times 90 = \boxed{n}$.

[B] Since $8 \times 7 = 56$, we know that $80 \times 70 = \boxed{n}$.

[C] Since $9 \times 8 = 72$, we know that $900 \times 8 = \boxed{n}$.

[D] Since $3 \times 6 = 18$, we know that $3 \times 600 = \boxed{n}$.

[E] Since $7 \times 5 = 35$, we know that $70 \times 50 = \boxed{n}$.

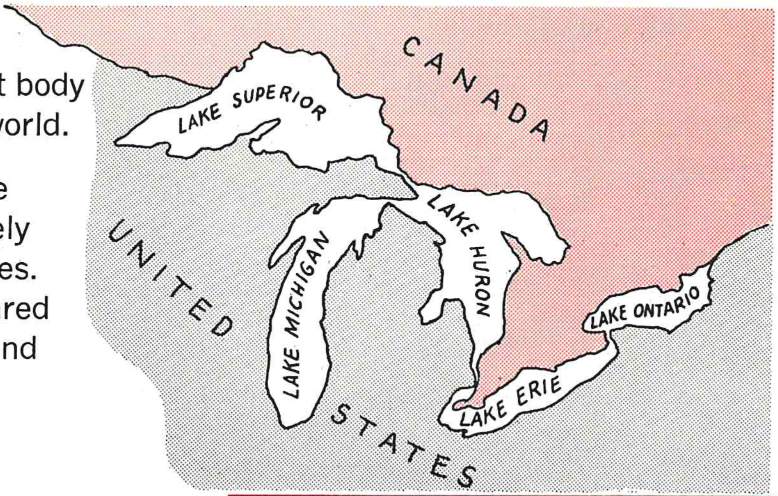
[F] Since $9 \times 7 = 63$, we know that $9 \times 700 = \boxed{n}$.

[G] $\begin{array}{r}60\\\times 5\\\hline\end{array}$ [H] $\begin{array}{r}80\\\times 7\\\hline\end{array}$ [I] $\begin{array}{r}90\\\times 9\\\hline\end{array}$ [J] $\begin{array}{r}60\\\times 50\\\hline\end{array}$ [K] $\begin{array}{r}80\\\times 70\\\hline\end{array}$ [L] $\begin{array}{r}90\\\times 90\\\hline\end{array}$

THE GREAT LAKES

Together the Great Lakes form the largest body of fresh water in the world.

Lake Michigan is the only lake that is entirely within the United States. The other four are shared by the United States and Canada.



Lake	Length (miles)	Greatest depth (ft)	Area (sq. mi)
Superior	350	1333	31,800
Michigan	307	923	22,400
Huron	206	750	23,000
Erie	241	210	9,910
Ontario	193	802	7,600

1. What is the area of Lake Erie and Lake Ontario together?
2. What is the area of Lake Superior, Lake Michigan, and Lake Huron together?
3. What is the total area of all five lakes?
4. How much greater is the area of Lake Huron than that of Lake Erie?
5. How much deeper is Lake Superior than Lake Huron?
6. The Nile River (world's longest) is about 21 times as long as Lake Ontario. About how long is the Nile?
7. The average depth of the Pacific Ocean is about 15 times as deep as the greatest depth of Lake Michigan. What is the average depth of the Pacific?
8. The distance around the earth is about 129 times the length of Lake Ontario. About how far is it around the earth?

Chapter review

Find the products.

1. $\begin{array}{r} 54 \\ \times 4 \\ \hline \end{array}$ **2.** $\begin{array}{r} 26 \\ \times 3 \\ \hline \end{array}$ **3.** $\begin{array}{r} 75 \\ \times 5 \\ \hline \end{array}$ **4.** $\begin{array}{r} 82 \\ \times 2 \\ \hline \end{array}$ **5.** $\begin{array}{r} 94 \\ \times 7 \\ \hline \end{array}$

6. $\begin{array}{r} 67 \\ \times 8 \\ \hline \end{array}$ **7.** $\begin{array}{r} 361 \\ \times 5 \\ \hline \end{array}$ **8.** $\begin{array}{r} 278 \\ \times 6 \\ \hline \end{array}$ **9.** $\begin{array}{r} 763 \\ \times 4 \\ \hline \end{array}$ **10.** $\begin{array}{r} 403 \\ \times 9 \\ \hline \end{array}$

11. $\begin{array}{r} 921 \\ \times 3 \\ \hline \end{array}$ **12.** $\begin{array}{r} 3462 \\ \times 5 \\ \hline \end{array}$ **13.** $\begin{array}{r} 5278 \\ \times 3 \\ \hline \end{array}$ **14.** $\begin{array}{r} 6439 \\ \times 7 \\ \hline \end{array}$ **15.** $\begin{array}{r} 8432 \\ \times 6 \\ \hline \end{array}$

16. $\begin{array}{r} 32 \\ \times 44 \\ \hline \end{array}$ **17.** $\begin{array}{r} 67 \\ \times 28 \\ \hline \end{array}$ **18.** $\begin{array}{r} 46 \\ \times 59 \\ \hline \end{array}$ **19.** $\begin{array}{r} 38 \\ \times 51 \\ \hline \end{array}$ **20.** $\begin{array}{r} 60 \\ \times 75 \\ \hline \end{array}$

21. $\begin{array}{r} 98 \\ \times 67 \\ \hline \end{array}$ **22.** $\begin{array}{r} 347 \\ \times 20 \\ \hline \end{array}$ **23.** $\begin{array}{r} 618 \\ \times 36 \\ \hline \end{array}$ **24.** $\begin{array}{r} 514 \\ \times 48 \\ \hline \end{array}$ **25.** $\begin{array}{r} 396 \\ \times 52 \\ \hline \end{array}$

26. $\begin{array}{r} 821 \\ \times 95 \\ \hline \end{array}$ **27.** $\begin{array}{r} 367 \\ \times 172 \\ \hline \end{array}$ **28.** $\begin{array}{r} 280 \\ \times 843 \\ \hline \end{array}$ **29.** $\begin{array}{r} 546 \\ \times 261 \\ \hline \end{array}$ **30.** $\begin{array}{r} 904 \\ \times 555 \\ \hline \end{array}$

31. $\begin{array}{r} 831 \\ \times 723 \\ \hline \end{array}$ **32.** $\begin{array}{r} 766 \\ \times 803 \\ \hline \end{array}$

33. $\begin{array}{r} 685 \\ \times 720 \\ \hline \end{array}$ **34.** $\begin{array}{r} 864 \\ \times 271 \\ \hline \end{array}$

35. 7 × 9 × 8

36. 9 × 6 × 7 × 7

37. 127 × 2 × 314

38. 326 × 218 × 2

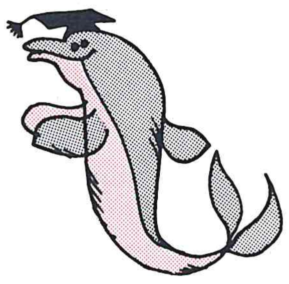
39. 34 × 26 × 5

40. 26 × 538 × 1

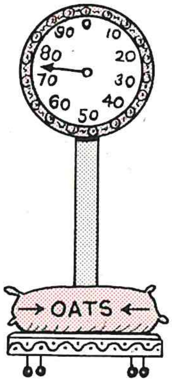
Think

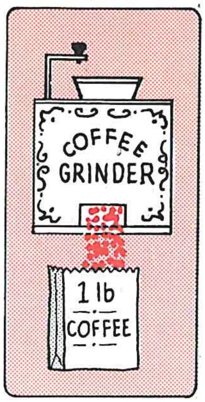
Study the pattern. Then copy the equations giving the missing numbers.

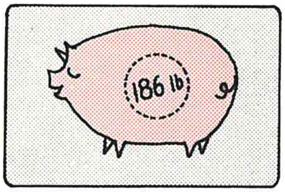
1 × 8 = 10 − 2
2 × 8 = 20 − 4
3 × 8 = 30 − 6
4 × 8 = 40 − 8
5 × 8 = ▯ − ▯
6 × 8 = ▯ − ▯
7 × 8 = ▯ − ▯
13 × 8 = ▯ − ▯

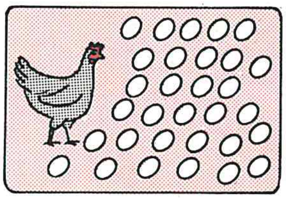


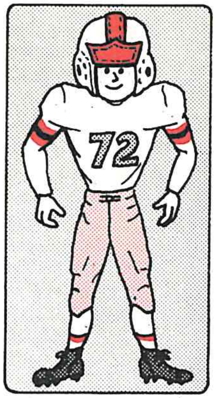
SHORT STORIES







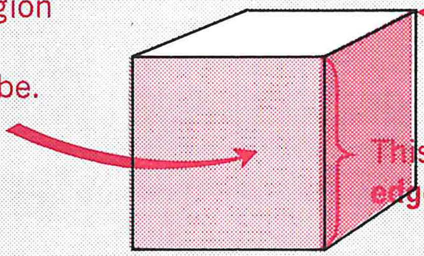




1. Oats. 48 sacks. 75 pounds each. How many pounds in all?
2. Coffee. 94 pounds. 16 ounces in each pound.
   How many ounces?
3. 440 yards. 36 inches in a yard. How many inches?
4. 24 hours in a day. 365 days in a year.
   How many hours in a year?
5. 640 acres in 1 square mile. National park
   covers 127 square miles. How many acres?
6. 27 hogs. Weigh about 186 pounds each. Total weight?
7. 156 girls. 87 boys. How many children?
8. 96 dozen eggs. How many eggs?
9. 215 children. 147 boys. How many girls?
10. 72 players. 8 on a team. How many teams?
11. 72 teams. 8 on a team. How many players?
12. 72 on each team. 8 teams. How many players?
13. First trip: 317 miles. Second trip: 189 miles. How
    much farther was the first trip than the second?
14. A mile is 8 furlongs. How many furlongs in 2763 miles?

GEOMETRY Unit 3

Cubes



This square region is one of the **faces** of the cube.

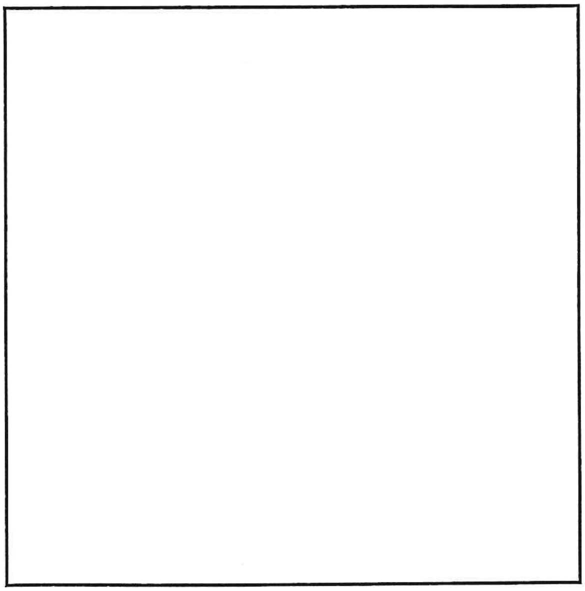
This point is one of the **vertices** of the cube. It is called a **vertex** of the cube.

This segment is one of the **edges** of the cube.

A CUBE

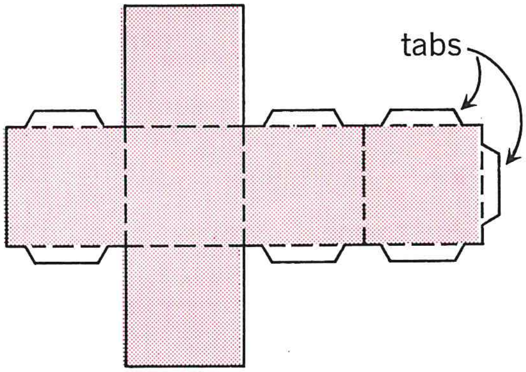
1. You can make a model of a cube as follows.

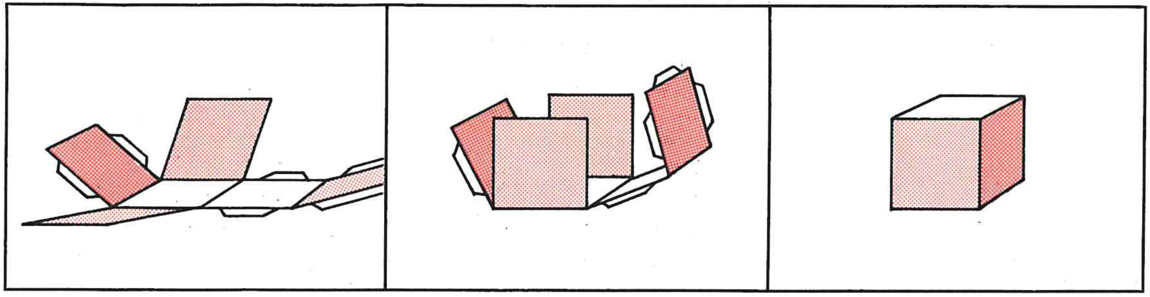
   [A] Cut a 3-inch square from a piece of cardboard. If you have done this carefully, your square should fit on this 3-inch square.



   [B] Use your square to draw a pattern like this on your paper.

   [C] Be sure to draw the tabs in the positions shown.





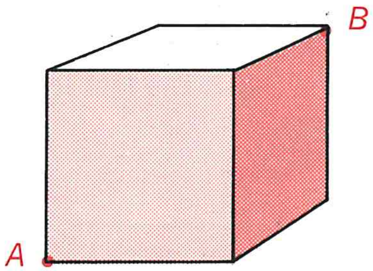
[D] Then cut out the pattern and fold as shown in the picture above. Use paste or cellophane tape to stick the tabs in place.

2. [A] How many faces does your cube have?

[B] How many edges does your cube have?

[C] How many vertices does your cube have?

3. Use your cube to help you answer these questions.



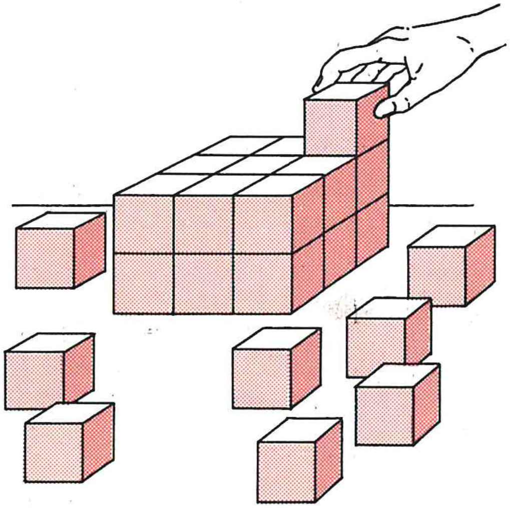
[A] What is the fewest number of edges you can move your finger along to get from *A* to *B*?

[B] What is the greatest number of edges you can move along to get from *A* to *B* if you never go along the same edge twice?

[C] Can you get from *A* to *B* by going along 4 different edges?

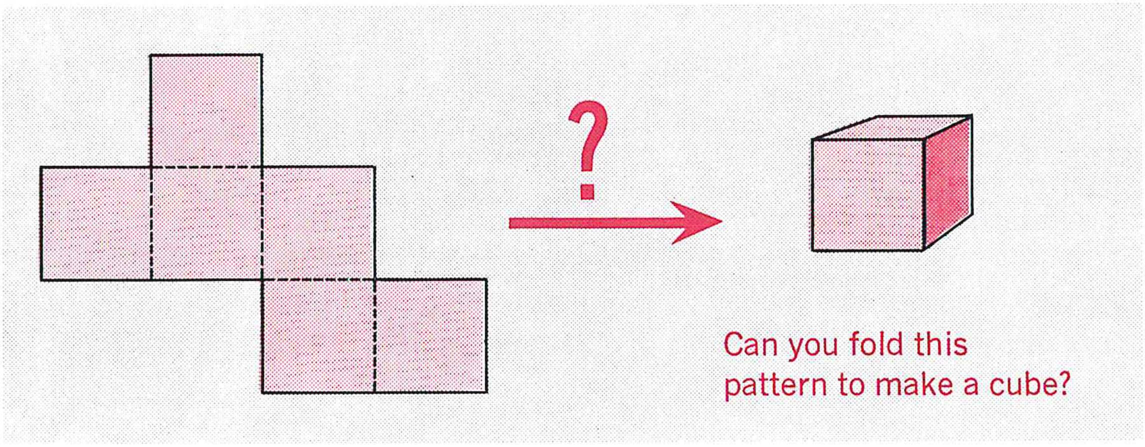
[D] Can you get from *A* to *B* by going along 5 different edges?

4. [A] Suppose you had 27 cubes stacked on your desk as shown (3 layers of 9 cubes each). If you could not pick up the stack, how many of the cubes could you not see at all?



[B] If you could pick up the stack and look at all sides, how many of the cubes could you not see at all?

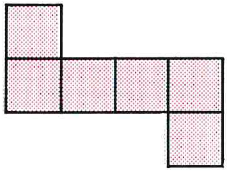
Patterns for cubes



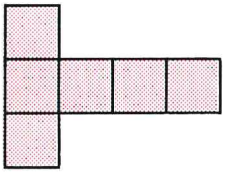
Can you fold this pattern to make a cube?

1. Each of the patterns shown on this page can be folded to make a cube. Select at least one of these patterns and use it to make a cube. It may be helpful to put tabs on the pattern as you did in the last lesson.

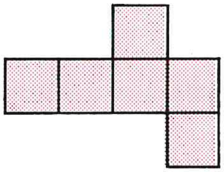
[A]



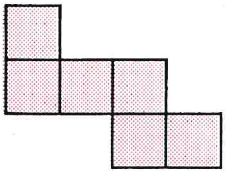
[B]



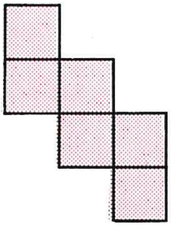
[C]



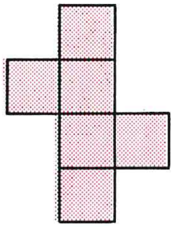
[D]



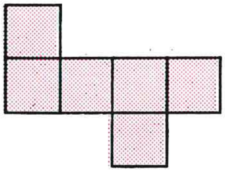
[E]



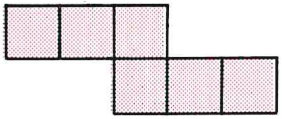
[F]



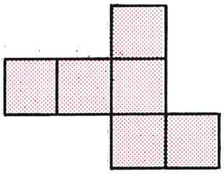
[G]



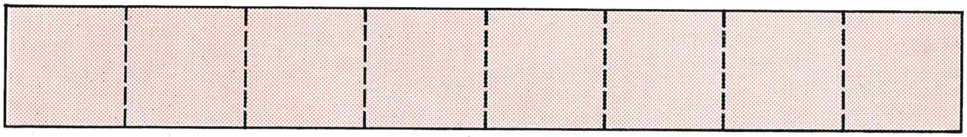
[H]



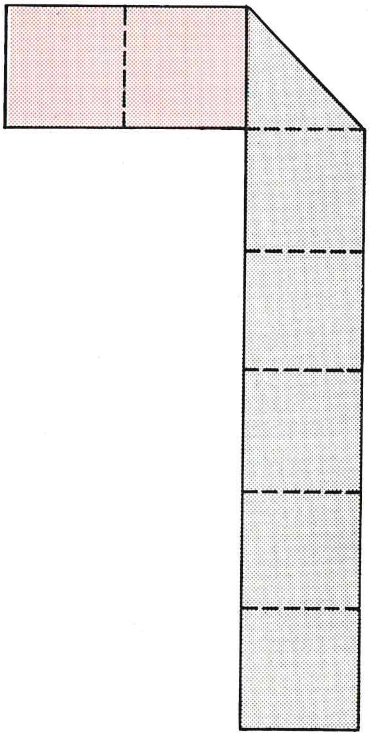
[I]



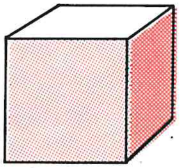
2. [A] Trace the pattern below and cut it out.



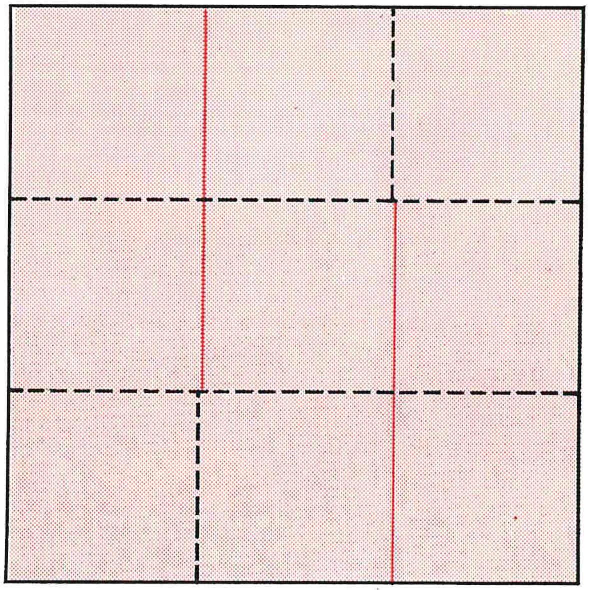
[B] Now fold the strip as in the figure.



[C] Continue folding so that you have a cube with each face the size of one of the squares on the strip.



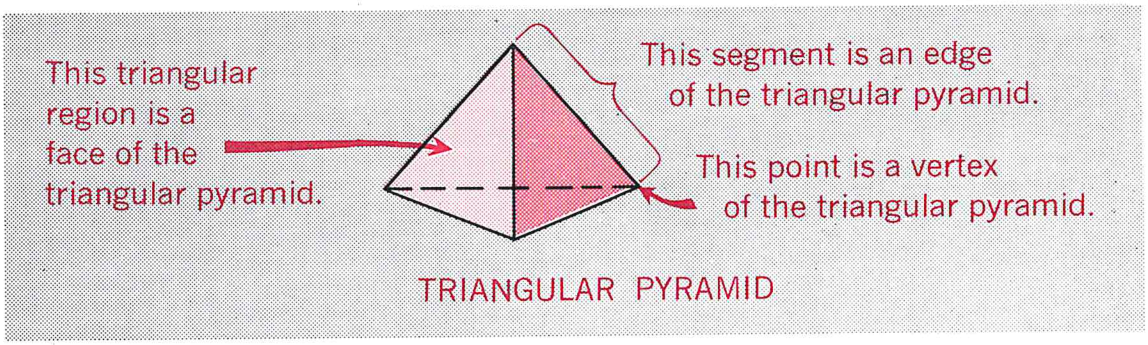
★**3.** Trace the square. Color the interior of the square. Cut along the solid red lines.



[A] Folding only along dashed lines, make a cube.

[B] Folding only along dashed lines, make a cube that has all its faces colored.

Triangular pyramids



This triangular region is a face of the triangular pyramid.

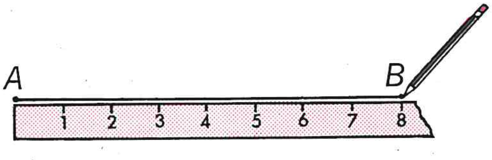
This segment is an edge of the triangular pyramid.

This point is a vertex of the triangular pyramid.

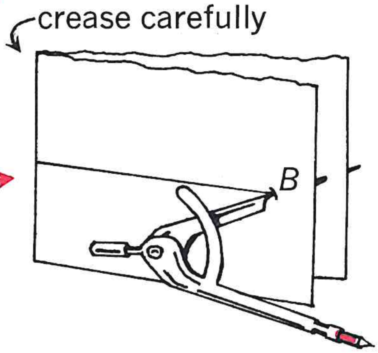
TRIANGULAR PYRAMID

1. You can make a model of a triangular pyramid in the following way.

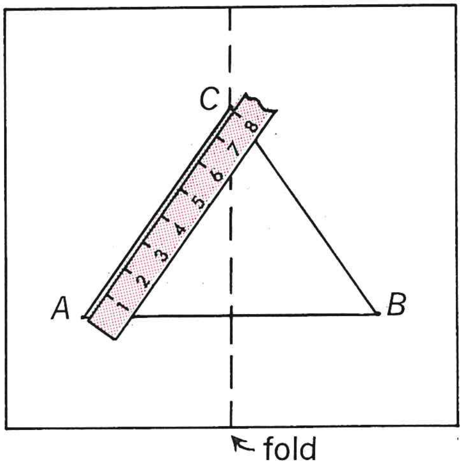
[**A**] Use your ruler to draw an 8-inch segment, *AB*, near the bottom of a sheet of paper.



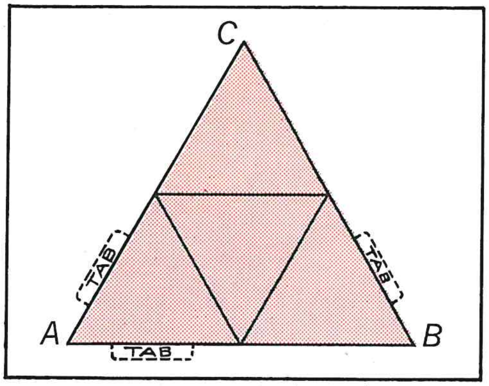
[**B**] Put the tip of your compass through both points *A* and *B* and fold the paper as shown.



[**C**] Use your ruler to find the point *C*, on the fold, that is 8 inches from *A*. Then draw lines from *A* and *B* to this point.

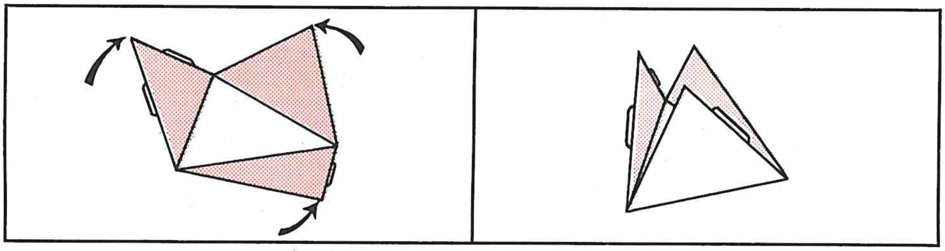


[D] Use your ruler to mark the point 4 inches from both *A* and *B*, the point 4 inches from both *A* and *C*, and the point 4 inches from both *C* and *B*. Connect these three points as shown.



[E] Be sure to make tabs as shown.

[F] Now cut out the figure carefully and fold as shown below. Use paste or tape to fasten the tabs.



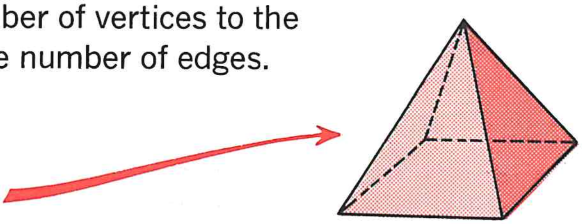
2. [A] How many faces does the triangular pyramid have?

   [B] How many edges does it have? [C] How many vertices does it have?

3. For each figure listed, add the number of vertices to the number of faces. Then subtract the number of edges. What is the answer?

   [A] triangular pyramid [B] cube

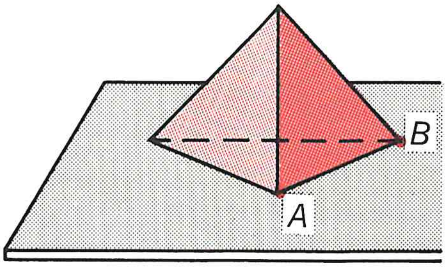
   [C] a pyramid with one square face



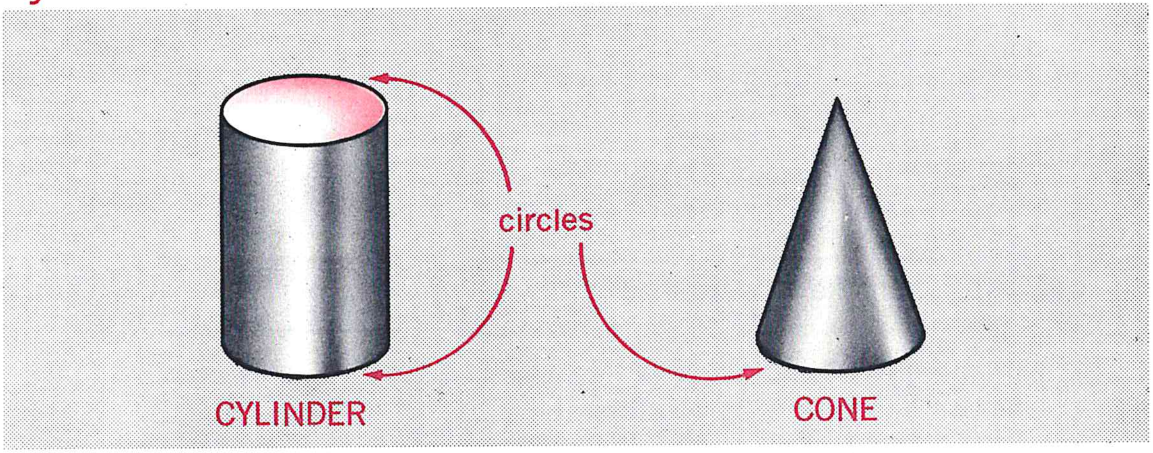
4. Place your triangular pyramid on your desk. Use it to answer the questions.

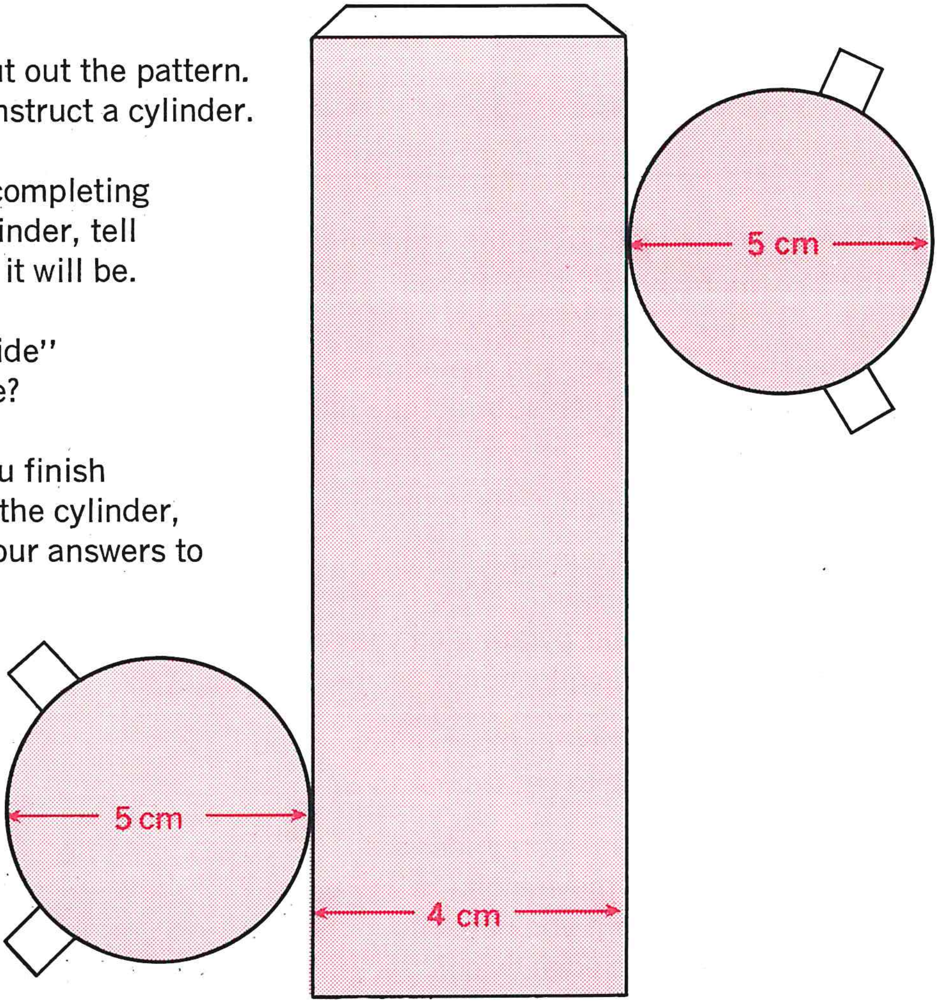
   [A] What is the fewest number of edges you can move along to get from *A* to *B*?

   [B] What is the greatest number of edges you can move along to get from *A* to *B* if you never go along the same edge twice?



Cylinders and cones





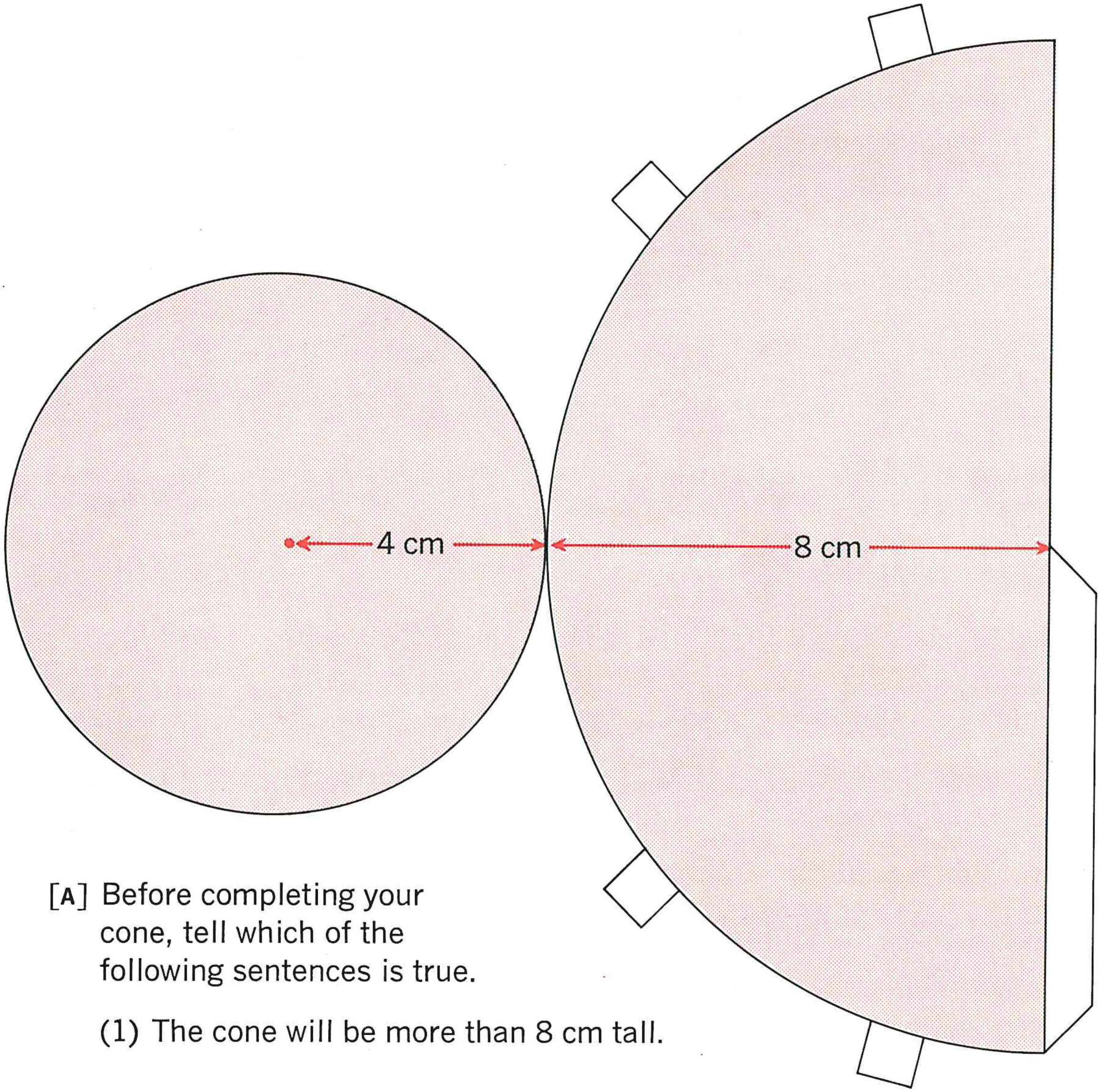
1. Copy and cut out the pattern. Use it to construct a cylinder.

[**A**] Before completing your cylinder, tell how tall it will be.

[**B**] How "wide" will it be?

[**C**] After you finish making the cylinder, check your answers to **A** and **B.**

2. Copy and cut out this pattern. Use it to construct a cone.



[**A**] Before completing your cone, tell which of the following sentences is true.

(1) The cone will be more than 8 cm tall.

(2) The cone will be 8 cm tall.

(3) The cone will be less than 8 cm tall.

[**B**] After you complete your cone, check and see if you selected the right statement.

CHAPTER 8 Dividing

Estimation

The exercises below will help you find quotients in division problems such as $5\overline{)47}$.

EXERCISES

1. Give the number pair that should go in each gray space.

Pairs: $\begin{matrix}1\\2\end{matrix}$ $\begin{matrix}2\\3\end{matrix}$ $\begin{matrix}3\\4\end{matrix}$ $\begin{matrix}4\\5\end{matrix}$ $\begin{matrix}5\\6\end{matrix}$ $\begin{matrix}6\\7\end{matrix}$ $\begin{matrix}7\\8\end{matrix}$ $\begin{matrix}8\\9\end{matrix}$ $\begin{matrix}9\\10\end{matrix}$

[A] ▨ $\times 5 < 47$
$\times 5 > 47$

(Answer: pair $\begin{matrix}9\\10\end{matrix}$)

[B] ▨ $\times 2 < 11$
$\times 2 > 11$

[C] ▨ $\times 3 < 26$
$\times 3 > 26$

[D] ▨ $\times 4 < 30$
$\times 4 > 30$

[E] ▨ $\times 5 < 43$
$\times 5 > 43$

[F] ▨ $\times 6 < 27$
$\times 6 > 27$

[G] ▨ $\times 4 < 18$
$\times 4 > 18$

[H] ▨ $\times 3 < 29$
$\times 3 > 29$

[I] ▨ $\times 7 < 46$
$\times 7 > 46$

[J] ▨ $\times 8 < 45$
$\times 8 > 45$

[K] ▨ $\times 9 < 40$
$\times 9 > 40$

[L] ▨ $\times 9 < 50$
$\times 9 > 50$

[M] ▨ $\times 8 < 54$
$\times 8 > 54$

[N] ▨ $\times 7 < 55$
$\times 7 > 55$

[O] ▨ $\times 6 < 51$
$\times 6 > 51$

[P] ▨ $\times 9 < 75$
$\times 9 > 75$

[Q] ▨ $\times 8 < 63$
$\times 8 > 63$

[R] ▨ $\times 7 < 68$
$\times 7 > 68$

[S] ▨ $\times 5 < 39$
$\times 5 > 39$

[T] ▨ $\times 6 < 53$
$\times 6 > 53$

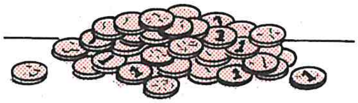
2. Find the **largest** whole number that will make the sentence true. Then copy the completed sentence.

[A] $n \times 5 < 47$
(Answer: $9 \times 5 < 47$)
[B] $n \times 3 < 22$
[C] $n \times 4 < 35$
[D] $n \times 5 < 47$
[E] $n \times 3 < 20$
[F] $n \times 6 < 38$
[G] $n \times 7 < 50$
[H] $n \times 7 < 65$
[I] $n \times 8 < 43$
[J] $n \times 9 < 80$
[K] $n \times 5 < 49$
[L] $n \times 4 < 19$
[M] $n \times 6 < 34$
[N] $n \times 3 < 29$
[O] $n \times 7 < 60$
[P] $n \times 9 < 51$
[Q] $n \times 8 < 15$
[R] $n \times 6 < 59$
[S] $n \times 7 < 40$
[T] $n \times 9 < 10$
[U] $n \times 8 < 66$
[V] $n \times 6 < 58$

How many?

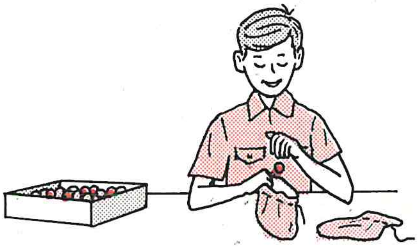
EXERCISES

1. [A] How many nickels can you get for 47 pennies?
[B] How many pennies will be left?



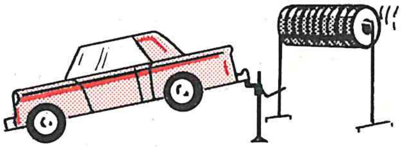
2. Joe had 42 marbles in a box. He put 5 marbles in each sack.

[A] How many sacks did he use?
[B] How many marbles were left over?



3. There were 38 new tires at a garage.

[A] If 4 tires were put on each car, how many cars could have all new tires?
[B] How many extra tires would there be?



4. There were 49 boys at the park. If they were divided into baseball teams with 9 on a team,

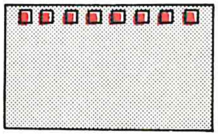
[A] how many teams could be made?
[B] how many extras would there be?

5. A carton holds 6 bottles of soda.

[A] How many cartons can you fill if you have 52 bottles?
[B] How many extra bottles will there be?

6. The coach asked 45 boys to line up in rows of 7.

[A] How many full rows did they make?
[B] How many boys were left?



7. Kay had 59 stamps. She pasted 8 stamps in each row.

[A] How many rows of 8 could she make?
[B] How many extra stamps would she have to start another row?

Quotients and remainders

A

How many fives in 47?

Think:

$$\begin{array}{rl} 9 & \leftarrow\!\urcorner \\ 5\overline{)\,47} & \\ -45 & \leftarrow 9\times 5 \\ \hline \end{array}$$

Since we can subtract as many as 9 fives from 47, the quotient is 9.

Write:

$$\begin{array}{r} 9 \\ 5\overline{)\,47} \\ -45 \\ \hline 2 \end{array}$$

B

How many sevens in 55?

Think:

$$\begin{array}{rl} 7 & \leftarrow\!\urcorner \\ 7\overline{)\,55} & \\ -49 & \leftarrow 7\times 7 \\ \hline \end{array}$$

Since we can subtract as many as 7 sevens from 55, the quotient is 7.

Write:

$$\begin{array}{r} 7 \\ 7\overline{)\,55} \\ -49 \\ \hline 6 \end{array}$$

The dividing has been completed correctly in example C. Study this example to review the meaning of the words **divisor, quotient, remainder** and **dividend.**

C

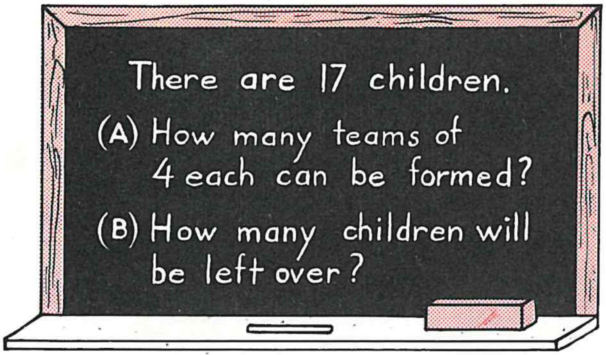
$$\begin{array}{rrl} & 8 & \leftarrow \text{quotient} \\ \text{divisor} \rightarrow & 6\overline{)50} & \leftarrow \text{dividend} \\ & -48 & \\ \hline & 2 & \leftarrow \text{remainder} \end{array}$$

DISCUSSION EXERCISES

1. Give the quotients for examples A and B.
   The quotient could not be 8 in example B. Why?

2. Give the remainders for examples A and B. In each example, how does the remainder compare with the divisor?

3. Study the pictures on the next page. Explain why Dick's first way of dividing 17 by 4 did not give the best answer to the problem about the teams.

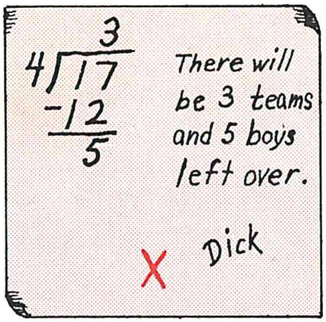


There are 17 children.

(A) How many teams of 4 each can be formed?

(B) How many children will be left over?

A



$$\begin{array}{r} 3 \\ 4\overline{)17} \\ -12 \\ \hline 5 \end{array}$$

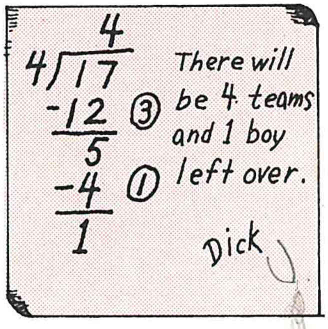
There will be 3 teams and 5 boys left over.

X Dick

▶ If the dividing has been completed correctly, then the remainder is less than the divisor.

Dick decided that he had not answered correctly. Instead of erasing all of his work, Dick corrected his work like this.

B



$$\begin{array}{rl} 4 & \\ 4\overline{)17} & \\ -12 & ③ \\ \hline 5 & \\ -4 & ① \\ \hline 1 & \end{array}$$

There will be 4 teams and 1 boy left over.

Dick

4. In solution B, how many fours were subtracted the first time? How many were subtracted the second time? How many fours were subtracted in all?

EXERCISES

1. If the quotient and remainder (R) are given correctly, write **C.** If not, give the correct quotient and remainder.

[A] $\begin{array}{r} 3,\ \text{R}2 \\ 4\overline{)14} \end{array}$ [B] $\begin{array}{r} 2,\ \text{R}4 \\ 3\overline{)10} \end{array}$ [C] $\begin{array}{r} 3,\ \text{R}3 \\ 5\overline{)18} \end{array}$ [D] $\begin{array}{r} 7,\ \text{R}5 \\ 4\overline{)33} \end{array}$ [E] $\begin{array}{r} 4,\ \text{R}10 \\ 9\overline{)46} \end{array}$

2. Copy each exercise. Find the quotient and remainder.

[A] $6\overline{)40}$ [B] $3\overline{)23}$ [C] $2\overline{)17}$ [D] $5\overline{)27}$ [E] $4\overline{)33}$ [F] $6\overline{)39}$

[G] $8\overline{)62}$ [H] $7\overline{)41}$ [I] $9\overline{)58}$ [J] $4\overline{)31}$ [K] $8\overline{)55}$ [L] $9\overline{)63}$

3. Copy each equation. Give the number instead of the $\boxed{n}$.

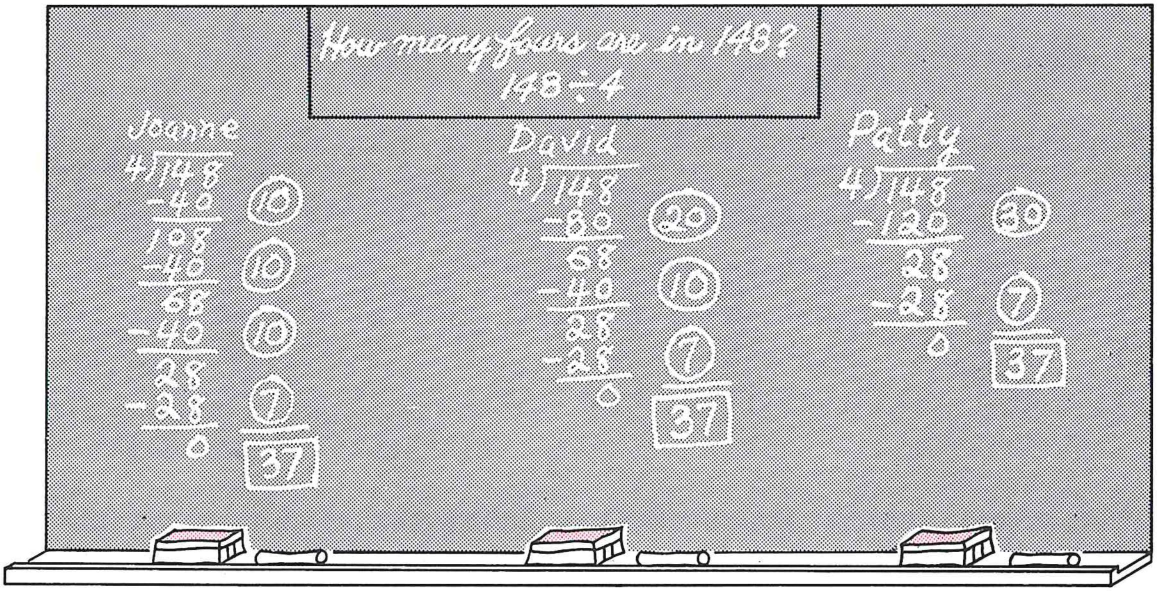
[A] $66 = (\boxed{n} \times 7) + 3$ [B] $29 = (9 \times 3) + \boxed{n}$ [C] $87 = (\boxed{n} \times 9) + 6$

4. Divide these numbers by 3 and list the remainders.
12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30

★ 5. List the possible remainders when dividing by 4.

Finding quotients by subtraction

Miss Brown had three children from her fourth grade class work the problem shown at the top of the chalkboard.



DISCUSSION EXERCISES

1. Explain the steps Joanne used to find the quotient.
2. In what way is David's work shorter than Joanne's?
3. How is Patty's work even shorter than David's?
4. Did all of the children get the same quotient?

EXERCISES

1. Find the quotients. The numbers in the rings tell how many fives were subtracted each time.

[A] 115 ÷ 5

$$\begin{array}{r l} 5\overline{)115} & \\ -50 & \textcircled{10} \\ \hline 65 & \\ -50 & \textcircled{10} \\ \hline 15 & \\ -15 & \textcircled{3} \\ \hline 0 & \end{array}$$

[B] 230 ÷ 5

$$\begin{array}{r l} 5\overline{)\ 230} & \\ -150 & \textcircled{30} \\ \hline 80 & \\ -50 & \textcircled{10} \\ \hline 30 & \\ -30 & \textcircled{6} \\ \hline 0 & \end{array}$$

[C] 185 ÷ 5

$$\begin{array}{r l} 5\overline{)\ 185} & \\ -100 & \textcircled{20} \\ \hline 85 & \\ -50 & \textcircled{10} \\ \hline 35 & \\ -35 & \textcircled{7} \\ \hline 0 & \end{array}$$

2. Copy each exercise. Give the number of sixes for each ring. Then give the quotient and remainder.

[A] 138 ÷ 6

$$\begin{array}{rl} 6\overline{)138} & \\ -60 & \bigcirc \\ \hline 78 & \\ -60 & \bigcirc \\ \hline 18 & \\ -18 & \bigcirc \\ \hline 0 & \end{array}$$

[B] 206 ÷ 6

$$\begin{array}{rl} 6\overline{)\ 206} & \\ -120 & \bigcirc \\ \hline 86 & \\ -60 & \bigcirc \\ \hline 26 & \\ -24 & \bigcirc \\ \hline 2 & \end{array}$$

[C] 252 ÷ 6

$$\begin{array}{rl} 6\overline{)\ 252} & \\ -120 & \bigcirc \\ \hline 132 & \\ -120 & \bigcirc \\ \hline 12 & \\ -12 & \bigcirc \\ \hline 0 & \end{array}$$

3. Copy each exercise. Give the number of threes for each ring. Then give the quotient and remainder.

[A] 105 ÷ 3

$$\begin{array}{rl} 3\overline{)105} & \\ -60 & \bigcirc \\ \hline 45 & \\ -30 & \bigcirc \\ \hline 15 & \\ -15 & \bigcirc \\ \hline 0 & \end{array}$$

[B] 168 ÷ 3

$$\begin{array}{rl} 3\overline{)168} & \\ -90 & \bigcirc \\ \hline 78 & \\ -60 & \bigcirc \\ \hline 18 & \\ -18 & \bigcirc \\ \hline 0 & \end{array}$$

[C] 145 ÷ 3

$$\begin{array}{rl} 3\overline{)145} & \\ -60 & \bigcirc \\ \hline 85 & \\ -60 & \bigcirc \\ \hline 25 & \\ -24 & \bigcirc \\ \hline 1 & \end{array}$$

4. Use subtraction as in exercises 1, 2, and 3 to help you find the quotient and remainder.

[A] 45 ÷ 3
[B] 104 ÷ 4
[C] 87 ÷ 5
[D] 144 ÷ 6
[E] 59 ÷ 2
[F] 84 ÷ 4
[G] 108 ÷ 3
[H] 179 ÷ 5
[I] 162 ÷ 6
[J] 142 ÷ 4
[K] 136 ÷ 8
[L] 200 ÷ 9

Think

When dividing you must do,
I'm really your best bet.
If you'll keep using me,
The quotient you will get.

WHO AM I?

Organizing your work

The example below will help you organize your work in finding quotients and remainders.

$136 \div 4$

Step 1

Think

$$\begin{array}{rl} 4\overline{)136} & \\ \underline{-80} & \leftarrow 20\times 4 \\ 56 & \end{array}$$

I can subtract
20 fours from 136.

Write

$$\begin{array}{rl} 4\overline{)136} & \\ \underline{\mathbf{-80}} & \boxed{\mathbf{20}} \\ \mathbf{56} & \end{array}$$

Step 2

Think

$$\begin{array}{rl} 4\overline{)136} & \\ \underline{-80} & \leftarrow 20\times 4 \\ 56 & \\ \underline{-40} & \leftarrow 10\times 4 \\ 16 & \end{array}$$

I can subtract
10 more fours.

Write

$$\begin{array}{rl} 4\overline{)136} & \\ \underline{-80} & \boxed{20} \\ 56 & \\ \underline{\mathbf{-40}} & \boxed{\mathbf{10}} \\ \mathbf{16} & \end{array}$$

Step 3

Think

$$\begin{array}{rl} 34 & \leftarrow \\ 4\overline{)136} & \\ \underline{-80} & \leftarrow 20\times 4 \\ 56 & \\ \underline{-40} & \leftarrow 10\times 4 \\ 16 & \\ \underline{-16} & \leftarrow 4\times 4 \\ 0 & \end{array}$$

I can subtract
4 more fours.

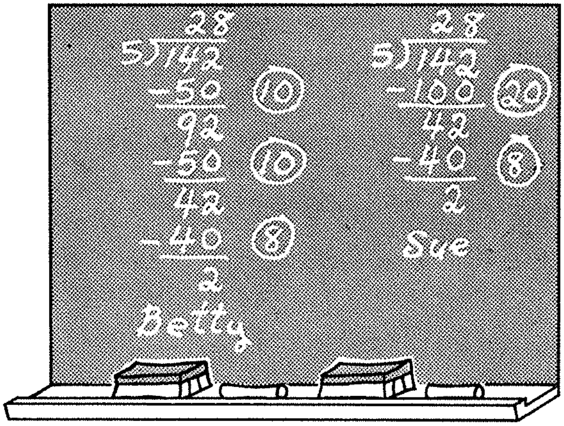
Write

$$\begin{array}{rl} \mathbf{34} & \\ 4\overline{)136} & \\ \underline{-80} & \boxed{20} \\ 56 & \\ \underline{-40} & \boxed{10} \\ 16 & \\ \underline{\mathbf{-16}} & \boxed{\mathbf{4}} \\ \mathbf{0} & \end{array}$$

DISCUSSION EXERCISES

1. What is the quotient for $136 \div 4$?
2. What is the remainder for $136 \div 4$?
3. Show how to organize your work if you begin by subtracting 30 fours.

EXERCISES



1. Betty and Sue each found the quotient and remainder for 142 ÷ 5.

[A] Did both girls get the same quotient?

[B] How many fives did Betty subtract the first time?

[C] How many fives did Sue subtract the first time?

[D] Whose work is shorter?

2. Copy each exercise and give the missing numbers.

[A] $$\begin{array}{rl} \square & \\ 4\overline{)152} & \\ -40 & \bigcirc \\ \hline 112 & \\ -40 & \bigcirc \\ \hline 72 & \\ -40 & \bigcirc \\ \hline 32 & \\ -32 & \bigcirc \\ \hline 0 & \end{array}$$

[B] $$\begin{array}{rl} \square & \\ 4\overline{)152} & \\ -80 & \bigcirc \\ \hline 72 & \\ -40 & \bigcirc \\ \hline 32 & \\ -32 & \bigcirc \\ \hline 0 & \end{array}$$

[C] $$\begin{array}{rl} \square & \\ 4\overline{)\ 152} & \\ -120 & \bigcirc \\ \hline 32 & \\ -32 & \bigcirc \\ \hline 0 & \end{array}$$

3. Did you get the same quotient for each part of exercise 2?

4. Find the quotients.

[A] $3\overline{)39}$ [B] $5\overline{)75}$ [C] $4\overline{)88}$ [D] $6\overline{)79}$ [E] $7\overline{)163}$

[F] $4\overline{)136}$ [G] $6\overline{)163}$ [H] $8\overline{)280}$ [I] $8\overline{)312}$ [J] $9\overline{)234}$

[K] $5\overline{)420}$ [L] $6\overline{)324}$ [M] $4\overline{)252}$ [N] $7\overline{)243}$ [O] $7\overline{)450}$

Multiples of 10 in estimation

The exercises below will help you find quotients in division problems such as $4\overline{)128}$.

EXERCISES

1. Give the number pair that should go in each gray space.

Pairs:									
	10	20	30	40	50	60	70	80	90
	20	30	40	50	60	70	80	90	100

[A] ▨ $\times 4 < 128$
▨ $\times 4 > 128$

(Answer: pair $\begin{matrix}30\\40\end{matrix}$)

[B] ▨ $\times 4 < 92$
▨ $\times 4 > 92$

[C] ▨ $\times 3 < 96$
▨ $\times 3 > 96$

[D] ▨ $\times 7 < 189$
▨ $\times 7 > 189$

[E] ▨ $\times 8 < 424$
▨ $\times 8 > 424$

[F] ▨ $\times 5 < 490$
▨ $\times 5 > 490$

[G] ▨ $\times 9 < 666$
▨ $\times 9 > 666$

[H] ▨ $\times 7 < 462$
▨ $\times 7 > 462$

[I] ▨ $\times 6 < 510$
▨ $\times 6 > 510$

[J] ▨ $\times 8 < 312$
▨ $\times 8 > 312$

[K] ▨ $\times 9 < 162$
▨ $\times 9 > 162$

[L] ▨ $\times 9 < 864$
▨ $\times 9 > 864$

[M] ▨ $\times 4 < 224$
▨ $\times 4 > 224$

[N] ▨ $\times 8 < 616$
▨ $\times 8 > 616$

[O] ▨ $\times 7 < 588$
▨ $\times 7 > 588$

[P] ▨ $\times 4 < 132$
▨ $\times 4 > 132$

[Q] ▨ $\times 9 < 657$
▨ $\times 9 > 657$

[R] ▨ $\times 5 < 325$
▨ $\times 5 > 325$

[S] ▨ $\times 3 < 264$
▨ $\times 3 > 264$

[T] ▨ $\times 8 < 608$
▨ $\times 8 > 608$

2. From the set {10, 20, 30, 40, 50, 60, 70, 80, 90}, find the **largest** number that will make each sentence true. Then copy the completed sentence.

[A] $n \times 4 < 128$
(Answer:
$30 \times 4 < 128$)

[B] $n \times 4 < 92$

[C] $n \times 3 < 96$

[D] $n \times 7 < 189$

[E] $n \times 3 < 252$

[F] $n \times 6 < 258$

[G] $n \times 7 < 448$

[H] $n \times 8 < 272$

[I] $n \times 9 < 387$

[J] $n \times 4 < 376$

[K] $n \times 5 < 390$

[L] $n \times 6 < 534$

[M] $n \times 7 < 546$

[N] $n \times 8 < 456$

[O] $n \times 9 < 612$

[P] $n \times 8 < 784$

[Q] $n \times 7 < 294$

[R] $n \times 9 < 747$

[S] $n \times 6 < 510$

Finding the quotient

1. Copy the problem. Complete the work of finding the quotient and remainder.

[A] ▥
$8\overline{)304}$
-240 (30)

[B] ▥
$5\overline{)327}$
-300 (60)

[C] ▥
$9\overline{)513}$
-450 (50)

[D] ▥
$6\overline{)294}$
-240 (40)

[E] ▥
$8\overline{)512}$
-480 (60)

[F] ▥
$5\overline{)215}$
-200 (40)

[G] ▥
$4\overline{)312}$
-280 (70)

[H] ▥
$3\overline{)233}$
-210 (70)

[I] ▥
$6\overline{)564}$
-540 (90)

[J] ▥
$2\overline{)178}$
-160 (80)

[K] ▥
$7\overline{)518}$
-490 (70)

[L] ▥
$9\overline{)891}$
-810 (90)

2. Copy each exercise and write the correct digits instead of the ▥.

[A] ▥
$5\overline{)213}$
−▥ (40)
▥
−▥ (▥)
▥

[B] ▥
$7\overline{)612}$
−▥ (80)
▥
−▥ (▥)
▥

[C] ▥
$4\overline{)156}$
−▥ (30)
▥
−▥ (▥)
▥

[D] ▥
$8\overline{)435}$
−▥ (50)
▥
−▥ (▥)
▥

3. Find the quotients and remainders.

[A] $3\overline{)66}$

[B] $4\overline{)94}$

[C] $7\overline{)189}$

[D] $4\overline{)97}$

[E] $8\overline{)424}$

[F] $5\overline{)494}$

[G] $7\overline{)462}$

[H] $6\overline{)513}$

[I] $9\overline{)234}$

[J] $4\overline{)370}$

[K] $6\overline{)246}$

[L] $8\overline{)615}$

Think

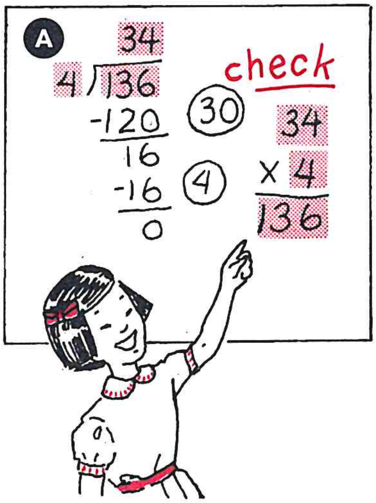
In the A figure below, the ▥ covers the same digit. What is this digit? What digit is covered by the ▥ in B?

Ⓐ
▥1
×▥
2▥▥

Ⓑ
▥1
×▥
3▥▥

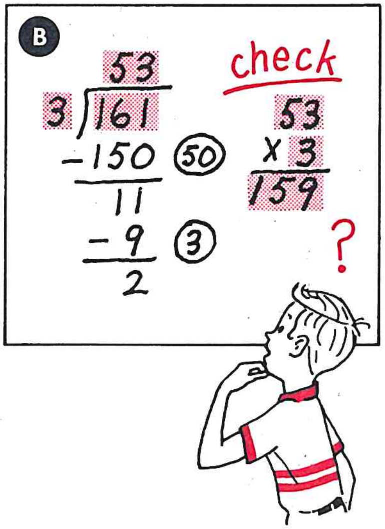
Finding and checking quotients

DISCUSSION EXERCISES



1. Jean checked problem A by multiplying. How did she know her quotient was correct?

2. Jack checked problem B. His product was 159. He thought it should be 161. Explain how Jack can finish checking.



EXERCISES

1. Find the quotients and remainders. Check each exercise.

[A] $5\overline{)305}$ [B] $3\overline{)249}$ [C] $4\overline{)85}$ [D] $6\overline{)198}$ [E] $4\overline{)338}$

[F] $7\overline{)327}$ [G] $8\overline{)363}$ [H] $9\overline{)605}$ [I] $6\overline{)339}$ [J] $4\overline{)117}$

[K] $3\overline{)288}$ [L] $7\overline{)620}$ [M] $2\overline{)179}$ [N] $9\overline{)335}$ [O] $6\overline{)288}$

[P] $5\overline{)385}$ [Q] $8\overline{)603}$ [R] $4\overline{)312}$ [S] $6\overline{)224}$ [T] $7\overline{)616}$

[U] $9\overline{)506}$ [V] $7\overline{)523}$ [W] $8\overline{)536}$ [X] $6\overline{)597}$ [Y] $5\overline{)99}$

★ 2. Give the digit that should go in the ▥. The x's are for other missing digits that you need not find.

[A]
$$\begin{array}{r} x\text{▥} \\ 9\overline{)\,xxx} \\ -xxx \\ \hline 9 \\ -x \\ \hline 0 \end{array}$$

[B]
$$\begin{array}{r} 60 \\ \text{▥}\overline{)\,x00} \\ -x00 \\ \hline 0 \end{array}$$

[C]
$$\begin{array}{r} x\text{▥} \\ 8\overline{)\,xxx} \\ -xxx \\ \hline 1x \\ -xx \\ \hline 0 \end{array}$$

[D]
$$\begin{array}{r} xx \\ x\overline{)\,31x} \\ -\text{▥}7x \\ \hline xx \\ -xx \\ \hline 0 \end{array}$$

EXERCISES

1. From the set $\{10, 20, 30, 40, 50, 60, 70, 80, 90\}$, find the largest number that will make each sentence true.

[A] Since $6 \times 4 < 25$, we know that $\boxed{n} \times 4 < 256$.
[B] Since $5 \times 3 < 16$, we know that $\boxed{n} \times 3 < 164$.
[C] Since $8 \times 7 < 59$, we know that $\boxed{n} \times 7 < 595$.
[D] Since $7 \times 6 < 43$, we know that $\boxed{n} \times 6 < 439$.
[E] Since $4 \times 2 < 9$, we know that $\boxed{n} \times 2 < 92$.
[F] Since $2 \times 5 < 13$, we know that $\boxed{n} \times 5 < 135$.
[G] Since $9 \times 6 < 55$, we know that $\boxed{n} \times 6 < 554$.
[H] Since $9 \times 8 < 74$, we know that $\boxed{n} \times 8 < 748$.
[I] Since $9 \times 9 < 83$, we know that $\boxed{n} \times 9 < 837$.
[J] Since $9 \times 7 < 65$, we know that $\boxed{n} \times 7 < 656$.
[K] Since $3 \times 2 < 7$, we know that $\boxed{n} \times 2 < 73$.
[L] Since $7 \times 8 < 63$, we know that $\boxed{n} \times 8 < 632$.
[M] Since $5 \times 6 < 34$, we know that $\boxed{n} \times 6 < 347$.
[N] Since $9 \times 3 < 28$, we know that $\boxed{n} \times 3 < 287$.
[O] Since $3 \times 7 < 27$, we know that $\boxed{n} \times 7 < 273$.

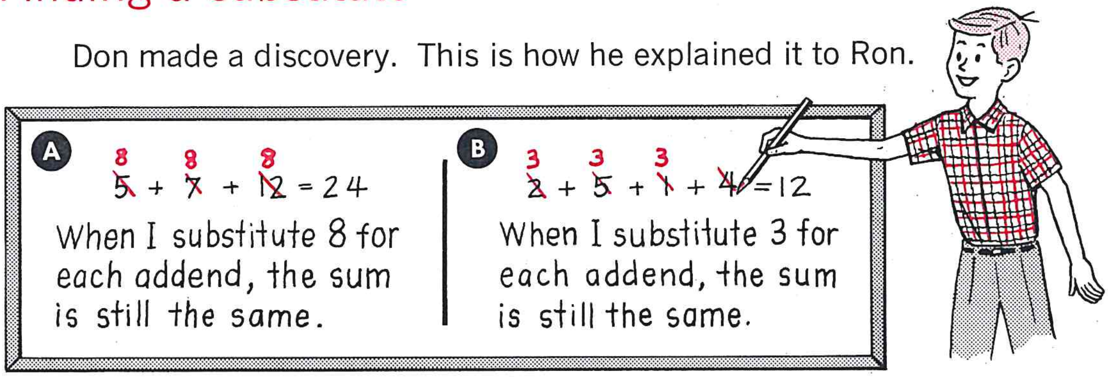
2. Find the quotients and remainders. Check each exercise.

[A] $4\overline{)256}$
[B] $3\overline{)164}$
[C] $7\overline{)595}$
[D] $6\overline{)439}$
[E] $2\overline{)92}$
[F] $5\overline{)135}$
[G] $6\overline{)554}$
[H] $8\overline{)748}$
[I] $9\overline{)837}$
[J] $7\overline{)656}$
[K] $2\overline{)73}$
[L] $8\overline{)632}$
[M] $6\overline{)347}$
[N] $3\overline{)287}$
[O] $7\overline{)273}$
[P] $5\overline{)271}$
[Q] $3\overline{)225}$
[R] $7\overline{)623}$
[S] $6\overline{)285}$
[T] $9\overline{)378}$
[U] $4\overline{)260}$
[V] $7\overline{)670}$
[W] $5\overline{)340}$
[X] $8\overline{)670}$
[Y] $9\overline{)531}$

3. If you divide a certain number by 7 and multiply the quotient by 7, the answer is 154. Give the number.

Finding a substitute

Don made a discovery. This is how he explained it to Ron.



A $\overset{8}{\cancel{5}} + \overset{8}{\cancel{7}} + \overset{8}{\cancel{12}} = 24$

When I substitute 8 for each addend, the sum is still the same.

B $\overset{3}{\cancel{2}} + \overset{3}{\cancel{5}} + \overset{3}{\cancel{1}} + \cancel{4} = 12$

When I substitute 3 for each addend, the sum is still the same.

Don listed these three numbers → 4, 5, 9 and asked Ron to find a number that could substitute for each and give the same sum.

Ron's first try looked like this → $\overset{7}{\cancel{4}} + \overset{7}{\cancel{5}} + \overset{7}{\cancel{9}} = 18$

DISCUSSION EXERCISES

1. When Ron substituted 7 for each addend, was the sum still the same?
2. Find a substitute that will give the same sum.

EXERCISES

1. Give the whole number (if there is one) that can substitute for each addend and give the same sum.

[A] $4+3+8=15$ (Answer: $\overset{5}{\cancel{4}}+\overset{5}{\cancel{3}}+\overset{5}{\cancel{8}}=15$ The number is 5.)

[B] $10+4=14$

[C] $6+4+5=15$

[D] $3+7+6+4=20$

[E] $2+12+10=24$

[F] $5+9+10+4=28$

[G] $8+11+9=28$

[H] $12+18=30$

[I] $2+3+10+9+11=35$

[J] $12+14+7=33$

[K] $100+86=186$

[L] $20+30+38+40=128$

[M] $26+30+46=102$

2. Give the number that can substitute for each number in the set and give the same sum. Do as little writing as possible.

[A] {2, 10} (Answer: $\overset{6}{\cancel{2}}+\overset{6}{\cancel{10}}=12$ The number is 6.)

[B] {1, 2, 6}

[C] {2, 5, 4, 5}

[D] {6, 13, 11}

[E] {6, 9, 13, 8}

[F] {7, 17}

[G] {60, 74}

[H] {73, 80, 93}

[I] {72, 53, 68, 71}

Finding the average

You have been finding the average of a set of numbers.

We can substitute 7 for each number in this set {10, 5, 6} and get the same sum.

$$\overset{7}{\cancel{10}}+\overset{7}{\cancel{5}}+\overset{7}{\cancel{6}}=21$$

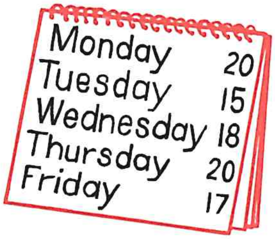
The average of 10, 5, and 6 is 7.

EXERCISES

1. Here is a list of the points Jim scored in each of 3 basketball games. Give the **average** number of points scored.

Game 1	Game 2	Game 3
12	7	11

2. Here are Jane's spelling scores for one week. Find the average of her scores.

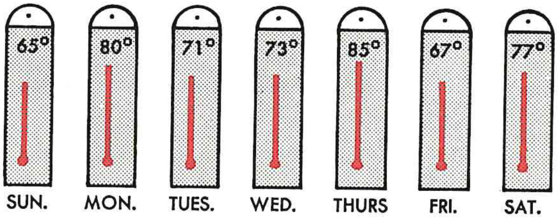


Monday	20
Tuesday	15
Wednesday	18
Thursday	20
Friday	17

3. For each exercise, find the **average** of the numbers in the set.

[A] {3, 7, 5, 5}
[B] {8, 12, 7}
[C] {4, 3, 5, 4}
[D] {15, 21}
[E] {34, 42}
[F] {93, 97}
[G] {21, 34, 44}
[H] {62, 79, 87}
[I] {52, 56, 51}
[J] {9, 8, 7, 6, 5, 4, 3}
[K] {81, 82, 91, 78}
[L] {91, 68, 75, 84, 52}

4. The Fahrenheit thermometers show the temperature for each day during one week. Give the average of these temperatures.



★ **5.** Here are Sue's and Carol's grades on 4 tests.

[A] Without finding the average score for either girl, show that their average scores are the same.

Sue	80	90	88	86
Carol	85	85	87	87

[B] Find the average of Sue's scores.

Multiples of 100 in estimation

The exercises below will help you find quotients in division problems such as $6\overline{)1944}$.

EXERCISES

1. Give the number pair that should go in each gray space.

Pairs:									
	100 200	200 300	300 400	400 500	500 600	600 700	700 800	800 900	900 1000

[A] ▨ $\times 6 < 1944$
▨ $\times 6 > 1944$

(Answer: pair $\begin{matrix}300\\400\end{matrix}$)

[B] ▨ $\times 3 < 711$
▨ $\times 3 > 711$

[C] ▨ $\times 5 < 643$
▨ $\times 5 > 643$

[D] ▨ $\times 4 < 2437$
▨ $\times 4 > 2437$

[E] ▨ $\times 2 < 1715$
▨ $\times 2 > 1715$

[F] ▨ $\times 7 < 2462$
▨ $\times 7 > 2462$

[G] ▨ $\times 8 < 3847$
▨ $\times 8 > 3847$

[H] ▨ $\times 9 < 5964$
▨ $\times 9 > 5964$

[I] ▨ $\times 4 < 3143$
▨ $\times 4 > 3143$

[J] ▨ $\times 5 < 4987$
▨ $\times 5 > 4987$

[K] ▨ $\times 8 < 4352$
▨ $\times 8 > 4352$

[L] ▨ $\times 9 < 1856$
▨ $\times 9 > 1856$

[M] ▨ $\times 7 < 4329$
▨ $\times 7 > 4329$

[N] ▨ $\times 6 < 5136$
▨ $\times 6 > 5136$

[O] ▨ $\times 3 < 2817$
▨ $\times 3 > 2817$

[P] ▨ $\times 9 < 5631$
▨ $\times 9 > 5631$

2. From the set {100, 200, 300, 400, 500, 600, 700, 800, 900}, find the **largest** number that will make each sentence true. Then copy the completed sentence.

[A] $n \times 6 < 1944$
(Answer: $300 \times 6 < 1944$)
[B] $n \times 3 < 1624$
[C] $n \times 6 < 2578$
[D] $n \times 4 < 2713$
[E] $n \times 5 < 4316$
[F] $n \times 8 < 6845$
[G] $n \times 7 < 6425$
[H] $n \times 3 < 1369$
[I] $n \times 9 < 4723$
[J] $n \times 4 < 2200$
[K] $n \times 8 < 7367$
[L] $n \times 9 < 8464$
[M] $n \times 6 < 3217$
[N] $n \times 7 < 3728$
[O] $n \times 5 < 1863$
[P] $n \times 3 < 2158$
[Q] $n \times 9 < 6583$
[R] $n \times 7 < 4217$
[S] $n \times 4 < 3614$
[T] $n \times 8 < 4686$

Finding 3-digit quotients

Study the examples.

A Think:

Since $300 \times 6 < 1944$,
$400 \times 6 > 1944$,

$$\begin{array}{r l}
324 & \leftarrow \\
6\overline{)\,1944} & \\
1800 & \leftarrow 300 \times 6 \\
\hline
144 & \\
120 & \leftarrow 20 \times 6 \\
\hline
24 & \\
24 & \leftarrow 4 \times 6 \\
\hline
0 &
\end{array}$$

we can subtract **300** sixes from 1944.

Then we can subtract **20** more sixes.

Finally, we can subtract **4** more sixes.

The quotient is 324.

Write:

$$\begin{array}{r l}
324 & \\
6\overline{)1944} & \\
1800 & (300) \\
\hline
144 & \\
120 & (20) \\
\hline
24 & \\
24 & (4) \\
\hline
0 &
\end{array}$$

B Think:

Since $500 \times 4 < 2014$,
$600 \times 4 > 2014$,

$$\begin{array}{r l}
503 & \leftarrow \\
4\overline{)\,2014} & \\
2000 & \leftarrow 500 \times 4 \\
\hline
14 & \\
12 & \leftarrow 3 \times 4 \\
\hline
2 &
\end{array}$$

we can subtract **500** fours from 2014.

Then we can subtract **3** more fours.

The quotient is 503.

The remainder is 2.

Write:

$$\begin{array}{r l}
503 & \\
4\overline{)2014} & \\
2000 & (500) \\
\hline
14 & \\
12 & (3) \\
\hline
2 &
\end{array}$$

EXERCISES

Copy each exercise and find the quotient and remainder. Check each problem.

1. $3\overline{)396}$
2. $5\overline{)1600}$
3. $6\overline{)1459}$
4. $7\overline{)2975}$
5. $9\overline{)3627}$
6. $2\overline{)1846}$
7. $4\overline{)3243}$
8. $8\overline{)3616}$
9. $3\overline{)2862}$
10. $5\overline{)3360}$
11. $9\overline{)2727}$
12. $6\overline{)5208}$
13. $5\overline{)4008}$
14. $7\overline{)4100}$
15. $8\overline{)5002}$

Dividing exercises

1. From the set $\{100, 200, 300, 400, 500, 600, 700, 800, 900\}$, find the largest number that will make each sentence true.

[A] Since $4 \times 3 < 14$, we know that $\boxed{n} \times 3 < 1461$.
[B] Since $3 \times 9 < 29$, we know that $\boxed{n} \times 9 < 2943$.
[C] Since $5 \times 8 < 42$, we know that $\boxed{n} \times 8 < 4296$.
[D] Since $5 \times 2 < 11$, we know that $\boxed{n} \times 2 < 1126$.
[E] Since $8 \times 6 < 51$, we know that $\boxed{n} \times 6 < 5118$.
[F] Since $6 \times 4 < 27$, we know that $\boxed{n} \times 4 < 2788$.
[G] Since $9 \times 7 < 67$, we know that $\boxed{n} \times 7 < 6776$.
[H] Since $6 \times 8 < 55$, we know that $\boxed{n} \times 8 < 5584$.
[I] Since $7 \times 9 < 65$, we know that $\boxed{n} \times 9 < 6538$.
[J] Since $8 \times 4 < 39$, we know that $\boxed{n} \times 4 < 3966$.
[K] Since $8 \times 3 < 25$, we know that $\boxed{n} \times 3 < 2575$.
[L] Since $9 \times 5 < 54$, we know that $\boxed{n} \times 5 < 5435$.
[M] Since $8 \times 9 < 73$, we know that $\boxed{n} \times 9 < 7386$.
[N] Since $8 \times 4 < 33$, we know that $\boxed{n} \times 4 < 3340$.
[O] Since $6 \times 2 < 13$, we know that $\boxed{n} \times 2 < 1328$.
[P] Since $3 \times 6 < 19$, we know that $\boxed{n} \times 6 < 1956$.

2. Find the quotients and remainders. Check each exercise.

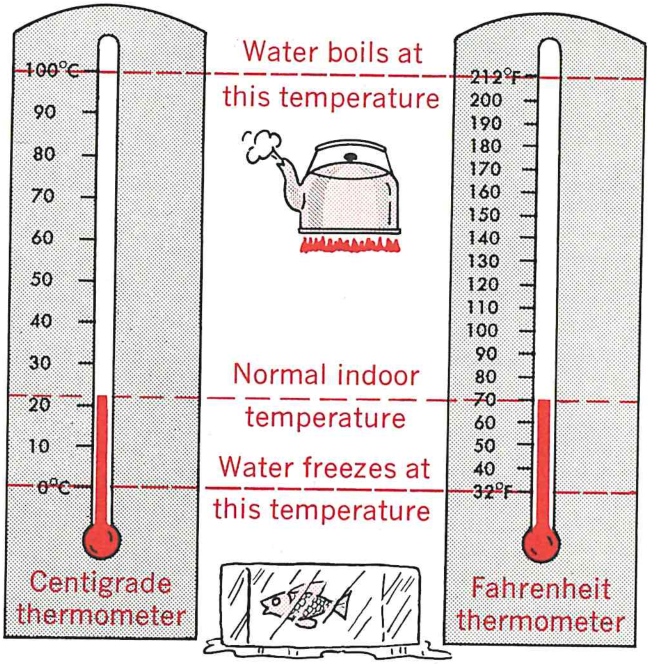
[A] $3\overline{)1461}$ [B] $9\overline{)2943}$
[C] $8\overline{)4296}$ [D] $2\overline{)1126}$
[E] $6\overline{)5118}$ [F] $4\overline{)2788}$
[G] $7\overline{)6776}$ [H] $8\overline{)5584}$
[I] $9\overline{)6538}$ [J] $4\overline{)3966}$
[K] $3\overline{)2575}$ [L] $5\overline{)5435}$

Think

On the coldest day of the winter the temperature was 15° below zero. That same year the temperature got as high as 92° in the summer. What was the difference between the coldest and hottest temperature that year?

Temperature

A **thermometer** is used to measure temperature. Two units often used to measure how hot or cold it is are the **Fahrenheit degree** and the **centigrade degree.** The thermometers show how these units compare. We read 32 **degrees centigrade** for 32°**C.** We read 212 **degrees Fahrenheit** for 212°**F.**



EXERCISES

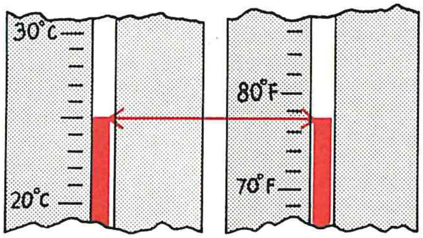
1. How many Fahrenheit degrees greater is the boiling point of water than the freezing point?

2. Use the thermometers shown to give the temperature in °C and °F.

3. What is the Fahrenheit temperature if

[**A**] the temperature is 30°C?

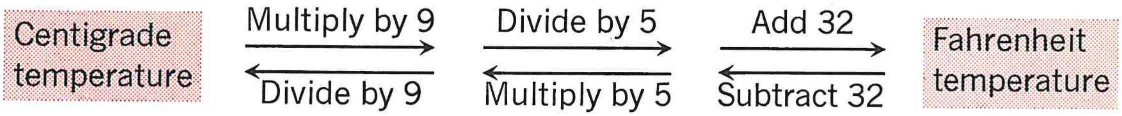
[**B**] the temperature is 20°C?



Centigrade thermometer

Fahrenheit thermometer

4. Study the diagram. Use it to find the missing numbers.



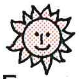
[**A**] Jill's temperature: 40°C, ||||||°F

[**B**] Hot summer day: 95°F, ||||||°C

[**C**] Mars at noon: about 10°C, ||||||°F

[**D**] Hot oven: 235°C, ||||||°F

[**E**] Alcohol boils: 167°F, ||||||°C

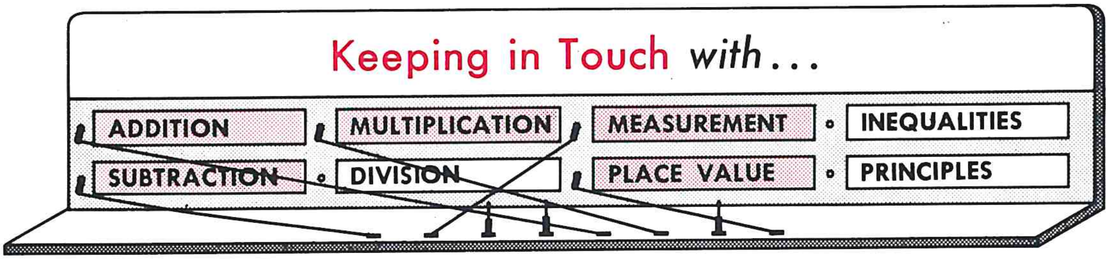


★[**F**] Sun's surface: 6000°C, ||||||°F

[**G**] Iron melts: about 1535°C, ||||||°F

★[**H**] Iron boils: about 2985°C, ||||||°F

Keeping in Touch *with*...



1. Give the correct number for each red digit.
In exercise **A** the 8 means 8000.

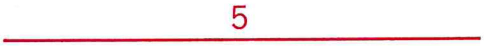
[A] 4,378,615
[C] 6,389,741
[E] 4,250,697
[B] 64,124,752
[D] 3,026,518
[F] 97,643,287

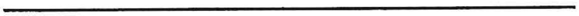
2. Find the sums, differences, and products.

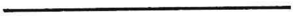
[A] 6728 +9357
[B] 8436 −2759
[C] 432 ×65
[D] 8463 +9287
[E] 7062 −888

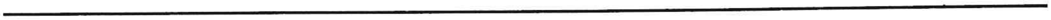
[F] 745 ×63
[G] 8469 +7531
[H] 8002 −6457
[I] 846 ×352
[J] 764 ×803

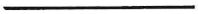
★ **3.** Look at the red segment and its length. Then use this to estimate the length of the other segments.

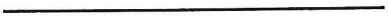


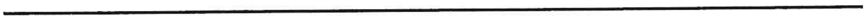
[A]


[B]


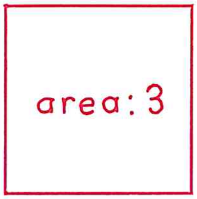
[C]


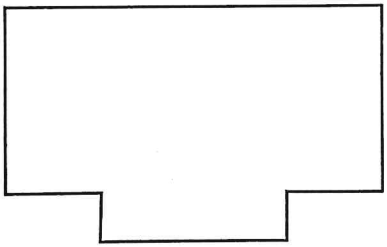
[D]


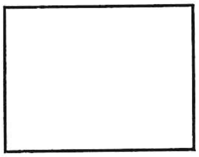
[E]


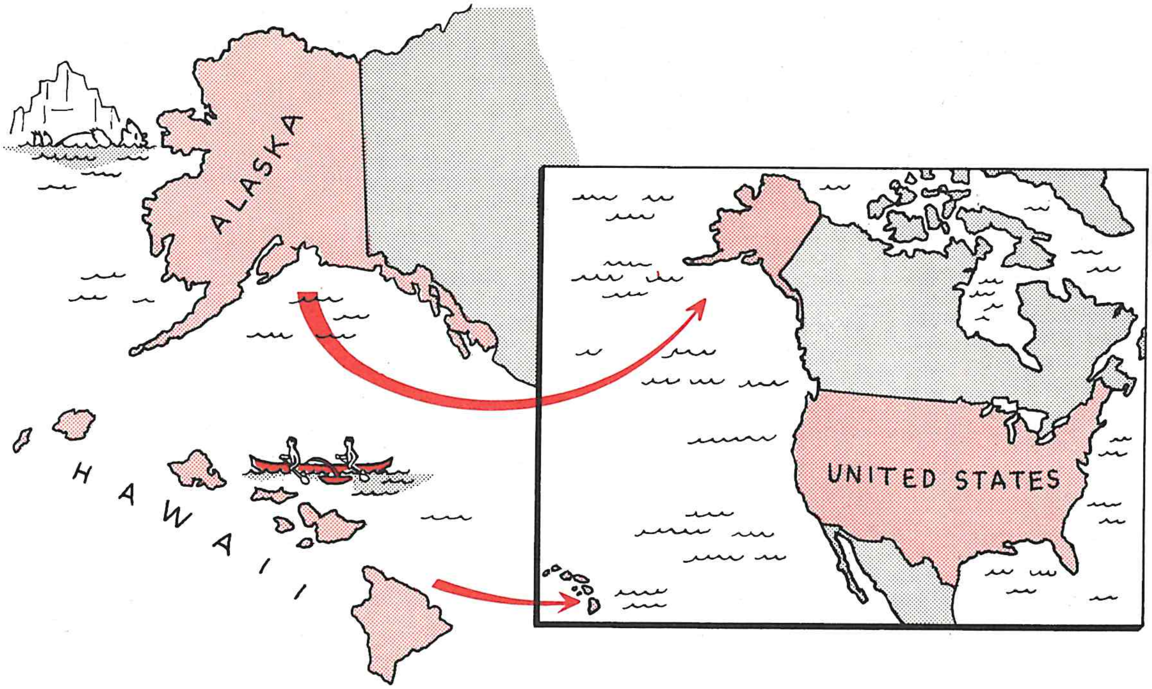
[F]


★ **4.** Look at the red figure and its area. Then use this to estimate the area of the other figures.



[A]


[B]




	Alaska	Hawaii
Area	586,400 sq miles	6,400 sq miles
Population (1960 census)	226,167	632,772
Average temperature	42° (Juneau)	75° (Honolulu)
Highest point	Mt. McKinley 20,320 feet	Mauna Kea (volcano) 13,796 feet

1. How many more square miles are in Alaska than in Hawaii? Give your answer without using pencil and paper.

2. [A] How many more people lived in Hawaii than in Alaska in 1960?

[B] How many lived in the two states together?

3. Mt. McKinley is the highest point in North America. How much higher is it than Mauna Kea in Hawaii?

★**4.** Alaska is more than two times the size of Texas. To find about how large Texas is, do the following: multiply the number of square miles in Alaska by 4 and then divide that number by 9.

More estimation

The exercises below will help you find quotients in division problems such as $40\overline{)290}$.

EXERCISES

1. Give the number pair that should go in each gray space.

Pairs:	1	2	3	4	5	6	7	8	9
	2	3	4	5	6	7	8	9	10

[A] $\square \times 40 < 290$
$\square \times 40 > 290$

(Answer: pair $\begin{matrix}7\\8\end{matrix}$)

[B] $\square \times 30 < 80$
$\square \times 30 > 80$

[C] $\square \times 20 < 130$
$\square \times 20 > 130$

[D] $\square \times 40 < 293$
$\square \times 40 > 293$

[E] $\square \times 50 < 237$
$\square \times 50 > 237$

[F] $\square \times 60 < 316$
$\square \times 60 > 316$

[G] $\square \times 70 < 445$
$\square \times 70 > 445$

[H] $\square \times 80 < 257$
$\square \times 80 > 257$

[I] $\square \times 90 < 650$
$\square \times 90 > 650$

[J] $\square \times 30 < 147$
$\square \times 30 > 147$

[K] $\square \times 60 < 512$
$\square \times 60 > 512$

[L] $\square \times 90 < 475$
$\square \times 90 > 475$

[M] $\square \times 80 < 740$
$\square \times 80 > 740$

[N] $\square \times 70 < 642$
$\square \times 70 > 642$

[O] $\square \times 60 < 555$
$\square \times 60 > 555$

2. Find the **largest** whole number that will make each sentence true. Then copy the completed sentence.

[A] $n \times 40 < 290$
(Answer:
$7 \times 40 < 290$)
[B] $n \times 20 < 56$
[C] $n \times 30 < 87$
[D] $n \times 50 < 168$
[E] $n \times 80 < 479$
[F] $n \times 90 < 392$
[G] $n \times 40 < 237$
[H] $n \times 70 < 584$
[I] $n \times 30 < 173$
[J] $n \times 60 < 437$
[K] $n \times 50 < 421$
[L] $n \times 80 < 637$
[M] $n \times 30 < 134$
[N] $n \times 90 < 736$
[O] $n \times 70 < 621$
[P] $n \times 40 < 213$
[Q] $n \times 60 < 487$
[R] $n \times 70 < 637$
[S] $n \times 50 < 363$
[T] $n \times 40 < 297$
[U] $n \times 90 < 834$
[V] $n \times 80 < 719$
[W] $n \times 70 < 227$
[X] $n \times 90 < 563$
[Y] $n \times 80 < 197$

Multiples of 10 as divisors

Study the example.

Think:

$$\begin{array}{r} 4 \\ 70\overline{)\,305} \\ -280 \\ \hline 25 \end{array} \quad \leftarrow 4 \times 70$$

Since $4 \times 70 < 305$,
$5 \times 70 > 305$,

we can subtract
4 seventies from 305.

The quotient is 4.

The remainder is 25.

Write:

$$\begin{array}{r} 4 \\ 70\overline{)305} \\ 280 \\ \hline 25 \end{array} \quad ④$$

EXERCISES

1. For each exercise, when you find the largest whole number that makes the sentence true, you will have found the quotient. Write the quotient and remainder for each division problem.

[A] ▥ $\times 30 < 192$ $\quad 30\overline{)192}$

[B] ▥ $\times 40 < 267$ $\quad 40\overline{)267}$

[C] ▥ $\times 60 < 317$ $\quad 60\overline{)317}$

[D] ▥ $\times 70 < 364$ $\quad 70\overline{)364}$

[E] ▥ $\times 80 < 490$ $\quad 80\overline{)490}$

[F] ▥ $\times 90 < 563$ $\quad 90\overline{)563}$

[G] ▥ $\times 20 < 89$ $\quad 20\overline{)89}$

[H] ▥ $\times 50 < 268$ $\quad 50\overline{)268}$

[I] ▥ $\times 80 < 736$ $\quad 80\overline{)736}$

2. Find the quotients and remainders.

[A] $20\overline{)190}$ [B] $30\overline{)180}$ [C] $40\overline{)297}$

[D] $70\overline{)220}$ [E] $80\overline{)230}$ [F] $50\overline{)415}$

[G] $70\overline{)579}$ [H] $40\overline{)317}$ [I] $70\overline{)518}$

[J] $80\overline{)650}$ [K] $60\overline{)463}$ [L] $90\overline{)645}$

Think

Study the pattern.
Solve the equations.

$$(1 \times 8) + 1 = 9$$
$$(12 \times 8) + 2 = 98$$
$$(123 \times 8) + 3 = 987$$
$$(1234 \times 8) + 4 = 9876$$
$$(12345 \times 8) + 5 = \boxed{n}$$
$$(123456 \times 8) + 6 = \boxed{n}$$

Larger quotients

EXERCISES

1. From the set {10, 20, 30, 40, 50, 60, 70, 80, 90}, find the largest number that makes the sentence true.
Then copy the completed sentence.

[A] $\boxed{n} \times 30 < 627$	[E] $\boxed{n} \times 60 < 3643$	[I] $\boxed{n} \times 80 < 3440$
[B] $\boxed{n} \times 20 < 1100$	[F] $\boxed{n} \times 40 < 3685$	[J] $\boxed{n} \times 50 < 4800$
[C] $\boxed{n} \times 50 < 1525$	[G] $\boxed{n} \times 70 < 3220$	[K] $\boxed{n} \times 90 < 5940$
[D] $\boxed{n} \times 40 < 2418$	[H] $\boxed{n} \times 30 < 750$	[L] $\boxed{n} \times 70 < 6175$

A Think:

$$\begin{array}{rl} 20 & \leftarrow \\ 30\overline{)\,627} & \\ 600 & \leftarrow 20 \times 30 \\ \hline 27 & \end{array}$$

Since $20 \times 30 < 627$
$30 \times 30 > 627$,

we can subtract
20 thirties from 627.
The quotient is 20.
The remainder is 27.

Write:

$$\begin{array}{rl} 20 & \\ 30\overline{)627} & \\ 600 & ⑳ \\ \hline 27 & \end{array}$$

B Think:

$$\begin{array}{rl} 89 & \leftarrow \\ 70\overline{)\,6230} & \\ 5600 & \leftarrow 80 \times 70 \\ \hline 630 & \\ 630 & \leftarrow 9 \times 70 \\ \hline 0 & \end{array}$$

Since $80 \times 70 < 6230$
$90 \times 70 > 6230$,

we can subtract
80 seventies from 6230.
Then we can subtract
9 more seventies.
The quotient is **89.**

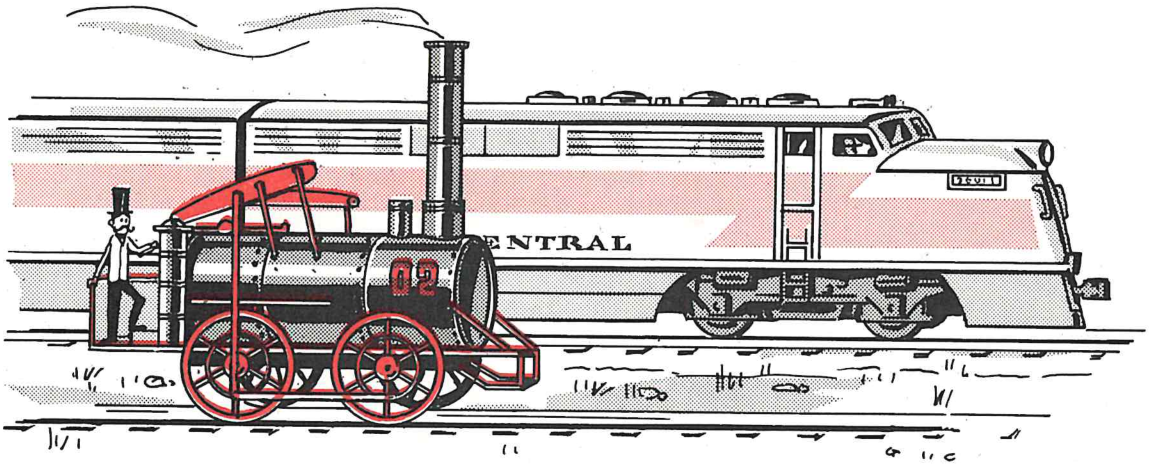
Write:

$$\begin{array}{rl} 89 & \\ 70\overline{)6230} & \\ 5600 & (80) \\ \hline 630 & \\ 630 & ⑨ \\ \hline 0 & \end{array}$$

2. Study examples A and B. Then find the quotients and remainders.

[A] $30\overline{)1200}$	[B] $20\overline{)816}$	[C] $40\overline{)2036}$	[D] $50\overline{)2536}$	[E] $20\overline{)1808}$
[F] $40\overline{)2835}$	[G] $90\overline{)5472}$	[H] $60\overline{)4231}$	[I] $30\overline{)690}$	[J] $70\overline{)2240}$
[K] $50\overline{)3200}$	[L] $40\overline{)2640}$	[M] $80\overline{)5200}$	[N] $70\overline{)5670}$	[O] $80\overline{)2723}$

Trains



1. A locomotive similar to the one shown in the picture above was invented in 1829. How many years ago was this?

2. The sleeping section of a modern passenger train is about 490 feet long. If each sleeping car is 70 feet long, how many cars are there?

3. A passenger coach holds 70 people. If 253 people are in passenger coaches on the train, how many full passenger coaches could there be? How many extra people would there be for a partly filled passenger coach?

4. A boxcar is 40 feet long. A train has a boxcar section that is about 920 feet long. How many boxcars are in the section?

5. There are 8 tractors on each flatcar except the last. There are 543 tractors in all.
   [A] How many full flatcars are there?
   [B] How many extra tractors are left for the last flatcar?

6. Rails about 40 feet long are put end to end to make one side of a long railroad track. A half mile of track is 2640 feet long. How many rails are needed to make one side of a half-mile track?

7. The average speed of a fast passenger train is 80 miles per hour. How many hours does it take to go 2080 miles?

Estimation exercises

The exercises will help you find quotients in division problems such as $39\overline{)223}$.

1. Give the number pair that should go in each gray space. Some hints are given to help you find the correct number pair in exercises **A** through **D.** Check both numbers by multiplying to be sure you chose the correct pair.

Pairs:	1	2	3	4	5	6	7	8	9
	1 2	2 3	3 4	4 5	5 6	6 7	7 8	8 9	9 10

[**A**] Think: n × 20 < 98
▨ × 21 < 98
× 21 > 98

[**B**] Think: n × 40 < 223
▨ × 39 < 223
× 39 > 223

[**C**] Think: n × 40 < 366
▨ × 43 < 366
× 43 > 366

[**D**] Think: n × 20 < 105
▨ × 17 < 105
× 17 > 105

[**E**] ▨ × 22 < 70
× 22 > 70

[**F**] ▨ × 61 < 145
× 61 > 145

[**G**] ▨ × 43 < 238
× 43 > 238

[**H**] ▨ × 19 < 129
× 19 > 129

[**I**] ▨ × 58 < 257
× 58 > 257

[**J**] ▨ × 32 < 223
× 32 > 223

[**K**] ▨ × 54 < 363
× 54 > 363

[**L**] ▨ × 37 < 190
× 37 > 190

[**M**] ▨ × 69 < 576
× 69 > 576

[**N**] ▨ × 81 < 573
× 81 > 573

[**O**] ▨ × 39 < 243
× 39 > 243

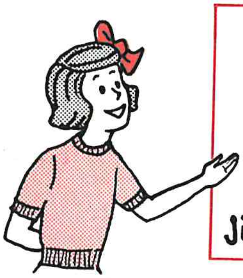
[**P**] ▨ × 76 < 310
× 76 > 310

2. Find the **largest** whole number that will make the sentence true. Then copy the completed sentence.

[**A**] n × 31 < 163
[**B**] n × 29 < 125
[**C**] n × 41 < 258
[**D**] n × 52 < 228
[**E**] n × 39 < 254
[**F**] n × 63 < 394
[**G**] n × 38 < 334
[**H**] n × 59 < 484
[**I**] n × 62 < 560
[**J**] n × 18 < 104
[**K**] n × 71 < 430
[**L**] n × 87 < 368
[**M**] n × 33 < 153
[**N**] n × 52 < 370
[**O**] n × 41 < 203
[**P**] n × 81 < 573
[**Q**] n × 39 < 243
[**R**] n × 68 < 575
[**S**] n × 92 < 545
[**T**] n × 89 < 725
[**U**] n × 43 < 265
[**V**] n × 76 < 310
[**W**] n × 34 < 310
[**X**] n × 26 < 209

2-digit divisors

Jill and Jane showed the class how they found the quotient for the division problem $41\overline{)258}$. Jane was able to find the quotient more quickly, but both girls did good work and found the correct quotient.



$$\begin{array}{rl} 6 & \\ 41\overline{)258} & \\ \underline{205} & \textcircled{5} \\ 53 & \\ \underline{41} & \textcircled{1} \\ 12 & \end{array}$$

Jill

$$\begin{array}{rl} 6 & \\ 41\overline{)258} & \\ \underline{246} & \textcircled{6} \\ 12 & \end{array}$$

Jane

DISCUSSION EXERCISES

1. Explain Jill's work.
2. How do you think Jane decided to use 6 as the quotient?
3. Dick asked the teacher if it would be correct to find the quotient this way. What do you think? Explain this way of finding the quotient.

Dick

$$\begin{array}{rl} 6 & \\ 41\overline{)258} & \\ \underline{82} & \textcircled{2} \\ 176 & \\ \underline{82} & \textcircled{2} \\ 94 & \\ \underline{82} & \textcircled{2} \\ 12 & \end{array}$$

EXERCISES

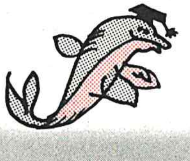
Find the quotients and remainders.

1. $21\overline{)126}$
2. $41\overline{)175}$
3. $52\overline{)156}$
4. $59\overline{)192}$
5. $69\overline{)560}$
6. $83\overline{)560}$
7. $71\overline{)500}$
8. $57\overline{)430}$
9. $12\overline{)96}$
10. $61\overline{)549}$
11. $89\overline{)443}$
12. $53\overline{)210}$

Think

Watch out for this one. There's a trick.

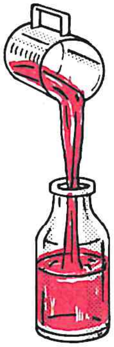
Which is heavier, a pound of cotton or a pound of iron?



Short stories

1. 60 minutes → 1 hour.
180 minutes. How many hours?

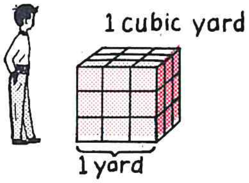
2. 260 minutes. How many full hours? How many extra minutes?



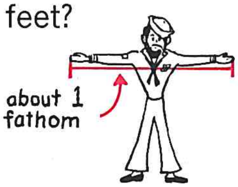
3. 32 ounces → 1 quart.
224 ounces.
How many quarts?

4. 200 ounces. How many full quarts? How many extra ounces?

5. 1 cubic yard → 27 cubic feet.
108 cubic feet.
How many cubic yards?



6. 1 fathom → 6 feet.
18 fathoms.
How many feet?

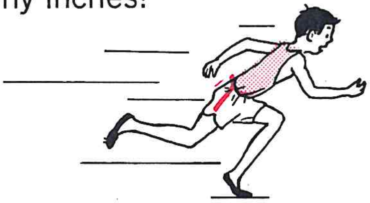


7. 12 inches → 1 foot.
A snake: 72 inches long.
How many feet long?

8. 24 hours → 1 day. 198 hours.
How many full days?
How many extra hours?

9. 36 inches → 1 yard.
A room: 324 inches wide.
How many yards wide?

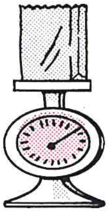
10. Run 220 yards. How many inches?



11. 64 tablespoons → 1 quart.
332 tablespoons.
How many full quarts?
How many extra tablespoons?

12. 60 seconds → 1 minute.
2058 seconds. How many full minutes? How many extra seconds?

13. 16 ounces → 1 pound.
128 ounces.
How many pounds?



★ **14.** 640 acres → 1 square mile.
State park: 3840 acres.
How many square miles?

★2-digit divisors and quotients

Here are three ways to find the quotient for the division problem $37\overline{)1184}$. Each example gives the correct quotient.

A

$$\begin{array}{r l}
32 & \leftarrow \\
37\overline{)1184} & \\
\underline{370} & \leftarrow 10 \times 37 \\
814 & \\
\underline{370} & \leftarrow 10 \times 37 \\
444 & \\
\underline{370} & \leftarrow 10 \times 37 \\
74 & \\
\underline{74} & \leftarrow 2 \times 37 \\
0 &
\end{array}$$

B

$$\begin{array}{r l}
32 & \leftarrow \\
37\overline{)1184} & \\
\underline{740} & \leftarrow 20 \times 37 \\
444 & \\
\underline{370} & \leftarrow 10 \times 37 \\
74 & \\
\underline{74} & \leftarrow 2 \times 37 \\
0 &
\end{array}$$

C

$$\begin{array}{r l}
32 & \leftarrow \\
37\overline{)1184} & \\
\underline{1110} & \leftarrow 30 \times 37 \\
74 & \\
\underline{74} & \leftarrow 2 \times 37 \\
0 &
\end{array}$$

DISCUSSION EXERCISES

1. Explain each example. Which way do you think is best?
2. Give the largest multiple of 10 for $\boxed{n} \times 37 < 1184$.
   Give two other multiples of 10 that make the sentence true.
3. When finding the quotient for $37\overline{)1184}$, it is quicker to first find the **largest** multiple of 10 that can be subtracted. Why?

EXERCISES

1. Give the largest multiple of 10 for each exercise.

[A] Think: $\boxed{n} \times 20 < 624$

$\boxed{n} \times 21 < 624$

[B] Think: $\boxed{n} \times 60 < 1357$

$\boxed{n} \times 59 < 1357$

[C] Think: $\boxed{n} \times 40 < 1548$

$\boxed{n} \times 36 < 1548$

[D] $\boxed{n} \times 42 < 1050$

[E] $\boxed{n} \times 38 < 2014$

[F] $\boxed{n} \times 51 < 3417$

[G] $\boxed{n} \times 69 < 2415$

[H] $\boxed{n} \times 72 < 3456$

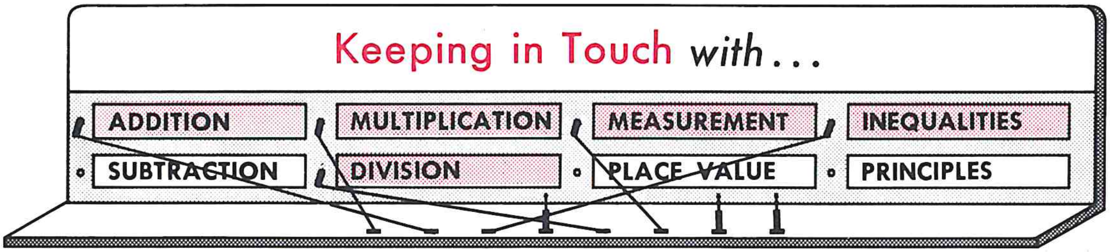
[I] $\boxed{n} \times 87 < 3262$

2. Find the quotients.

[A] $21\overline{)630}$ [B] $59\overline{)1357}$ [C] $42\overline{)1050}$ [D] $31\overline{)372}$ [E] $42\overline{)966}$

[F] $54\overline{)1674}$ [G] $87\overline{)3262}$ [H] $26\overline{)832}$ [I] $91\overline{)1911}$ [J] $78\overline{)1560}$

Keeping in Touch with . . .



1. Write the sign (<, >, =) that should go in each ◍.

[**A**] 25 + 24 ◍ 23 + 26

[**B**] 99 × 7 ◍ 700

[**C**] 51 × 4 ◍ 200

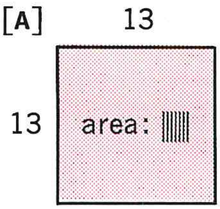
[**D**] 840 ÷ 6 ◍ 840 ÷ 7

[**E**] 24 × 36 ◍ 36 × 24

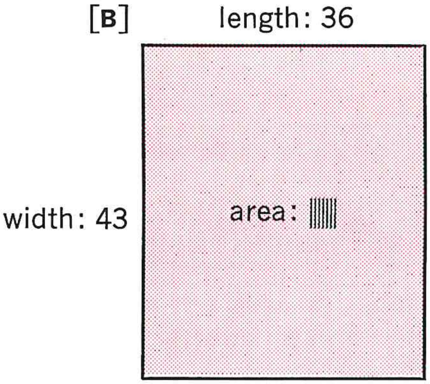
[**F**] 387 + 49 ◍ 48 + 388

2. Pretend that a unit has been chosen and the lengths, widths, and areas of the rectangles are as given. Find the missing numbers.

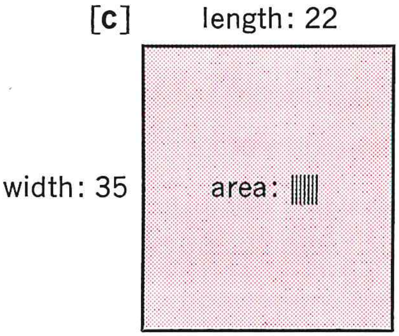
[**A**]



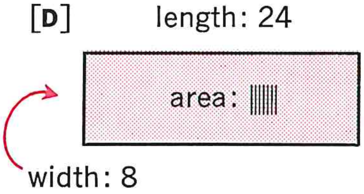
[**B**]



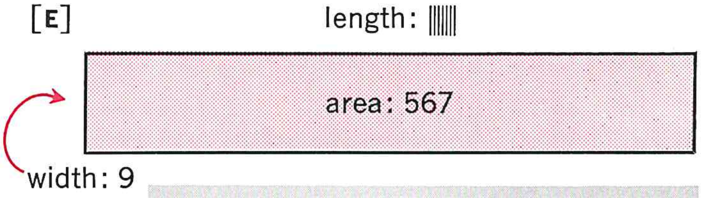
[**C**]



[**D**]



[**E**]



3. Find the sums.

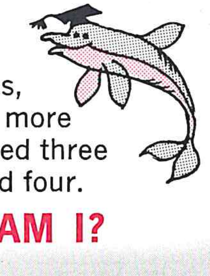
[**A**]
$$\begin{array}{r}23\\40\\56\\35\\\underline{49}\end{array}$$

[**B**]
$$\begin{array}{r}613\\492\\876\\37\\\underline{4023}\end{array}$$

[**C**]
$$\begin{array}{r}8\\407\\8932\\63\\\underline{4006}\end{array}$$

[**D**] 13 + 258 + 87 + 5976 + 503

Think

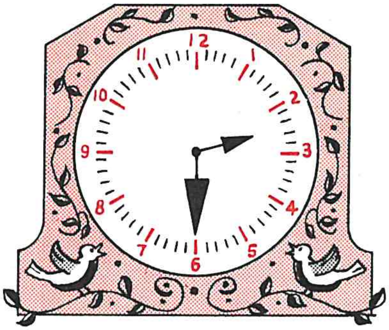


If I were four less,
Then I'd be four more
Than four hundred three
Plus two hundred four.

WHO AM I?

CLOCK PROBLEMS

For exercises 1–4, it is 2:30 in the afternoon.



1. In how many hours will it be

[A] 5:30 in the afternoon?
[B] 7:30 in the evening?
[C] midnight?

2. In how many minutes will it be

[A] 3:30 in the afternoon?
[B] 6:00 in the evening?
[C] 4:47 in the afternoon?
[D] 7:10 in the evening?
[E] 11:20 at night?
[F] 2:30 the next afternoon?

3. Will the minute hand overtake the hour hand

[A] in the next 30 minutes?
[B] in the next 40 minutes?
[C] in the next 45 minutes?
[D] in the next 50 minutes?

4. How many times will the minute hand pass the hour hand in the next

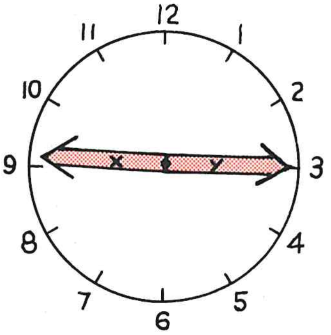
[A] 2 hours?
[B] 3 hours?
[C] 4 hours?
[D] 12 hours?

5. It is 5:37 in the afternoon.

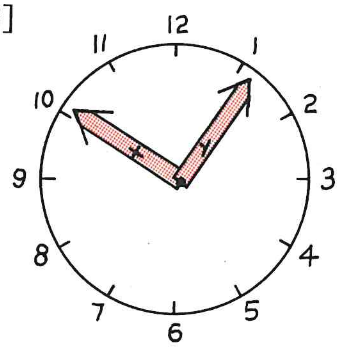
[A] How many minutes has it been since 2:45 in the afternoon?
[B] How many minutes will it be before a television program at 7:45 in the evening?
[C] How long has it been since breakfast at 7:30 in the morning?

★**6.** If the hands of a clock point in the directions shown, which hand is the minute hand? Answer **x** or **y**.

[A]



[B]



Chapter review

1. Copy each exercise and find the quotients and remainders. Check your work.

[A] $3\overline{)60}$ [B] $4\overline{)72}$ [C] $5\overline{)68}$

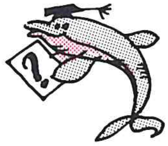
[D] $6\overline{)258}$ [E] $7\overline{)430}$ [F] $8\overline{)664}$

[G] $9\overline{)2106}$ [H] $7\overline{)3759}$ [I] $6\overline{)5239}$

[J] $32\overline{)224}$ [K] $68\overline{)476}$ [L] $91\overline{)549}$

★[M] $35\overline{)1505}$ ★[N] $53\overline{)2491}$ ★[O] $76\overline{)6764}$

Think



I'm smaller than divisors.
We never are the same.
They say it comes out even
When zero is my name.

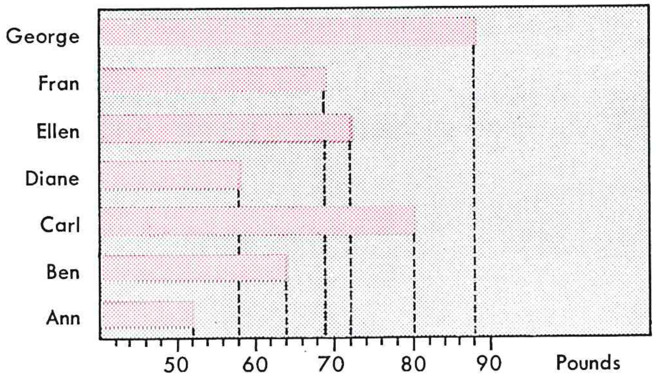
WHO AM I?

2. [A] 35 nines can be subtracted from 315. Give the quotient for $315 \div 9$.

[B] Start with 612. Subtract 50×9. Then subtract 10×9. Finally subtract 8×9. Give the quotient for $612 \div 9$.

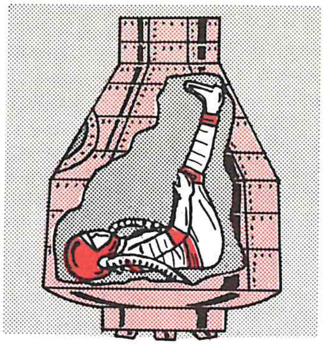
[C] Start with 5799. Subtract 40×87. Subtract 20×87. Subtract 6×87. Give the quotient for $5799 \div 87$. Give the remainder.

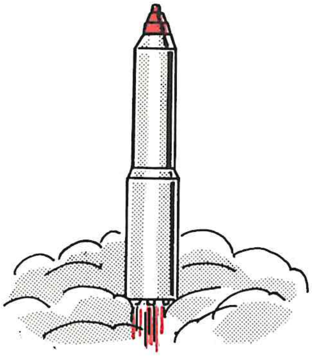
3. The bar graph shows the weights of 7 fourth-grade children. Give the average weight of these children.

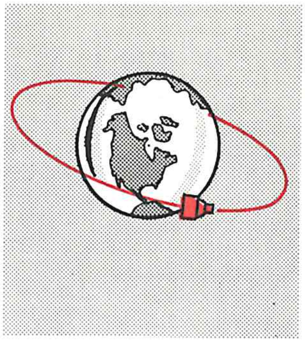


4. The U.S.S. Skate was the first atomic submarine to cross the Atlantic Ocean both ways submerged. The Skate went about 3159 miles in 9 days. About how many miles was this each day?

ASTRONAUTS AND SPACE FLIGHT







1. The ages, weights, and heights of the first 7 United States astronauts (when they joined Project Mercury) are given in the table. Give these averages:

[**A**] average age of the first 4 astronauts in the list

[**B**] average weight of the 5 youngest astronauts

[**C**] average height in inches of the 2 lightest astronauts

Name	Age	Height	Weight
Carpenter	34	5 ft $10\frac{1}{2}$ in.	160
Cooper	32	5 ft 9 in.	150
Glenn	37	5 ft $10\frac{1}{2}$ in.	180
Grissom	33	5 ft 7 in.	155
Schirra	36	5 ft 10 in.	185
Shepard	35	5 ft 11 in.	160
Slayton	35	5 ft $10\frac{1}{2}$ in.	160

2. A satellite in orbit travels about 17,500 miles per hour. How far does it travel in 8 hours?

3. A satellite in orbit travels about 5 miles per second. The airline distance from San Francisco to New York is about 2570 miles.

[**A**] About how many seconds would it take the satellite to go over the United States?

[**B**] About how many minutes?

4. In one of the manned satellite flights it took about 88 minutes for the satellite to make one orbit of the earth.

[**A**] If 1936 minutes had passed since launch time, about how many orbits would have been made?

★ [**B**] If the astronaut slept for 8 hours while in orbit, about how many complete orbits did he make while asleep?

CHAPTER 9 Number theory

Even and odd numbers

$$\begin{array}{cccccc}
 & \text{WHOLE NUMBERS} & & \text{EVEN NUMBERS} & & \text{ODD NUMBERS} \\
2 \times & 0 & \longrightarrow & 0 & \xrightarrow{+1} & 1 \\
2 \times & 1 & \longrightarrow & 2 & \xrightarrow{+1} & 3 \\
2 \times & 2 & \longrightarrow & 4 & \xrightarrow{+1} & 5 \\
2 \times & 3 & \longrightarrow & 6 & \xrightarrow{+1} & 7 \\
2 \times & 4 & \longrightarrow & 8 & \xrightarrow{+1} & 9 \\
2 \times & 5 & \longrightarrow & 10 & \xrightarrow{+1} & 11 \\
2 \times & 6 & \longrightarrow & 12 & \xrightarrow{+1} & 13 \\
2 \times & 7 & \longrightarrow & 14 & \xrightarrow{+1} & 15 \\
2 \times & 8 & \longrightarrow & 16 & \xrightarrow{+1} & 17 \\
2 \times & 9 & \longrightarrow & 18 & \xrightarrow{+1} & 19 \\
 & \vdots & & \vdots & & \vdots \\
2 \times & N & \longrightarrow & 2 \times N & \xrightarrow{+1} & (2 \times N) + 1 \\
 & \vdots & & \vdots & & \vdots
\end{array}$$

▶ A number that is
2 × (a whole number)
is called an **EVEN NUMBER**.

▶ A number that is
1 more than
an even number
is called an **ODD NUMBER**.

EXERCISES

1. Study this sequence of **even numbers.**

$$0, 2, 4, 6, 8, \underbrace{10, 12, 14, 16, 18}_{\text{tens}}, \underbrace{2▥, 2▥, 2▥, 2▥, 2▥}_{\text{twenties}}, \ldots$$

[A] Give the last digits of the even numbers in the twenties.

[B] Give the last digits of the even numbers in the thirties.

[C] Can you give an even number that ends in a digit other than 0, 2, 4, 6, or 8?

2. Every whole number that is not even is odd.

[A] Give the last digits of the odd numbers in the twenties.

[B] Can you give an odd number that ends in a digit other than 1, 3, 5, 7, or 9?

EXERCISES

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

1. [A] How many sums are there in the addition table?

[B] How many of the sums are **even numbers?**

[C] How many of the sums are **odd numbers?**

2. [A] How many products are there in the multiplication table?

[B] How many of the products are **even numbers?**

[C] How many of the products are **odd numbers?**

3. [A] In the multiplication table, the 2 row contains only even numbers. Give the other rows that contain only even numbers.

[B] Are there any rows that contain only odd numbers?

[C] Give a row that helps show that an even number times any number gives a product that is an even number.

[D] Does an odd number times any number give a product that is an odd number?

[E] Write **O** (odd) or **E** (even) for each space in the tables at the right.

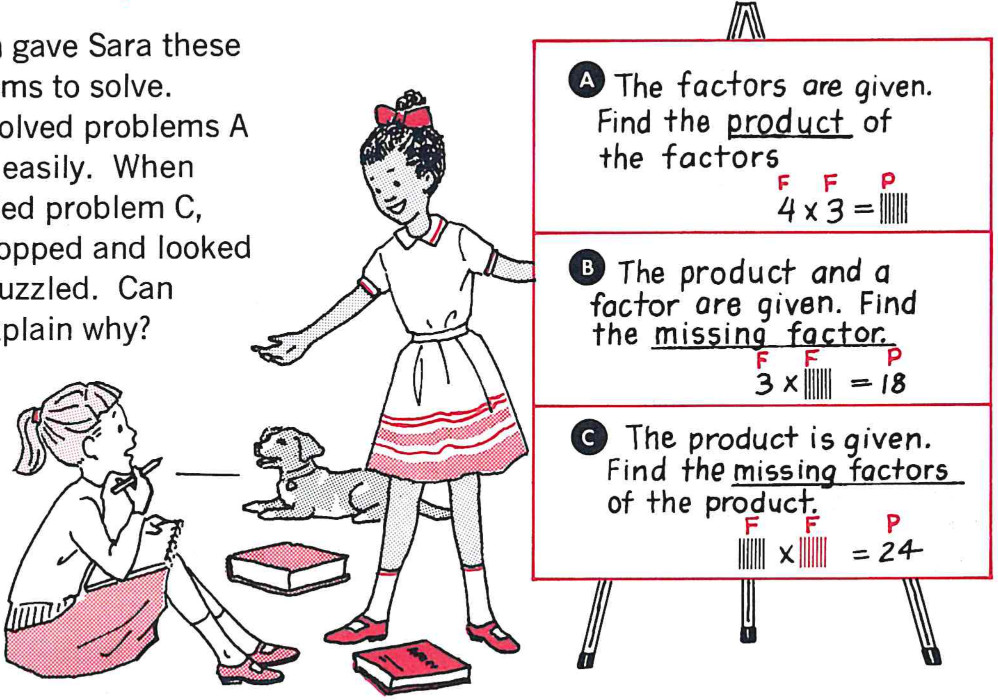
+	E	O
E (even)	[A]	[B]
O (odd)	[C]	[D]

×	E	O
E	[E]	[F]
O	[G]	[H]

4. Which is more, the sum of the first 10 odd numbers or the sum of the first 10 even numbers?

Factors and products

Joan gave Sara these problems to solve. Sara solved problems A and B easily. When she tried problem C, she stopped and looked very puzzled. Can you explain why?



Ⓐ The factors are given. Find the product of the factors

$4 \times 3 = ▥$

Ⓑ The product and a factor are given. Find the missing factor.

$3 \times ▥ = 18$

Ⓒ The product is given. Find the missing factors of the product.

$▥ \times ▥ = 24$

EXERCISES

1. Give as many different pairs of numbers as you can that could serve as factors of 24.

2. List the equations that have **no** whole-number solution. Then find the missing factors in the other equations.

[A] $\boxed{n} \times 1 = 12$
[B] $\boxed{n} \times 2 = 12$
[C] $\boxed{n} \times 3 = 12$
[D] $\boxed{n} \times 4 = 12$
[E] $\boxed{n} \times 5 = 12$
[F] $\boxed{n} \times 6 = 12$
[G] $\boxed{n} \times 7 = 12$
[H] $\boxed{n} \times 8 = 12$
[I] $\boxed{n} \times 9 = 12$
[J] $\boxed{n} \times 10 = 12$
[K] $\boxed{n} \times 11 = 12$
[L] $\boxed{n} \times 12 = 12$
[M] $\boxed{n} \times 1 = 32$
[N] $\boxed{n} \times 2 = 32$
[O] $\boxed{n} \times 3 = 32$
[P] $\boxed{n} \times 4 = 32$
[Q] $\boxed{n} \times 5 = 32$
[R] $\boxed{n} \times 6 = 32$

3. Write as many different multiplication equations as you can that have these numbers as **products.** Write equations that have just two factors.

[A] 6 (Example: $1 \times 6 = 6$) [B] 18 [C] 20 [D] 28
[E] 32 [F] 30 [G] 36 [H] 48 [I] 100

Finding factors

Ⓐ Since $\boxed{n} \times 7 = 28$ has the solution 4,
we know that 4 and 7 are factors of 28.

Ⓑ Since $\boxed{n} \times 6 = 28$ has no whole-number solution,
we know that 6 is not a factor of 28.

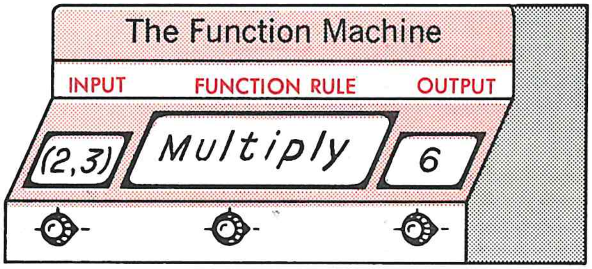
EXERCISES

1. Solve the equation and give two factors of the product.

[**A**] $\boxed{n} \times 5 = 15$
(Answer: 3, 5)
[**B**] $\boxed{n} \times 7 = 63$
[**C**] $\boxed{n} \times 5 = 60$
[**D**] $\boxed{n} \times 12 = 48$
[**E**] $\boxed{n} \times 15 = 75$
[**F**] $\boxed{n} \times 13 = 39$
[**G**] $\boxed{n} \times 9 = 72$
[**H**] $\boxed{n} \times 18 = 72$
[**I**] $\boxed{n} \times 36 = 72$
[**J**] $\boxed{n} \times 1 = 32$
[**K**] $\boxed{n} \times 2 = 32$
[**L**] $\boxed{n} \times 4 = 32$
[**M**] $\boxed{n} \times 16 = 48$
[**N**] $\boxed{n} \times 2 = 48$

2. [**A**] Is 4 a factor of 11?
[**B**] Is 8 a factor of 63?
[**C**] Is 13 a factor of 42?
[**D**] Is 17 a factor of 51?

3. Using the rule shown, you can put a **pair** of numbers into the function machine and get a single output number. Some output numbers are given below. Give as many pairs as you can that would produce each output number.



[**A**] 4 [Answers: (1, 4) and (2, 2)] [**B**] 12 [**C**] 6 [**D**] 5
[**E**] 8 [**F**] 18 [**G**] 11 [**H**] 30 [**I**] 36 [**J**] 50

4. List all the factors of each number.

[**A**] 12 (Answer: 1, 2, 3, 4, 6, 12) [**B**] 6 [**C**] 5
[**D**] 8 [**E**] 18 [**F**] 11 [**G**] 30 [**H**] 36
[**I**] 50 [**J**] 45 [**K**] 1 [**L**] 37 [**M**] 100

Common factors

Ⓐ The factors of 8: {1, 2, 4, 8}

The factors of 12: {1, 2, 3, 4, 6, 12}

1, 2, and 4 are all factors of both 8 and 12.

1, 2, and 4 are common factors of 8 and 12.

Ⓑ The factors of 6: {1, 2, 3, 6}

The factors of 9: {1, 3, 9}

1 and 3 are common factors of 6 and 9.

EXERCISES

1. [A] List the factors of 9. [B] List the factors of 12.
[C] List the common factors of 9 and 12.

2. [A] List the factors of 6. [B] List the factors of 8.
[C] List the common factors of 6 and 8.

3. [A] List the factors of 12. [B] List the factors of 16.
[C] List the common factors of 12 and 16.

4. [A] List the factors of 8. [B] List the factors of 9.
[C] List the common factors of 8 and 9.

5. [A] List the factors of 12. [B] List the factors of 18.
[C] List the common factors of 12 and 18.

6. Give the number that is a factor of every number.

7. Give two numbers greater than 10 that have no common factor except 1.

★ **8.** Give a number less than 70 that is a factor of one of the numbers 70 and 105, and not of the other. Do as little computing as possible.

Greatest common factors

Ⓐ The factors of 8: {1, 2, 4, 8}

The factors of 12: {1, 2, 3, 4, 6, 12}

4 is the **largest number** that is a factor of both 8 and 12.

4 is the greatest common factor of 8 and 12.

Ⓑ The factors of 6: {1, 2, 3, 6}

The factors of 9: {1, 3, 9}

3 is the greatest common factor of 6 and 9.

EXERCISES

1. Use your answers to the exercises on page 236 to help you find
   - [A] the **greatest common factor** of 9 and 12.
   - [B] the greatest common factor of 6 and 8.
   - [C] the greatest common factor of 12 and 16.
   - [D] the greatest common factor of 8 and 9.
   - [E] the greatest common factor of 12 and 18.

2. [A] List the factors of 20. [B] List the factors of 16.
   [C] List the common factors of 20 and 16.
   [D] What is the greatest common factor of 20 and 16?

3. [A] List the factors of 24. [B] List the factors of 32.
   [C] List the common factors of 24 and 32.
   [D] What is the greatest common factor of 24 and 32?

★ 4. Give an odd number and an even number between 10 and 20 that have 3 as the greatest common factor.

How many factors?

DISCUSSION EXERCISES

▶ Every number (except 1) has at least two factors.

Since $1 \times 11 = 11$, we know that 1 and 11 are factors of 11.
Since $1 \times 28 = 28$, we know that 1 and 28 are factors of 28.
Since $1 \times 37 = 37$, we know that 1 and 37 are factors of 37.

1. Can you describe the two factors that any number (except 1) is sure to have?

▶ Some numbers have more than two factors.

A Here is a set of numbers that have 2 as a factor.

B Here is a set of numbers that have 3 as a factor.

C Here is a set of numbers that have 4 as a factor.

×	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
2 →	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34
3 →	0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51
4 →	0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68
5	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85
6	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102
7	0	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119

2. 34 is in the 2 row of the table above. Use this fact and exercise 1 to show that 34 has more than two factors.

3. 42 is in the 3 row of the table. Show that 42 has more than two factors by giving three factors of 42.

▶ Some numbers have **only** two factors.

4. 1 and 37 are factors of 37. Can you give others?

5. Give five numbers that have only two factors.

Prime numbers

Here is a multiplication table with the 0 row and column and the 1 row and column left out.

×	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54		
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68			
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80				
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90					
7	14	21	28	35	42	49	56	63	70	77	84	91	98						
8	16	24	32	40	48	56	64	72	80	88	96	104							
9	18	27	36	45	54	63	72	81	90	99	108								
10	20	30	40	50	60	70	80	90	100	110									
11	22	33	44	55	66	77	88	99	110										
12	24	36	48	60	72	84	96	108											
13	26	39	52	65	78	91	104												
14	28	42	56	70	84	98													
15	30	45	60	75	90														
16	32	48	64	80															
17	34	51	68																
18	36	54																	
19	38																		
20																			

121
12 14 31
131

Think of this table as going on without end.

EXERCISES

1. List the numbers from 2 to 40. Use the table to **mark out** the numbers in your list that have **more than two factors.** What you have left is the set of **prime numbers** less than 40.

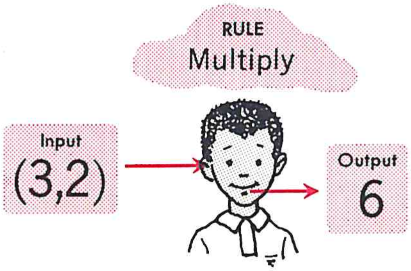
   ▶ Numbers that have **exactly** 2 factors are called **PRIME NUMBERS.**

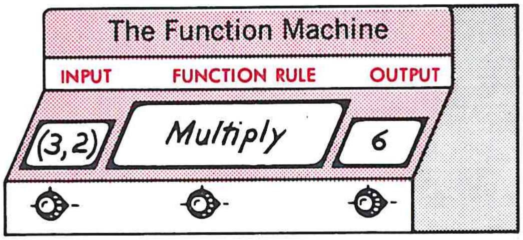
2. List the prime numbers between 40 and 50.

3. Tell why these numbers are not prime.
   [A] 52 [B] 54 [C] 56 [D] 55 [E] 51 ★[F] 57

4. Which of these numbers are prime?
   [A] 53 [B] 58 [C] 59 [D] 63 [E] 61 [F] 77

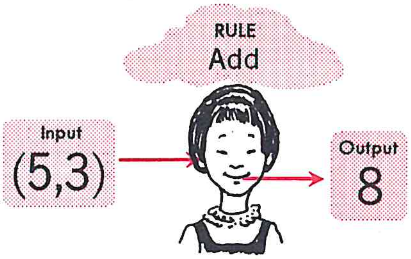
The function machine

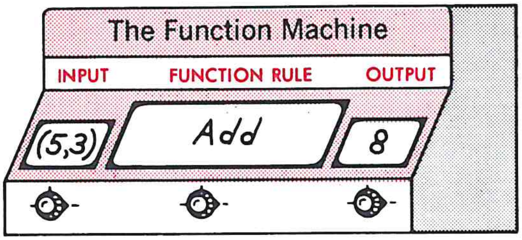




1. Give a pair of **prime numbers** that will produce each of these output numbers. [A] 4 [Answer: (2, 2)]

[B] 10 [C] 9 [D] 14 [E] 15 [F] 21 [G] 22 [H] 25 [I] 26 [J] 33

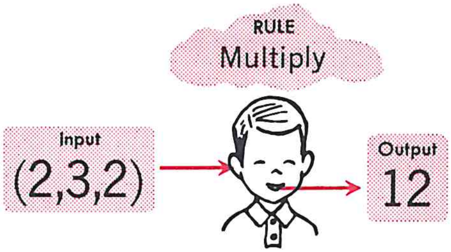


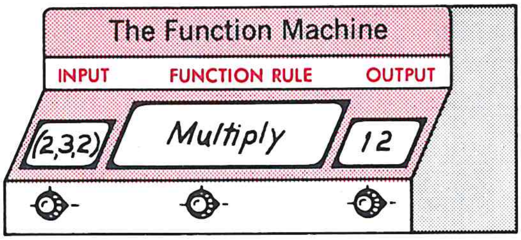


2. Give a pair of **prime numbers** that will produce each output number. [A] 9 [Answer: (7, 2)]

[B] 4 [C] 12 [D] 6 [E] 14 [F] 10 [G] 15 [H] 16 [I] 18 [J] 20
[K] 21 [L] 22 [M] 24 [N] 25 [O] 26 [P] 28 [Q] 30 [R] 32 [S] 33

Using the rule shown, you can put three numbers into the function machine and get a single output number.





3. Give the output number for each number triple.

[A] (1, 4, 2) [B] (2, 4, 2) [C] (5, 3, 0) [D] (1, 1, 1) [E] (3, 4, 5)

EXERCISES

1. The whole numbers greater than 1 that are not prime numbers are called **composite** numbers.

[A] List the composite numbers less than 40.

[B] Give a composite number between 90 and 100.

2. [A] List all the factors of 12.

[B] List the prime factors of 12.

[C] List the factors of 30.

[D] List the prime factors of 30.

Think

I'm such a sad odd number.
I cannot be a prime.
Since I have just one factor,
They leave me out each time.

WHO AM I?

3. [A] The equation $30 = 5 \times 2 \times 3$ shows that 30 is a product of prime numbers. Write an equation to show that 35 is a product of prime numbers.

[B] Write an equation to show that 36 is a product of prime numbers.

4. Give the word or number for each blank.

[A] Every prime number is odd except _?_.

[B] The number _?_ is a factor of every number.

[C] Each prime number has exactly _?_ factors.

[D] The only factor of 1 is _?_.

[E] Since the only factors of 73 are 1 and 73, 73 is a _?_ number.

[F] Since 69 has four factors, 69 is a _?_ number.

[G] 57 is not a prime number because $3 \times \boxed{n} = 57$.

[H] _?_ is the only prime number between 61 and 71.

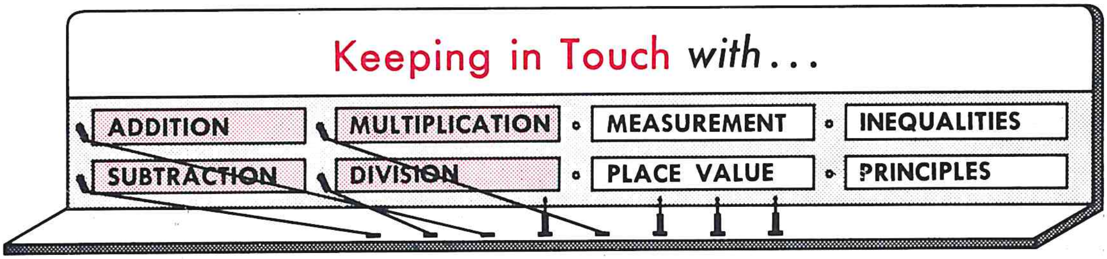
★ [I] _?_ is the only prime number between 79 and 89.

★ [J] _?_ is the only prime number between 103 and 109.

★ [K] _?_ is the only prime number between 113 and 131.

★ **5.** The numbers 3 and 5 are called **twin primes** because their difference is 2. Another pair of twin primes is 41 and 43. Give four more pairs of twin primes.

Keeping in Touch *with . . .*



1. Solve the equations.

[A] $10 \times 10 = \boxed{n}$ [D] $100 \times 100 = \boxed{n}$ [G] $10{,}000 \div 10 = \boxed{n}$
[B] $100 \times 10 = \boxed{n}$ [E] $100 \div 10 = \boxed{n}$ [H] $10{,}000 \div 100 = \boxed{n}$
[C] $10 \times 1000 = \boxed{n}$ [F] $1000 \div 10 = \boxed{n}$ [I] $10{,}000 \div 1000 = \boxed{n}$

2. Give the missing numbers.

[A] Since $5 \times 7 = 35$, we call ▨ a multiple of 7.
[B] 48 is a multiple of 6 because $\boxed{n} \times 6 = 48$.
[C] ▨ is a multiple of 2 because $30 \times 2 = 60$.
[D] Since $5 \times 9 = 45$, 45 is a multiple of both 5 and ▨.
[E] Since $63 + 78 = 141$, we know that $141 - 78 = \boxed{n}$.
[F] Since $156 - 79 = 77$, we know that $77 + 79 = \boxed{n}$.
[G] Since $7 \times 38 = 266$, we know that $266 \div 7 = \boxed{n}$.
[H] Since $147 \div 3 = 49$, we know that $49 \times 3 = \boxed{n}$.
[I] Since $39 + 39 = 78$, we know that $78 - (39 + 39) = \boxed{n}$.
[J] Since $11 \times 12 = 132$, we know that $132 \div (11 \times 12) = \boxed{n}$.
[K] Since $39 + 61 = 100$, we know that $39 + 28 + 61 = \boxed{n}$.

3. Find the sums and differences.

[A] $\begin{array}{r} 35 \\ +89 \\ \hline \end{array}$ [B] $\begin{array}{r} 132 \\ -59 \\ \hline \end{array}$ [C] $\begin{array}{r} 5076 \\ +3938 \\ \hline \end{array}$ [D] $\begin{array}{r} 8206 \\ -39 \\ \hline \end{array}$ [E] $\begin{array}{r} 30{,}042 \\ -5{,}978 \\ \hline \end{array}$

4. Find the products.

[A] $\begin{array}{r} 9 \\ \times 6 \\ \hline \end{array}$ [B] $\begin{array}{r} 38 \\ \times 7 \\ \hline \end{array}$ [C] $\begin{array}{r} 434 \\ \times 9 \\ \hline \end{array}$

[D] $\begin{array}{r} 32 \\ \times 25 \\ \hline \end{array}$ [E] $\begin{array}{r} 627 \\ \times 34 \\ \hline \end{array}$ [F] $\begin{array}{r} 347 \\ \times 562 \\ \hline \end{array}$

5. Find the quotients.

[A] $7\overline{)294}$ [B] $8\overline{)3448}$ [C] $23\overline{)2093}$

Think

Multiply me by myself,
Or find my sum with two.
Your answer is the same
No matter which you do.

WHO AM I?

all a/o

Chapter review

1. Answer **E** if the number is even and **O** if it is odd.

[A] 68
[B] 83
[C] 2001
[D] 3958
[E] 7642
[F] 7050

2. The first even number is 0. The second is 2. The third is 4. The fourth is 6. The fifth is 8.

[A] What is the sixth even number?
[B] What is the ninth even number?
[C] What is the seventeenth even number?

Think

I'm slightly less than 30,
But more than 22.
They say that I am perfect,
And here is why they do.
Just add up all my factors,
And then divide by two.
You'll find that I'm the answer.
I'll say no more to you.

WHO AM I?

3. The first odd number is 1. The second is 3.

[A] What is the third odd number?
[B] What is the eighth odd number?
[C] What is the fourteenth odd number?

4. List the composite numbers between 20 and 30.

5. [A] List the factors of 18. [B] List the factors of 24.
[C] List the common factors of 18 and 24.
[D] What is the greatest common factor of 18 and 24?

6. [A] List the factors of 30. [B] List the factors of 42.
[C] List the common factors of 30 and 42.
[D] List the common **prime** factors of 30 and 42.
[E] What is the greatest common factor of 30 and 42?

7. [A] Write an equation to show that 38 is the product of two prime numbers.
[B] Write an equation to show that 39 is the product of two prime numbers.

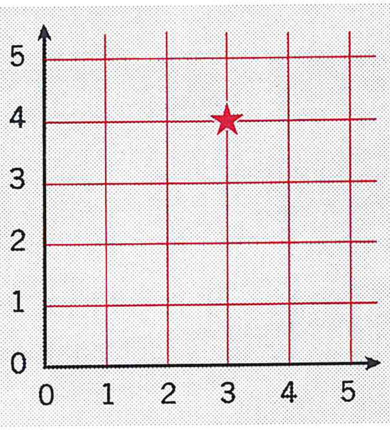
COORDINATE GEOMETRY Unit 4

Using coordinates

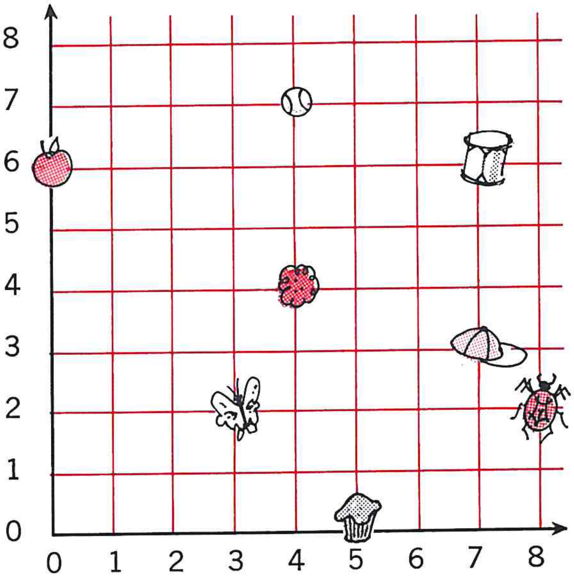
Where is the star?

"3 over and 4 up"

Its coordinates are (3, 4).



In this lesson, you will use number pairs (coordinates) to describe locations.



1. Give the missing numbers. Then give the coordinates.

[**A**] The butterfly is 3 over and ▥ up.
The coordinates for the butterfly are _?_.

[**B**] The cap is ▥ over and 3 up.
The coordinates for the the cap are _?_.

[**C**] The beetle is 8 over and ▥ up.
The coordinates for the beetle are _?_.

[**D**] The drum is ▥ over and 6 up.
The coordinates for the drum are _?_.

[**E**] The cupcake is 5 over and ▥ up. The coordinates for the cupcake are _?_.

[**F**] The apple is 0 over and ▥ up. The coordinates for the apple are _?_.

[**G**] What are the coordinates for the flower?

[**H**] What are the coordinates for the ball?

2. [A] Give the coordinates for K.

[B] What letter is 11 over and 3 up?

[C] What letter has coordinates (5, 5)?

[D] Give the coordinates for C.

[E] What do you find at (9, 0)?

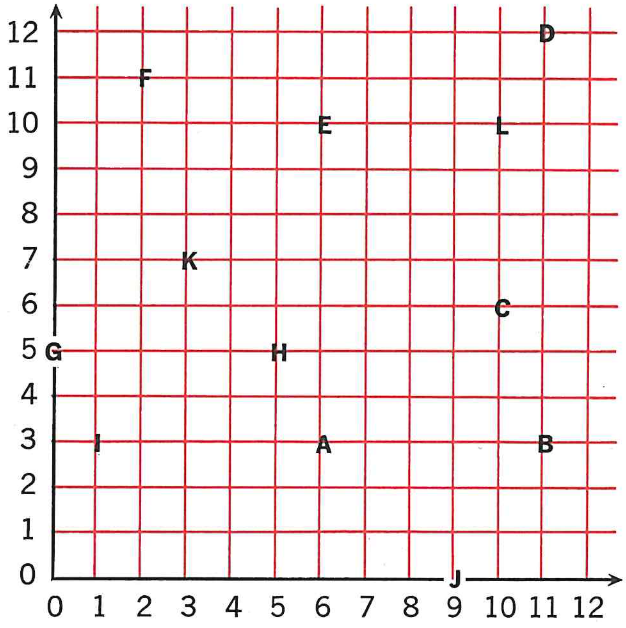
[F] What letter has 2 as its first coordinate?

[G] What letter is 12 up?

[H] What letter is at (0, 5)?

[I] What letter has coordinates (6, 3)?

[J] Give the coordinates of **I**.



★ **3.** If the coordinates of **W** are (5, 6),

[A] what are the coordinates of **X**?

[B] what are the coordinates of **Y**?

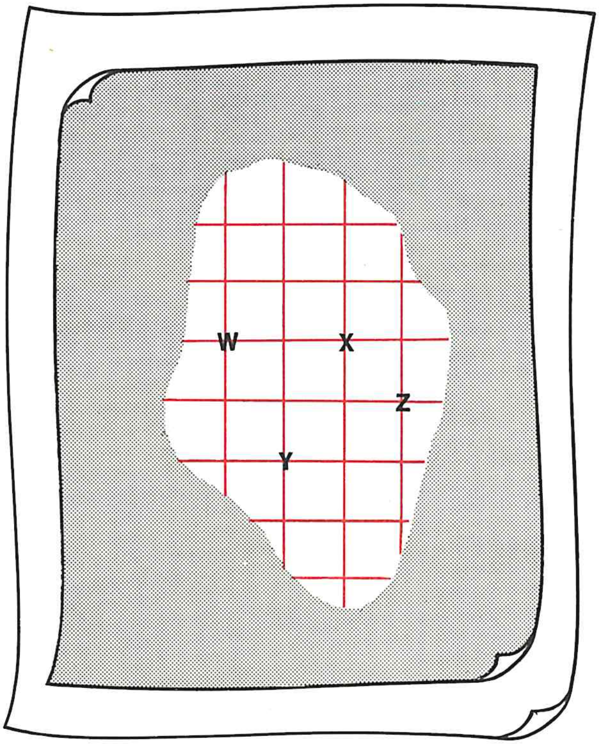
[C] what are the coordinates of **Z**?

★ **4.** If the coordinates of **Z** are (7, 5),

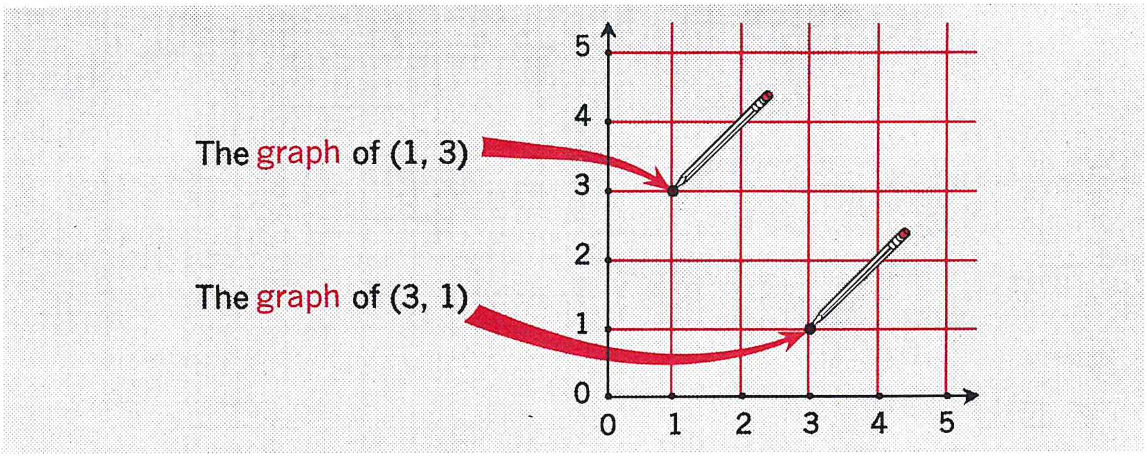
[A] what are the coordinates of **W**?

[B] what are the coordinates of **X**?

[C] what are the coordinates of **Y**?

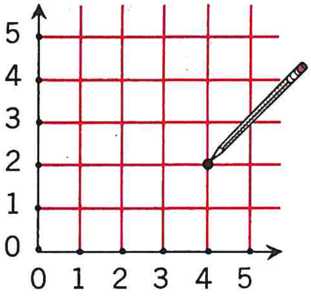


Graphing number pairs

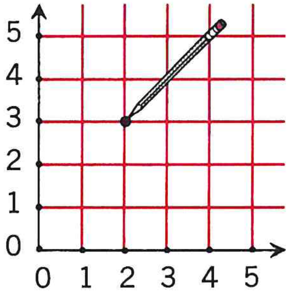


1. Each figure below shows a point being graphed. Give the coordinates of that point.

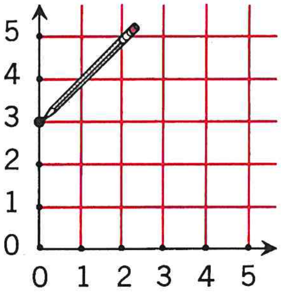
[A]



[B]



[C]



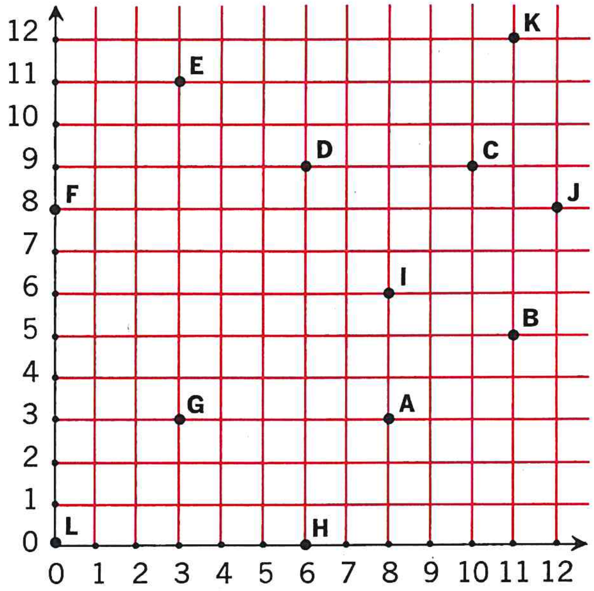
2. Give the coordinates for each point **A** through **H**.

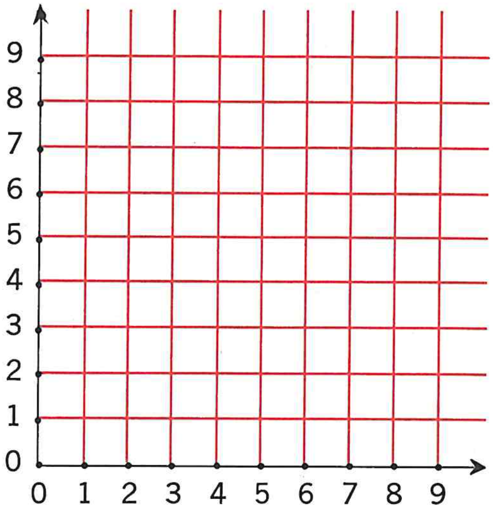
3. [A] What point has coordinates (11, 12)?

[B] What point is at (8, 6)?

[C] (0, 0) locates what point?

[D] Give the point for (12, 8).





4. The figure at the right is called a pair of coordinate axes. Each number line is called an axis. The red lines are given to help locate points. Draw a pair of coordinate axes on your paper.

5. Use your coordinate axes from exercise 4 to graph each of the following coordinates. Label the first point A, the second B and so on.

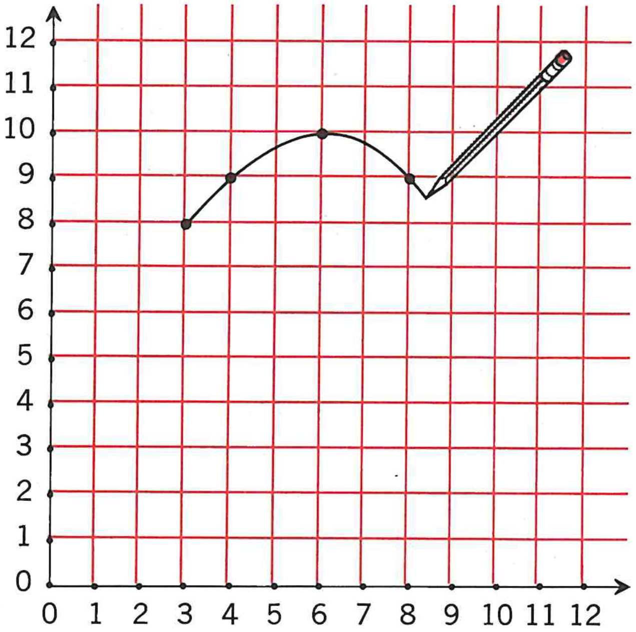
[A] (6, 2)   [C] (7, 7)   [E] (4, 0)
[B] (2, 6)   [D] (3, 8)   [F] (0, 5)

6. Draw a pair of coordinate axes like those in exercise 4.

[A] Graph the points that have coordinates with 6 as their first number. {(6, 0), (6, 1), (6, 2), . . .}

[B] On the same figure, graph the points that have 4 as their second number.

[C] What are the coordinates of the point that is in both sets?



7. Graph each of the following points.

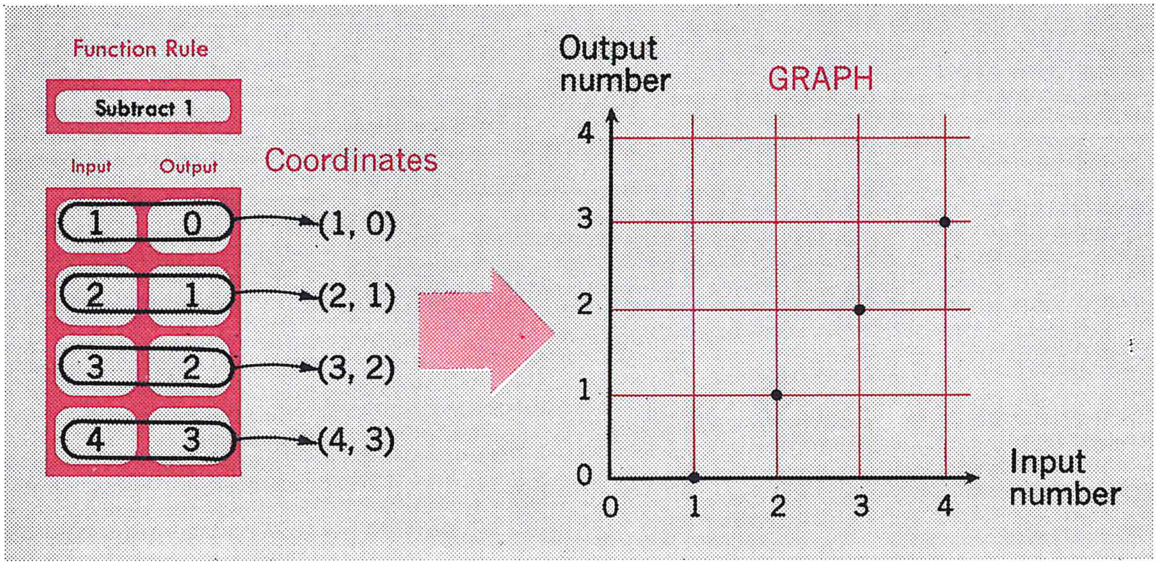
(3, 8), (4, 9), (6, 10), (8, 9), (9, 8), (10, 6), (9, 4), (8, 3), (6, 2), (4, 3), (3, 4), (2, 6)

[A] Draw a simple closed curve connecting all the points as in the figure.

[B] Give the coordinates of a point inside the curve.

[C] Give the coordinates of a point outside the curve.

Graphing functions

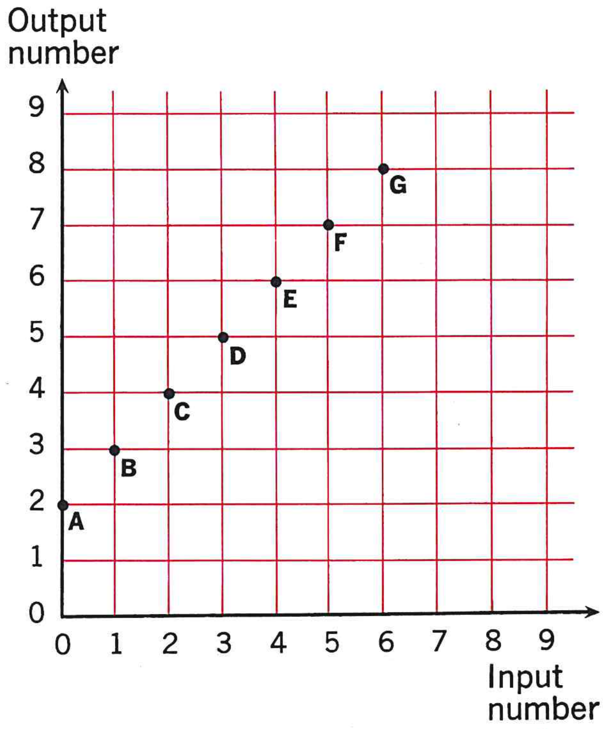


1. Give the missing coordinates. Then give the coordinates for each point in the graph. The two sets of coordinates should be the same.

Function Rule

Add 2

Input	Output		Coordinates
0	2	⟶	(0, 2)
1	3	⟶	(1, ▥)
2	4	⟶	(▥, ▥)
3	5	⟶	(▥, ▥)
4	6	⟶	(▥, ▥)
5	7	⟶	(▥, ▥)
6	8	⟶	(▥, ▥)



2. What would you expect the coordinates of a point **H** to be?

3. Copy the function tables and give the missing numbers. Then list the sets of coordinates for the tables. Finally, draw the graph for each set of coordinates.

[A] Function Rule

Subtract 2

Input	Output
2	▥
3	▥
4	▥
5	▥
6	▥
7	▥
8	▥
9	▥

[B] Function Rule

Divide by 2

Input	Output
0	▥
2	▥
4	▥
6	▥
8	▥
10	▥
12	▥
14	▥

[C] Function Rule

Subtract from 9

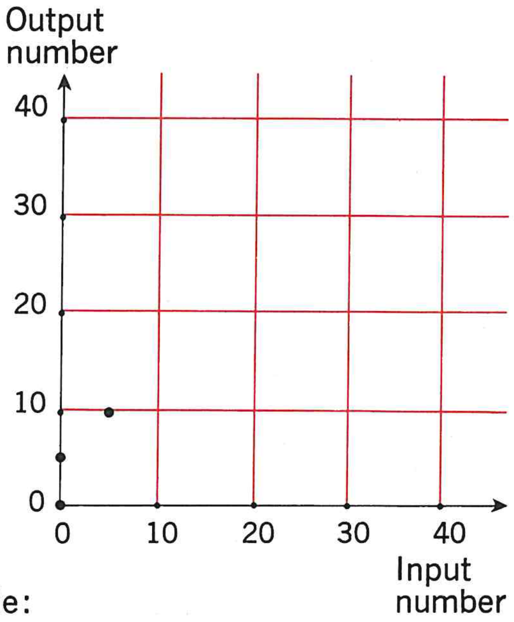
Input	Output
0	▥
1	▥
2	▥
3	▥
4	▥
5	▥
6	▥
7	▥

★**4.** Copy and complete the table. List the coordinates. Copy and complete the graph of this function.

Function Rule

Add 5

Input	Output		Coordinates
0	5	⟶	(0, 5)
5	▥	⟶	(5, ▥)
10	▥	⟶	(10, ▥)
15	▥	⟶	(15, ▥)
20	▥	⟶	(20, ▥)
25	▥	⟶	(25, ▥)
30	▥	⟶	(30, ▥)
35	▥	⟶	(35, ▥)

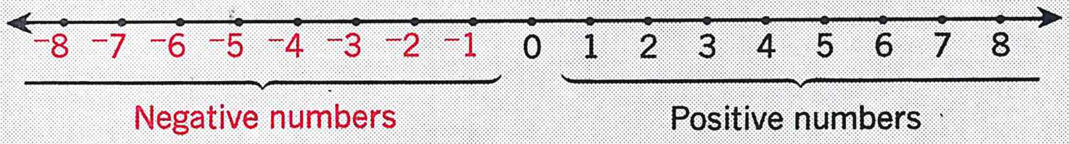


★**5.** Make your own table for this function rule:

Multiply the number by itself.

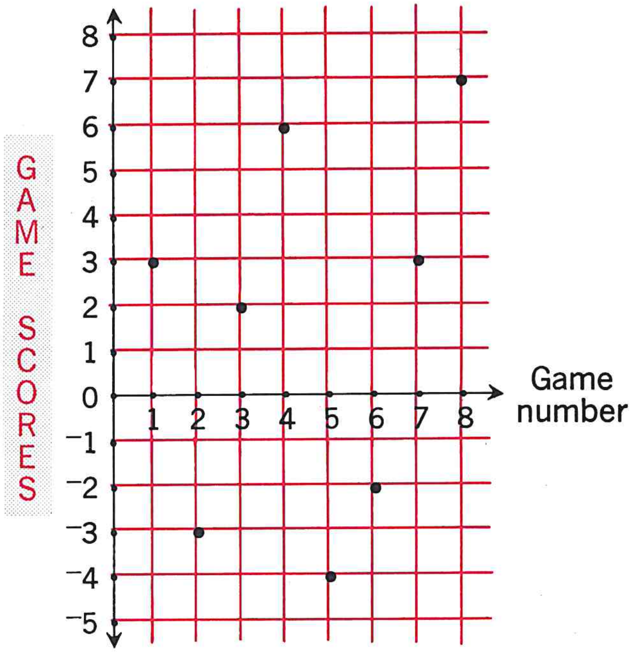
Use input numbers 0, 1, 2, 3, 4.
Draw the graph for this function.

★Negative numbers and graphing



Read: Negative one, negative two, negative three, . . .

1. Carl and Janet were playing a game where you could get scores less than zero. They used **negative** numbers for their scores **below zero** and positive numbers for scores above zero. The graph shows Janet's scores on the first 8 games.



[A] What was Janet's score on the first game? What are the coordinates of that point?

[B] What was Janet's score on the second game? What are the coordinates of that point?

[C] What are the coordinates for Janet's lowest score?

[D] In what game did Janet score −2?

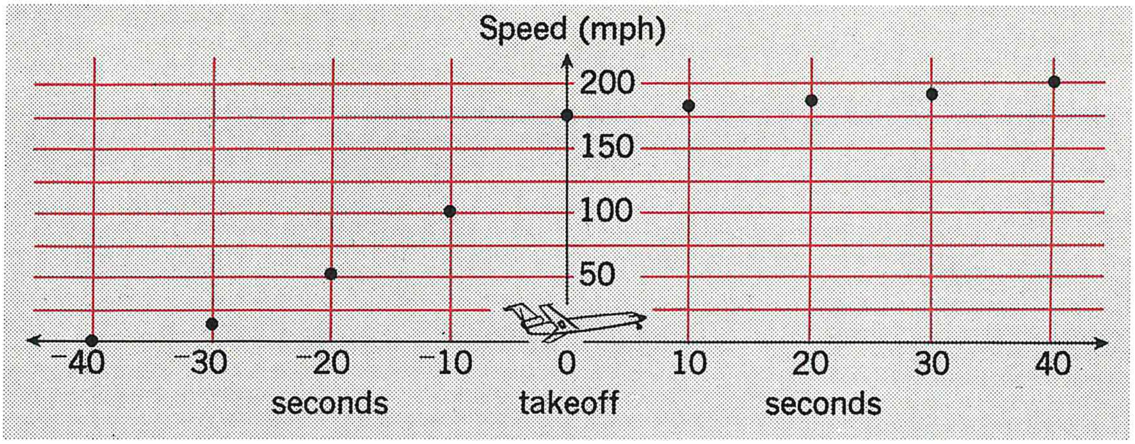
[E] In what game did Janet make her highest score?

[F] In game 3, Carl scored 3 points below Janet. What was his score?

[G] In game 5, Carl scored 10 points higher than Janet. What was his score?

[H] In game 1, Carl was 5 points below Janet. What are the coordinates for Carl's score in game 1?

2. The graph below shows the approximate changes in speed of a modern jet aircraft during takeoff. Negative numbers indicate seconds before takeoff and positive numbers indicate seconds after takeoff.



[A] What was the speed 20 seconds before takeoff?
What are the coordinates of that point?

[B] What was the speed at takeoff?
What are the coordinates of that point?

[C] Give the coordinates for the speed 40 seconds after

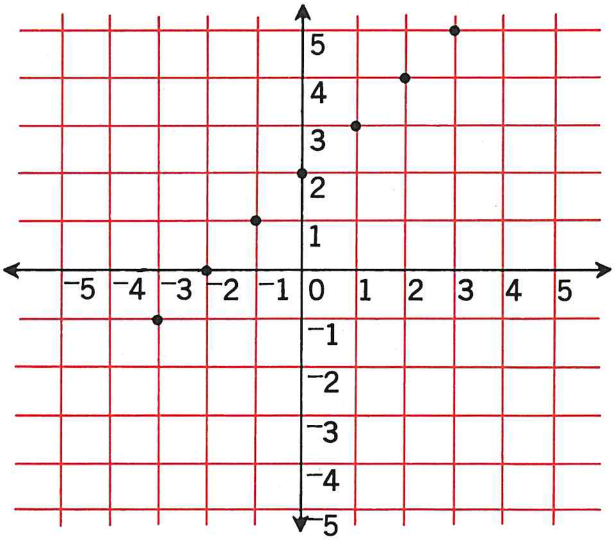
[D] What do coordinates $(^{-}10, 100)$ tell you?

3. Complete the function table. The graph may help you.

Function Rule

Add 2

Input	Output
3	5
2	4
1	▥
0	▥
$^{-}1$	▥
$^{-}2$	▥
$^{-}3$	▥

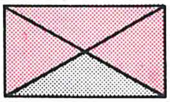
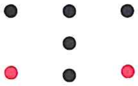
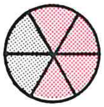
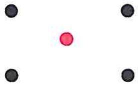


4. Use the table or graph to help you solve these equations.

[A] $2 + {}^{-}2 = \boxed{n}$ [B] $2 + {}^{-}1 = \boxed{n}$ [C] $2 + {}^{-}3 = \boxed{n}$ [D] $2 + {}^{-}4 = \boxed{n}$

CHAPTER 10 Fractions

Number pairs and fractions

We see	We think	We write	We say
	3 red parts 4 parts in all	$\frac{3}{4}$ of the region is red.	Three fourths of the region is red.
	2 red dots 7 dots in all	$\frac{2}{7}$ of the dots are red.	Two sevenths of the dots are red.
	4 red parts 6 parts in all	$\frac{4}{6}$ of the region is red.	Four sixths of the region is red.
	1 red dot 5 dots in all	$\frac{1}{5}$ of the dots are red.	One fifth of the dots are red.

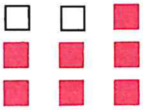
↑ In this column, we think about a number pair.

↑ In this column, we write a fraction for the number pair.

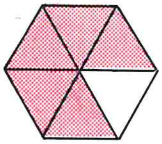
EXERCISES

For each picture, write a fraction to tell what part is shaded.

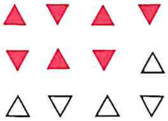
1.



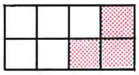
2.



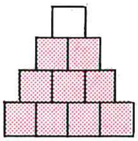
3.



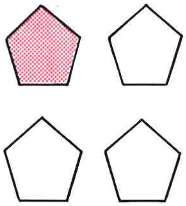
4.



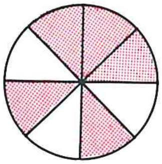
5.



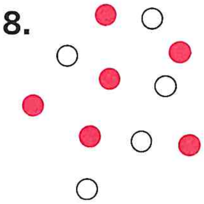
6.



7.



8.



ORAL EXERCISES

1. Read the examples.

[A] For $\frac{1}{2}$, we say one half.

[B] For $\frac{3}{8}$, we say three eighths.

[C] For $\frac{2}{3}$, we say two thirds.

[D] For $\frac{6}{12}$, we say six twelfths.

[E] For $\frac{7}{10}$, we say seven tenths.

[F] For $\frac{3}{4}$, we say three fourths.

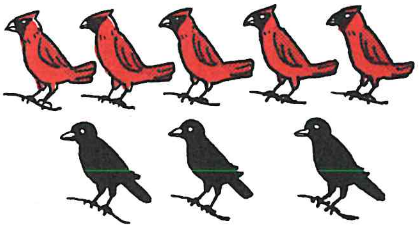
2. Read the fractions.

[A] $\frac{5}{8}$
[B] $\frac{7}{9}$
[C] $\frac{3}{5}$
[D] $\frac{3}{6}$
[E] $\frac{2}{4}$
[F] $\frac{1}{7}$
[G] $\frac{7}{15}$
[H] $\frac{9}{10}$
[I] $\frac{1}{3}$
[J] $\frac{1}{14}$
[K] $\frac{15}{16}$
[L] $\frac{6}{7}$
[M] $\frac{1}{4}$
[N] $\frac{1}{2}$
[O] $\frac{7}{8}$
[P] $\frac{1}{10}$
[Q] $\frac{13}{20}$
[R] $\frac{5}{12}$
[S] $\frac{2}{10}$
[T] $\frac{15}{1000}$
[U] $\frac{7}{100}$
[V] $\frac{68}{100}$
[W] $\frac{8}{10}$
[X] $\frac{6}{1000}$

EXERCISES

1. Write a fraction for each number-pair story.

[A] Tom said, "5 of the 8 birds in our yard are cardinals." What fraction of the birds are cardinals?



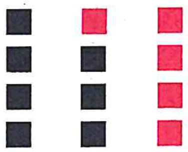
[B] Sue has 9 coins in her purse. 4 of them are nickels. What fraction of the coins are nickels?

[C] There are 27 children in Miss Small's class. 15 of them are girls. What fraction of the children are girls?

2. Write a number-pair story about each fraction.

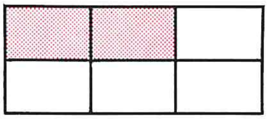
[A] $\frac{1}{4}$ [B] $\frac{2}{3}$ [C] $\frac{5}{6}$ [D] $\frac{1}{2}$ [E] $\frac{7}{8}$

Number pairs

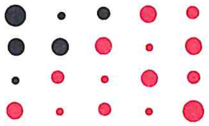
We think about a pair of numbers. ⬇		We write a fraction for the number pair. ⬇
Black squares: 7 All squares: 12	⇨	$\frac{7}{12}$ of the squares are black.
5 out of 12 are red.	⇨	$\frac{5}{12}$ of the squares are red.
First row: 3 All squares: 12	⇨	$\frac{3}{12}$ of the squares are in the first row.
4 out of 12 are in the last column.	⇨	$\frac{4}{12}$ of the squares are in the last column.

EXERCISES

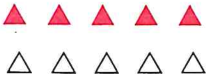
Give the missing numbers. Then give a fraction to answer each question.

1. 

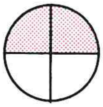
[A] ▥ parts are red.
[B] ▥ parts in all.
[C] What part of the region is red?

2. 

[A] ▥ dots are black.
[B] ▥ dots in all.
[C] What part of the dots are black?

3. 

[A] ▥ triangles are red.
[B] ▥ triangles in all.
[C] What part of the triangles are red?

4. 

[A] ▥ sections are red.
[B] ▥ sections in all.
[C] What part of the region is red?

Number-pair exercises

1. Write a fraction for each exercise.

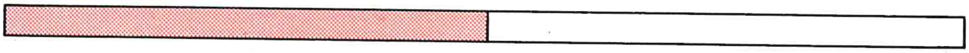
[A] 2 out of 5
(Answer: $\frac{2}{5}$)
[B] 7 out of 10
[C] 9 out of 30
[D] 1 out of 2
[E] 6 out of 18
[F] two thirds
[G] 2 out of 3
[H] twelve hundredths
[I] 12 out of a hundred
[J] nine tenths
[K] 9 times out of 10

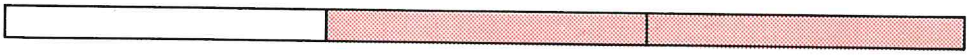
2. Ann has 12 goldfish. 5 of the fish have black spots. What fraction of the fish have black spots?

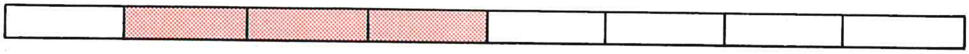
3. Jim said, "Two thirds of my goldfish have black spots." Jim has 3 goldfish. How many have black spots?

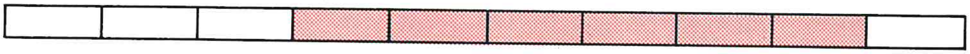
4. Write a story using the number pair in exercise 1B. Now write a story about exercise 1K.

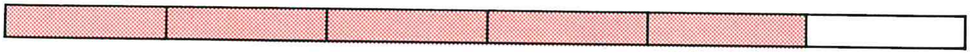
5. What fraction of each bar is shaded?

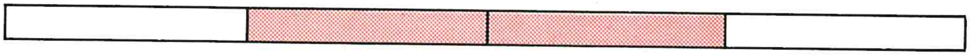
[A]


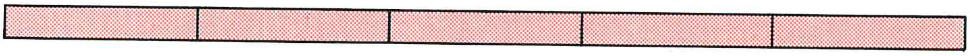
[B]


[C]


[D]


[E]


[F]


★ [G]


Think

There is a trick to this one. See if you can figure it out.

Sara said, "I have two coins in my hand. Together they total 30¢. One of them is not a nickel. What coins do I have?"

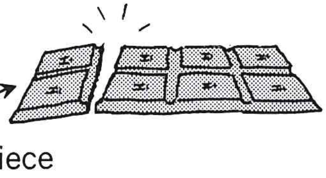
DISCUSSION EXERCISES

1. Explain how you found your answers to exercise 5.

2. Explain what you think $\frac{3}{3}$ means.

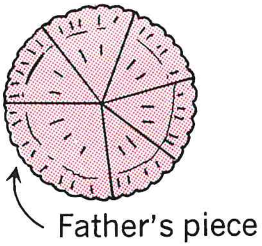
Parts of an object

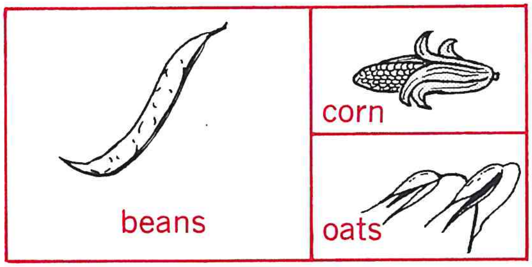
DISCUSSION EXERCISES



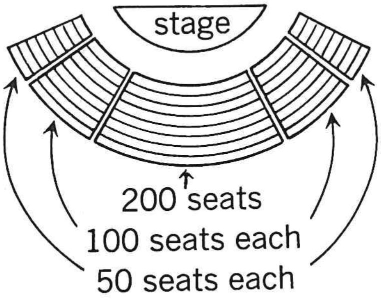
1. Bobby gave his little brother Alan part of his candy bar. Did Alan get half of the candy?

2. Mother cut the dinner pie into 6 pieces like this. Was Father's piece $\frac{1}{6}$ of the pie?





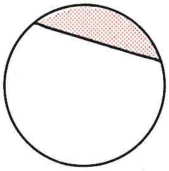
3. Mr. White planted part of his field with beans, part of it with corn, and the rest with oats. Would you say that $\frac{1}{3}$ of the field was planted with oats?



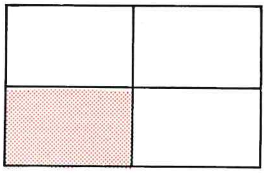
4. There are 5 sections of seats in the theater. Would you say that the center section has $\frac{1}{5}$ of the seats?

EXERCISES

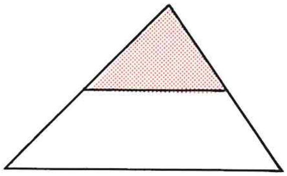
Answer **true** or **false** for each exercise.

1. 

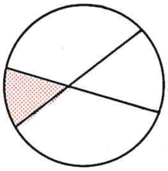
$\frac{1}{2}$ of the region is red.

2. 

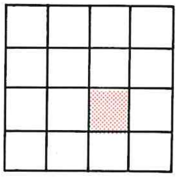
$\frac{1}{4}$ of the region is red.

3. 

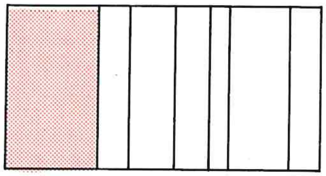
$\frac{1}{2}$ of the region is red.

4. 

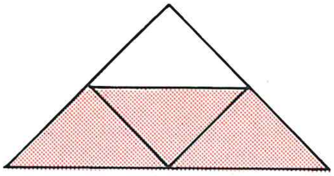
$\frac{1}{4}$ of the region is red.

5. 

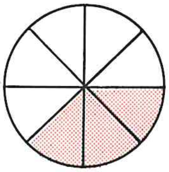
$\frac{1}{16}$ of the region is red.

6. 

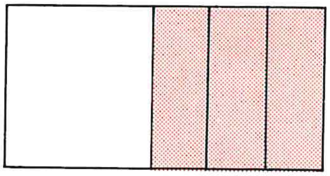
$\frac{1}{7}$ of the region is red.

7. 

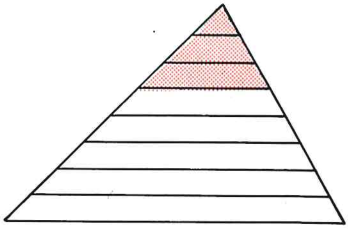
$\frac{3}{4}$ of the region is red.

8. 

$\frac{3}{8}$ of the region is red.

9. 

$\frac{3}{4}$ of the region is red.

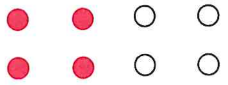
10. 

$\frac{3}{8}$ of the region is red.

Fractions and sets

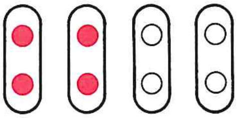
We often use fractions to compare part of a set with the whole set. By thinking of the same set in different ways, we may find that more than one fraction can be used for the comparison.

Study these examples.



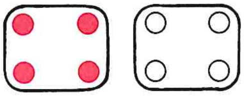
4 out of 8 dots are red.

$\frac{4}{8}$ of the dots are red.



2 of the 4 sets have red dots.

$\frac{2}{4}$ of the dots are red.



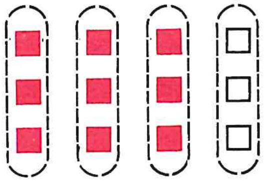
1 of the 2 sets has red dots.

$\frac{1}{2}$ of the dots are red.

DISCUSSION EXERCISES

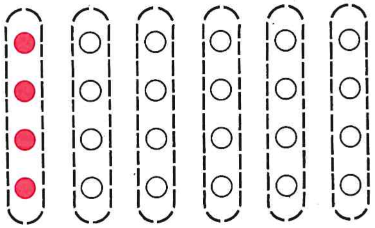
1. [A] Explain how you might think if you said, "$\frac{9}{12}$ of the squares are red."

   [B] Explain how you might think if you said, "$\frac{3}{4}$ of the squares are red."

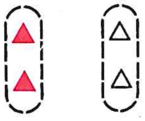


2. [A] Explain how you might think if you said, "$\frac{4}{24}$ of the dots are red."

   [B] Explain how you might think if you said, "$\frac{1}{6}$ of the dots are red."



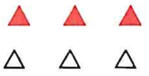
3. Explain two different ways you might think about what part of the set of triangles is red.



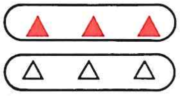
EXERCISES

1. First give the missing number. Then give the fraction for the number pair.

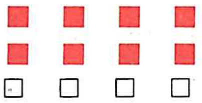
[A] 3 of the ||||| triangles are red.
||||| of the triangles are red.



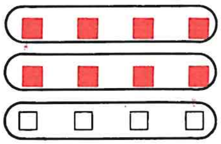
[B] 1 of the ||||| sets has red triangles.
||||| of the triangles are red.



[C] 8 of the ||||| squares are red.
||||| of the squares are red.

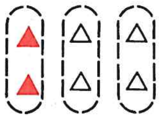


[D] ||||| of the 3 sets have red squares.
||||| of the squares are red.

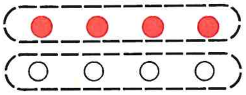


2. For each set, give at least two different fractions to tell what part of the set is red.

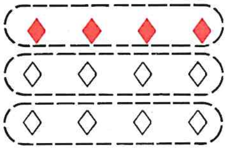
[A]



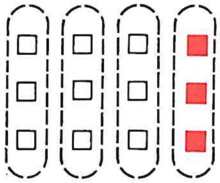
[B]



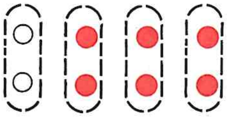
[C]



[D]

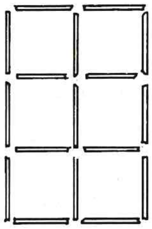


[E]



Think

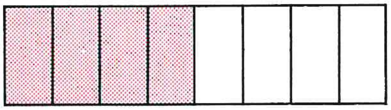
Arrange 17 pencils or sticks to make 6 squares. ⟶
Now remove just 5 of the pencils so you have 3 squares.



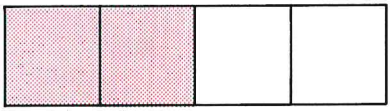
Fractions and parts of an object

We often use fractions to compare part of an object with the whole object. By dividing the same object in different ways, we can find more than one fraction for comparison.

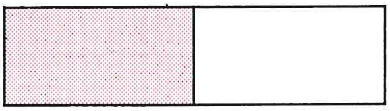
Study these examples.



4 parts out of 8 are red.
$\frac{4}{8}$ of the region is red.

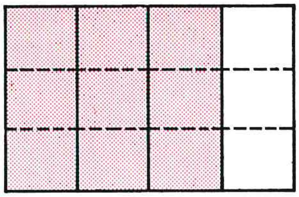


2 parts out of 4 are red.
$\frac{2}{4}$ of the region is red.



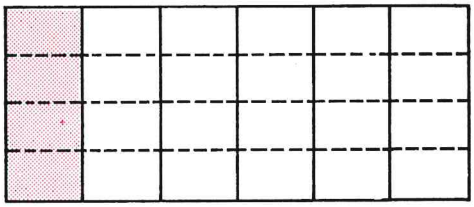
1 part out of 2 is red.
$\frac{1}{2}$ of the region is red.

DISCUSSION EXERCISES



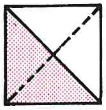
1. [A] Explain how you might think if you said, "$\frac{9}{12}$ of the region is red."

   [B] Explain how you might think if you said, "$\frac{3}{4}$ of the region is red."



2. [A] Explain how you might think if you said, "$\frac{4}{24}$ of the region is red."

   [B] Explain how you might think if you said, "$\frac{1}{6}$ of the region is red."

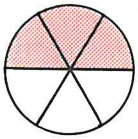


3. Explain two different ways you might think about the part of the small square region that is shaded.

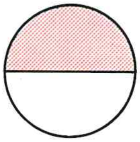
EXERCISES

1. First give the missing number. Then give the fraction for the number pair.

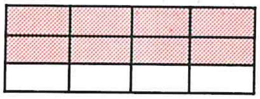
[A] 3 of the ▥ parts are red.
▥ of the region is red.



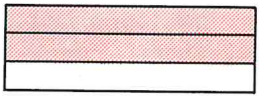
[B] 1 of the ▥ parts is red.
▥ of the region is red.



[C] 8 of the ▥ parts are red.
▥ of the region is red.

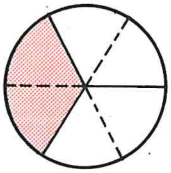


[D] ▥ of the 3 parts are red.
▥ of the region is red.

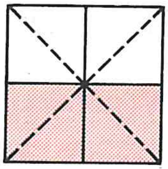


2. Give at least two fractions to tell what part of each region is red.

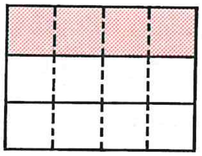
[A]



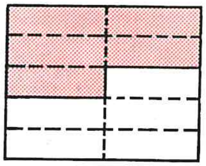
[B]



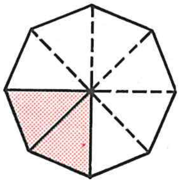
[C]



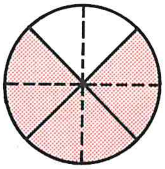
[D]



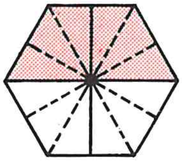
[E]



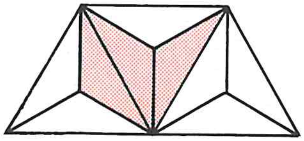
[F]



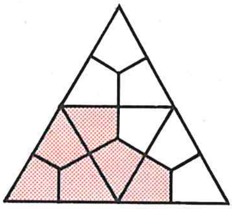
[G]



★[H]

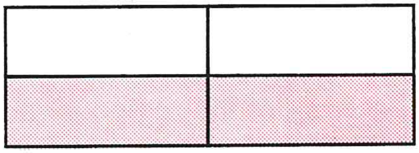


★[I]



Equivalent fractions

DISCUSSION EXERCISES

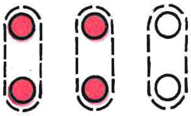


1. [A] Explain what you are thinking if you say, "$\frac{1}{2}$ of the region is shaded."

[B] Explain what you are thinking if you say, "$\frac{2}{4}$ of the region is shaded."

▶ Such pairs of fractions are called equivalent fractions.

$\frac{1}{2}$ is equivalent to $\frac{2}{4}$



2. [A] Explain what you are thinking if you say "$\frac{2}{3}$ of the dots are red."

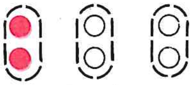
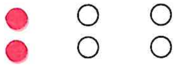
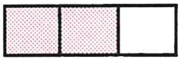
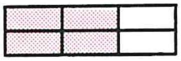
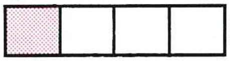
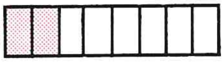
[B] Explain what you are thinking if you say "$\frac{4}{6}$ of the dots are red."

▶ Such pairs of fractions are called equivalent fractions.

$\frac{2}{3}$ is equivalent to $\frac{4}{6}$

EXERCISES

1. In each row of the table, copy the sentence and give the missing numerator.

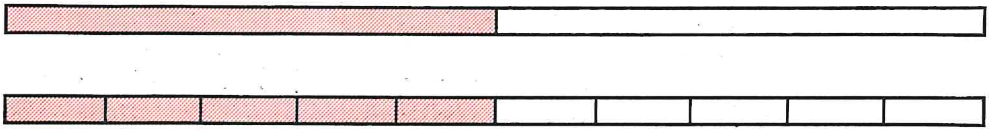
[A]		$\frac{\square}{3}$	is equivalent to	$\frac{\square}{6}$	
[B]		$\frac{\square}{3}$	is equivalent to	$\frac{\square}{6}$	
[C]		$\frac{\square}{4}$	is equivalent to	$\frac{\square}{8}$	

2. $\frac{2}{4}$ and $\frac{3}{6}$ are equivalent to $\frac{1}{2}$. Give another fraction that is equivalent to $\frac{1}{2}$.

3. Give three fractions that are equivalent to $\frac{1}{3}$.

4. Give the fraction suggested by each bar. Then copy the sentence and give the missing fractions.

[A]

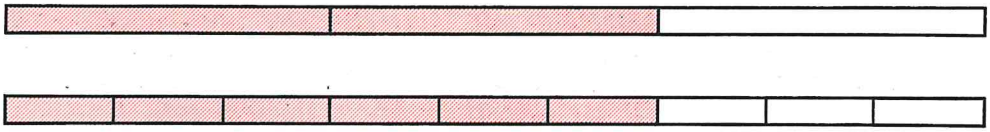


$\frac{\square}{2}$

$\frac{\square}{10}$

$\frac{\square}{2}$ is equivalent to $\frac{\square}{10}$

[B]

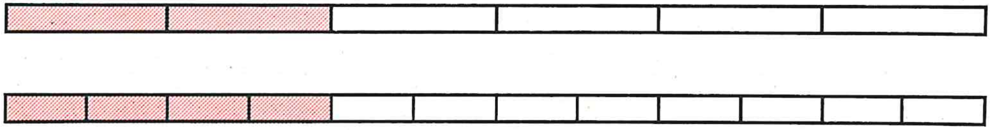


$\frac{\square}{3}$

$\frac{\square}{9}$

$\frac{\square}{3}$ is equivalent to $\frac{\square}{9}$

[C]

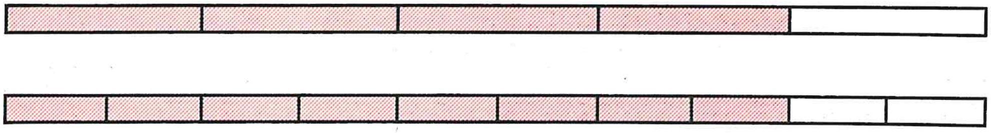


$\frac{\square}{6}$

$\frac{\square}{12}$

$\frac{\square}{6}$ is equivalent to $\frac{\square}{12}$

[D]



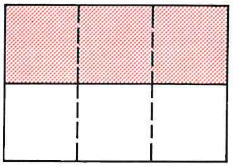
$\frac{\square}{5}$

$\frac{\square}{10}$

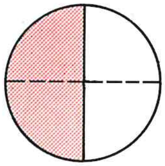
$\frac{\square}{5}$ is equivalent to $\frac{\square}{10}$

5. Write the pair of equivalent fractions suggested by each picture.

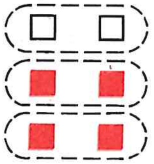
[A]



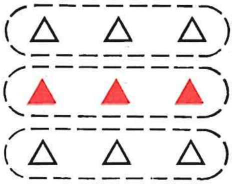
[B]



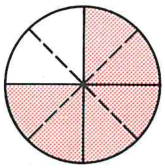
[C]



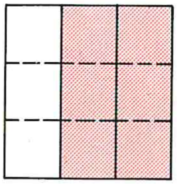
[D]



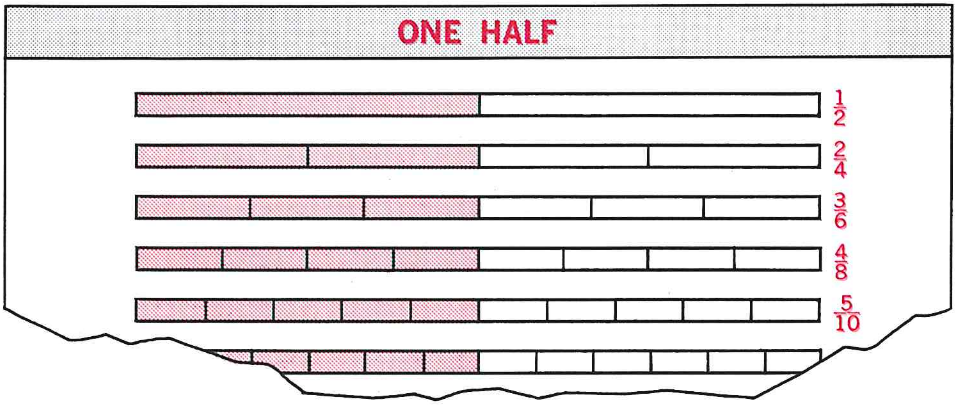
[E]



[F]

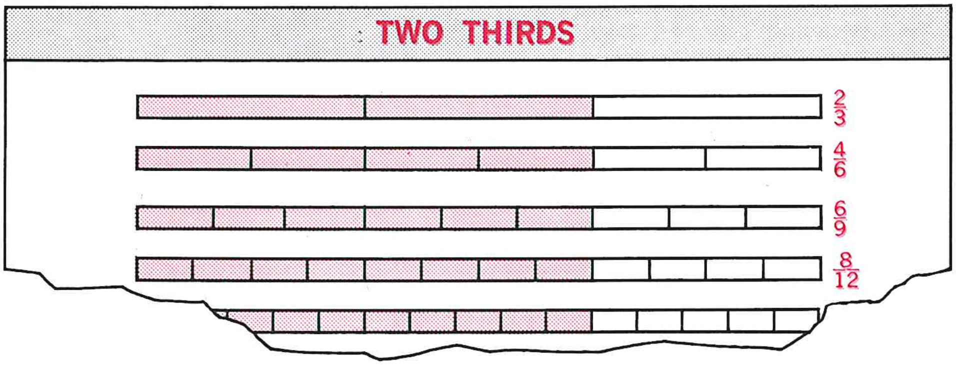


Sets of equivalent fractions



This chart shows some fractions that are equivalent to $\frac{1}{2}$:

$$\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \ldots\right\}.$$



This chart shows some fractions that are equivalent to $\frac{2}{3}$:

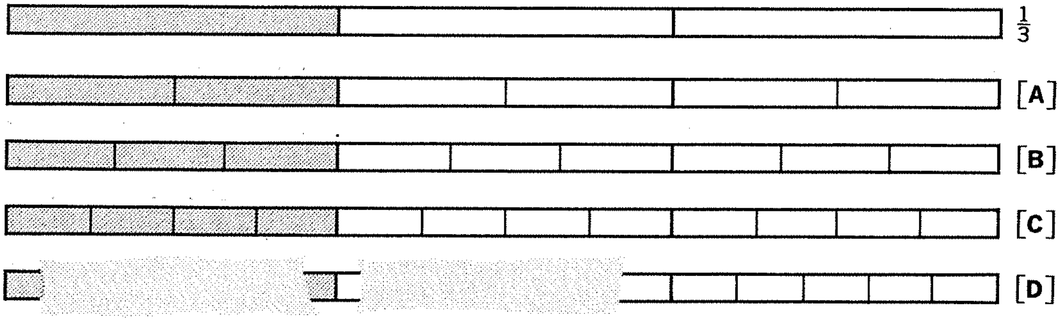
$$\left\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots\right\}.$$

DISCUSSION EXERCISES

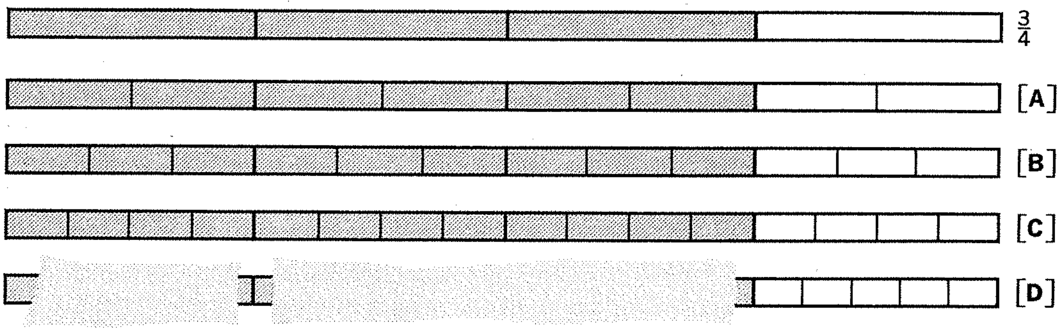
1. What fraction comes after $\frac{5}{10}$ in the ONE-HALF chart?
2. Give the next three fractions in the set
$$\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \ldots\right\}.$$
3. What fraction comes after $\frac{8}{12}$ in the TWO-THIRDS chart?
4. Give the next three fractions in the set
$$\left\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots\right\}.$$

EXERCISES

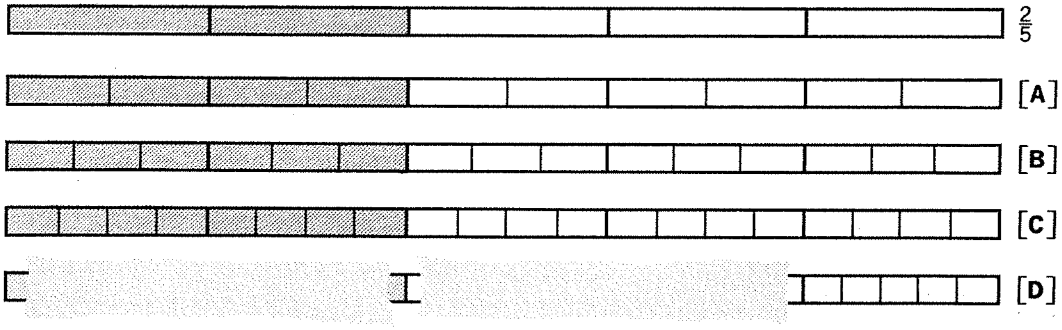
1. Study the chart and give the missing fractions.



2. Study the chart and give the missing fractions.



3. Study the chart and give the missing fractions.



★**4.** Give the next three fractions for each set of equivalent fractions.

[A] $\left\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \ldots\right\}$

[B] $\left\{\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \ldots\right\}$

[C] $\left\{\frac{1}{6}, \frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \ldots\right\}$

[D] $\left\{\frac{1}{8}, \frac{2}{16}, \frac{3}{24}, \ldots\right\}$

[E] $\left\{\frac{7}{10}, \frac{14}{20}, \frac{21}{30}, \ldots\right\}$

[F] $\left\{\frac{1}{12}, \frac{2}{24}, \frac{3}{36}, \ldots\right\}$

Building sets of equivalent fractions

In the last lesson, you studied sets of equivalent fractions like these.

$$\frac{1}{2}, \quad \frac{2}{4}, \quad \frac{3}{6}, \quad \frac{4}{8}, \quad \frac{5}{10} \qquad\qquad \frac{2}{3}, \quad \frac{4}{6}, \quad \frac{6}{9}, \quad \frac{8}{12}, \quad \frac{10}{15}$$

Examples A and B will help you understand how to build these sets of equivalent fractions.

A

$$\begin{tikzcd}
\frac{1}{2} \arrow[d] & \frac{2}{4} \arrow[d] & \frac{3}{6} \arrow[d] & \frac{4}{8} \arrow[d] & \frac{5}{10} \arrow[d] & \frac{6}{12} & \frac{7}{14} & \\
\frac{1\times1}{1\times2} & \frac{2\times1}{2\times2} & \frac{3\times1}{3\times2} & \frac{4\times1}{4\times2} & \frac{5\times1}{5\times2} \arrow[r] & \boxed{\frac{6\times1}{6\times2}} \arrow[u] \arrow[r] & \boxed{\frac{7\times1}{7\times2}} \arrow[u] \arrow[r] & {}
\end{tikzcd}$$

B

$$\begin{tikzcd}
\frac{2}{3} \arrow[d] & \frac{4}{6} \arrow[d] & \frac{6}{9} \arrow[d] & \frac{8}{12} \arrow[d] & \frac{10}{15} \arrow[d] & \frac{12}{18} & \frac{14}{21} & \\
\frac{1\times2}{1\times3} & \frac{2\times2}{2\times3} & \frac{3\times2}{3\times3} & \frac{4\times2}{4\times3} & \frac{5\times2}{5\times3} \arrow[r] & \boxed{\frac{6\times2}{6\times3}} \arrow[u] \arrow[r] & \boxed{\frac{7\times2}{7\times3}} \arrow[u] \arrow[r] & {}
\end{tikzcd}$$

DISCUSSION EXERCISES

1. Give the missing fractions.

$$\begin{tikzcd}
\frac{1\times3}{1\times4}, \arrow[d] & \frac{2\times3}{2\times4}, \arrow[d] & \frac{3\times3}{3\times4}, \arrow[d] & \frac{4\times3}{4\times4}, \arrow[d] & . \arrow[d] & . \arrow[d] & . \arrow[d] \\
\frac{3}{4} & \frac{6}{8} & [A] & [B] & [C] & [D] & [E]
\end{tikzcd}$$

2. Explain how you got your answers for **C**, **D**, and **E** in exercise 1.

EXERCISES

1. Find the missing fractions.

$\frac{1\times1}{1\times6}$,	$\frac{2\times1}{2\times6}$,	$\frac{3\times1}{3\times6}$,	$\frac{4\times1}{4\times6}$,	.	.
↓	↓	↓	↓	↓	↓
$\frac{1}{6}$	$\frac{2}{12}$	[A]	[B]	[C]	[D]
$\frac{1\times3}{1\times8}$,	$\frac{2\times3}{2\times8}$,	$\frac{3\times3}{3\times8}$,	$\frac{4\times3}{4\times8}$,	.	.
↓	↓	↓	↓	↓	↓
$\frac{3}{8}$	$\frac{6}{16}$	[E]	[F]	[G]	[H]
$\frac{1\times4}{1\times5}$,	$\frac{2\times4}{2\times5}$,	$\frac{3\times4}{3\times5}$,	$\frac{4\times4}{4\times5}$,	.	.
↓	↓	↓	↓	↓	↓
$\frac{4}{5}$	$\frac{8}{10}$	[I]	[J]	[K]	[L]
$\frac{1\times1}{1\times4}$,	$\frac{2\times1}{2\times4}$,	$\frac{3\times1}{3\times4}$,	.	.	.
↓	↓	↓	↓	↓	↓
$\frac{1}{4}$	[M]	[N]	[O]	[P]	[Q]

2. Find the next three fractions for each set of equivalent fractions.

[A] $\left\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\right\}$

[B] $\left\{\frac{1}{8}, \frac{2}{16}, \frac{3}{24}, \frac{4}{32}, \ldots\right\}$

[C] $\left\{\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \ldots\right\}$

[D] $\left\{\frac{2}{7}, \frac{4}{14}, \frac{6}{21}, \ldots\right\}$

[E] $\left\{\frac{5}{9}, \frac{10}{18}, \frac{15}{27}, \ldots\right\}$

[F] $\left\{\frac{1}{10}, \frac{2}{20}, \frac{3}{30}, \ldots\right\}$

Think

Find the missing numbers. $\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots, \frac{[A]}{50}, \ldots, \frac{[B]}{100}\right\}$

$\left\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots, \frac{[C]}{90}, \ldots, \frac{[D]}{300}\right\}$ $\left\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \ldots, \frac{[E]}{40}, \ldots, \frac{[F]}{100}, \ldots, \frac{[G]}{400}\right\}$

Numerator and denominator

$\frac{2}{3}$ ⟵ NUMERATOR ⟶ $\frac{6}{8}$

⟵ DENOMINATOR ⟶

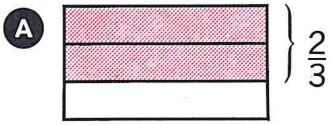
The numerator of $\frac{2}{3}$ is 2.

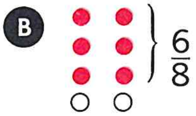
The denominator of $\frac{2}{3}$ is 3.

The numerator of $\frac{6}{8}$ is 6.

The denominator of $\frac{6}{8}$ is 8.

In these examples the **numerator** tells how many parts or dots are red. The **denominator** tells how many parts or dots in all.





We use denominators to describe certain groups of fractions. For example:

$\frac{1}{3}$ and $\frac{2}{3}$ are called thirds.

$\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, $\frac{5}{8}$, $\frac{6}{8}$, $\frac{7}{8}$ are called eighths.

EXERCISES

1. Give the word or numeral for each blank.

[A] In $\frac{4}{5}$, the numerator is _?_.

[B] In $\frac{7}{9}$, 9 is the _?_.

[C] $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$ are all _?_.

[D] There are _?_ sixths in $\frac{5}{6}$.

[E] In $\frac{3}{4}$, 3 is the _?_.

[F] In $\frac{6}{10}$, the denominator is _?_.

2. Write a fraction for each exercise.

[A] Denominator: 7
Numerator: 2

[B] Numerator: 4
Denominator: 10

[C] Denominator: 12
Numerator: 8

[D] The denominator is 10 and is 2 times the numerator.

[E] The numerator is 6, and the denominator is 3 more than 6.

[F] The denominator is 100, and the numerator is half that.

EXERCISES

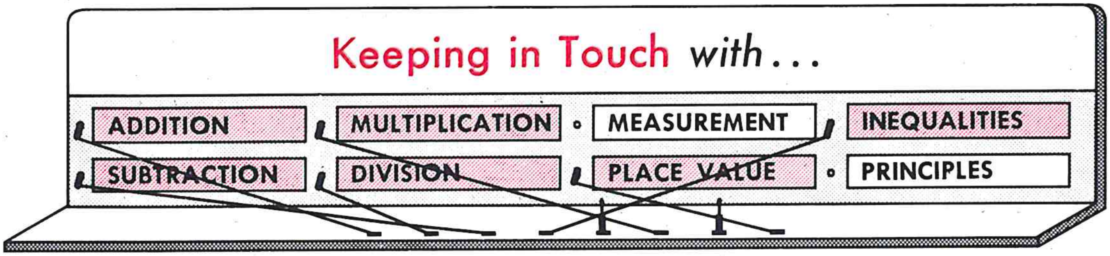
Copy each exercise on your paper. Write the missing numerators and denominators to form a set of equivalent fractions.

1.	$\frac{1}{2}$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{4}{8}$	$\frac{[A]}{10}$	$\frac{[B]}{12}$	$\frac{7}{[C]}$	$\frac{[D]}{16}$	$\frac{9}{[E]}$	$\frac{[F]}{100}$
2.	$\frac{1}{3}$	$\frac{2}{6}$	$\frac{3}{9}$	$\frac{4}{12}$	$\frac{[A]}{15}$	$\frac{6}{[B]}$	$\frac{[C]}{21}$	$\frac{[D]}{24}$	$\frac{9}{[E]}$	$\frac{100}{[F]}$
3.	$\frac{2}{3}$	$\frac{4}{6}$	$\frac{6}{9}$	$\frac{8}{12}$	$\frac{10}{[A]}$	$\frac{[B]}{18}$	$\frac{[C]}{21}$	$\frac{16}{[D]}$	$\frac{18}{[E]}$	$\frac{[F]}{300}$
4.	$\frac{1}{4}$	$\frac{2}{8}$	$\frac{3}{12}$	$\frac{4}{16}$	$\frac{[A]}{20}$	$\frac{6}{[B]}$	$\frac{[C]}{28}$	$\frac{8}{[D]}$	$\frac{[E]}{36}$	$\frac{20}{[F]}$
5.	$\frac{3}{4}$	$\frac{6}{8}$	$\frac{9}{12}$	$\frac{12}{16}$	$\frac{15}{[A]}$	$\frac{[B]}{24}$	$\frac{[C]}{28}$	$\frac{24}{[D]}$	$\frac{27}{[E]}$	$\frac{[F]}{200}$
6.	$\frac{3}{5}$	$\frac{6}{10}$	$\frac{9}{15}$	$\frac{12}{20}$	$\frac{15}{[A]}$	$\frac{18}{[B]}$	$\frac{[C]}{35}$	$\frac{[D]}{40}$	$\frac{27}{[E]}$	$\frac{300}{[F]}$
7.	$\frac{1}{10}$	$\frac{2}{20}$	$\frac{3}{30}$	$\frac{4}{40}$	$\frac{5}{[A]}$	$\frac{6}{[B]}$	$\frac{[C]}{70}$	$\frac{[D]}{80}$	$\frac{[E]}{90}$	$\frac{100}{[F]}$
8.	$\frac{3}{10}$	$\frac{6}{20}$	$\frac{9}{30}$	$\frac{12}{40}$	$\frac{[A]}{50}$	$\frac{18}{[B]}$	$\frac{[C]}{70}$	$\frac{24}{[D]}$	$\frac{27}{[E]}$	$\frac{[F]}{1000}$

9. Choose the best answer. If the denominator of a fraction is 3 times its numerator, the fraction is equivalent to

[A] $\frac{3}{6}$ [B] $\frac{1}{3}$ [C] $\frac{2}{3}$ [D] $\frac{6}{9}$

★**10.** The sum of the numerator and denominator of a fraction is 10. The denominator is 4 times the numerator. Give the fraction.



1. Give the correct sign (= or ≠) for each ●. The sign ≠ means "is **not** equal to."

[A] 63,427 + 1000 ● 64,427
[B] 100 × 1000 ● 1,000,000
[C] 872,000 ÷ 10 ● 8720
[D] 1,000,000 ● 999,999 + 1
[E] 10,000 − 1 ● 99,000
[F] 6285 × 100 ● 628,500
[G] 70 ÷ 1000 ● 70,000
[H] 832,070 − 10,000 ● 831,070
[I] 10 × 10 × 10 ● 10,000
[J] 10,000 ● 100 × 100
[K] 10 × 10 × 10 × 10 ● 100 × 10
[L] 6322 − 302 ● 6020

2. Find the sums.

[A] $\begin{array}{r}2\\7\\8\\+\;6\\\hline\end{array}$ [B] $\begin{array}{r}32\\58\\46\\+\;71\\\hline\end{array}$ [C] $\begin{array}{r}428\\106\\750\\+\;623\\\hline\end{array}$ [D] $\begin{array}{r}7465\\8321\\2405\\+\;3106\\\hline\end{array}$ [E] $\begin{array}{r}82{,}471\\93{,}265\\47{,}721\\+\;65{,}132\\\hline\end{array}$

3. Find the differences.

[A] $\begin{array}{r}32\\-8\\\hline\end{array}$ [B] $\begin{array}{r}75\\-37\\\hline\end{array}$ [C] $\begin{array}{r}128\\-49\\\hline\end{array}$ [D] $\begin{array}{r}604\\-29\\\hline\end{array}$ [E] $\begin{array}{r}7028\\-4639\\\hline\end{array}$

4. Find the sums and differences.

[A] 6243 + 721 + 38,642 + 29
[B] 6302 − 587

5. Find the products.

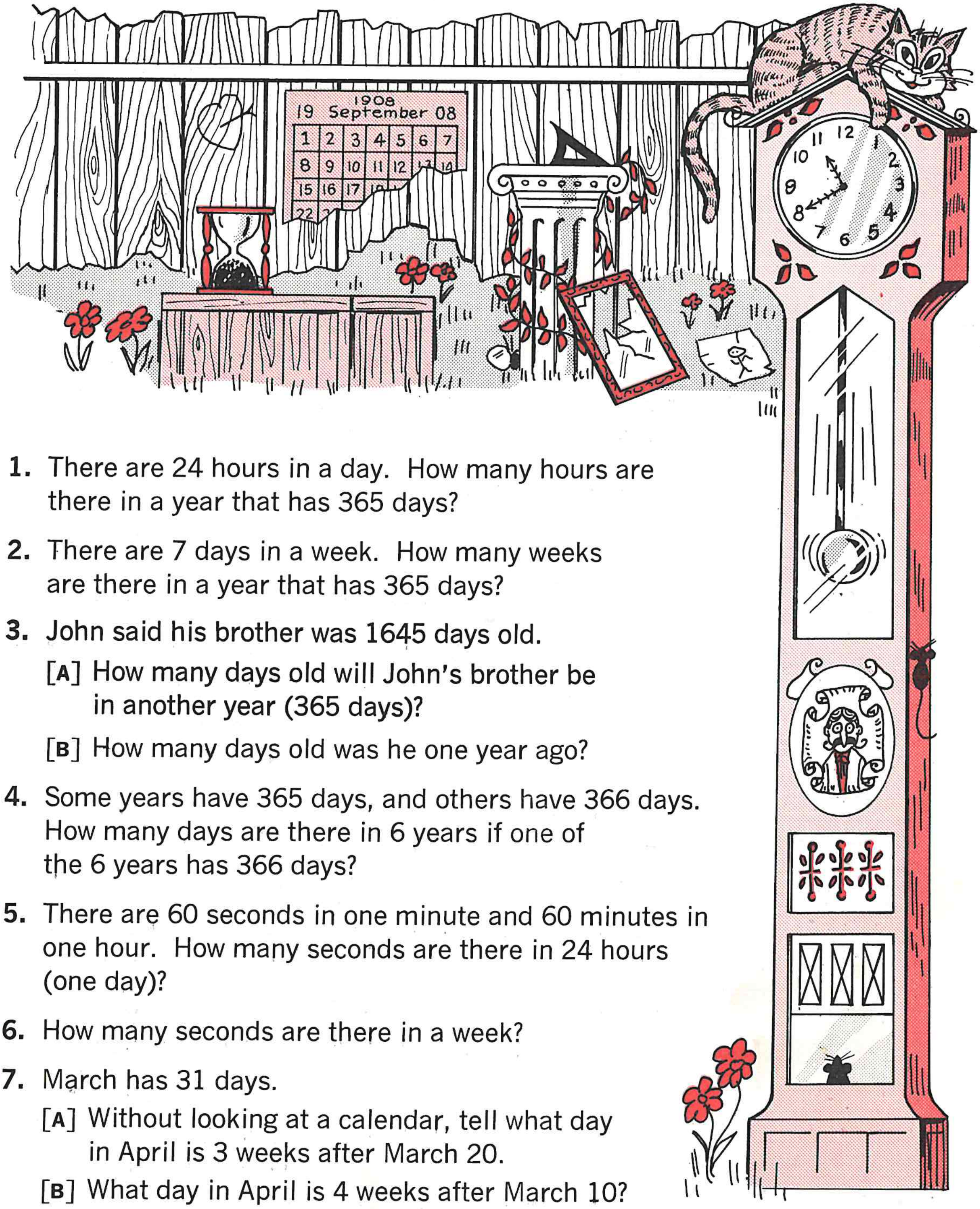
[A] $\begin{array}{r}27\\\times 6\\\hline\end{array}$ [B] $\begin{array}{r}32\\\times 12\\\hline\end{array}$ [C] $\begin{array}{r}58\\\times 26\\\hline\end{array}$

6. Find the quotients and remainders.

[A] 283 ÷ 6
[B] 4286 ÷ 7
[C] 316 ÷ 13
[D] 228 ÷ 24

Think

The can contains only red beads and black beads. The beads are mixed up and, without looking, Jan drew out 100 beads, 90 red and 10 black. Guess at the number of black beads in the can at first.



1. There are 24 hours in a day. How many hours are there in a year that has 365 days?

2. There are 7 days in a week. How many weeks are there in a year that has 365 days?

3. John said his brother was 1645 days old.

   [A] How many days old will John's brother be in another year (365 days)?

   [B] How many days old was he one year ago?

4. Some years have 365 days, and others have 366 days. How many days are there in 6 years if one of the 6 years has 366 days?

★ 5. There are 60 seconds in one minute and 60 minutes in one hour. How many seconds are there in 24 hours (one day)?

★ 6. How many seconds are there in a week?

★ 7. March has 31 days.

   [A] Without looking at a calendar, tell what day in April is 3 weeks after March 20.

   [B] What day in April is 4 weeks after March 10?

Fractions (numerator > or = denominator)

You have been learning about fractions that have numerators that are less than the denominators.

$\frac{3}{4}$ 3 is less than 4.	$\frac{7}{10}$ 7 is less than 10.

You will also learn about fractions that have:

Ⓐ A numerator the same as the denominator. ⟶ $\frac{5}{5}$

Ⓑ A numerator greater than the denominator. ⟶ $\frac{9}{6}$

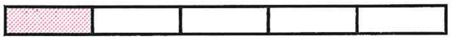
Fractions like those shown in A and B are sometimes called improper fractions.

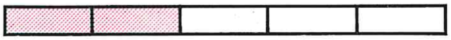
EXERCISES

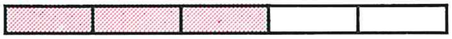
1. Which of these fractions are improper fractions?

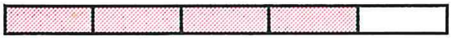
$\frac{3}{4}, \frac{4}{3}, \frac{6}{8}, \frac{7}{8}, \frac{8}{8}, \frac{9}{8}, \frac{5}{10}, \frac{10}{5}, \frac{1}{1}, \frac{7}{6}, \frac{12}{12}, \frac{11}{9}$

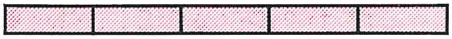
2. Give a fraction to tell what part of each bar is red.

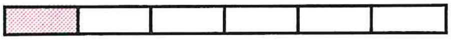
[A] 

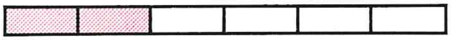
[B] 

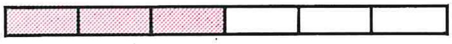
[C] 

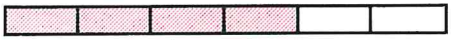
[D] 

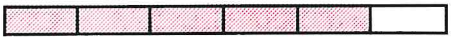
[E] 

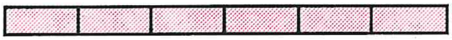
[F] 

[G] 

[H] 

[I] 

[J] 

[K] 

3. Write the next three fractions for each exercise.

[A] $\frac{4}{8}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}, \ldots$

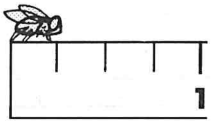
[B] $\frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \frac{9}{10}, \ldots$

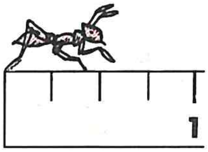
More about fractions

We often use fractions to compare an object to a unit. In the exercises below, we compare objects to the inch.

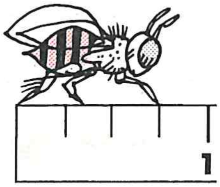
DISCUSSION EXERCISES

1. You can use the fraction $\frac{1}{4}$ to compare the fly with the inch. What fractions can you use to compare the ant with the inch?

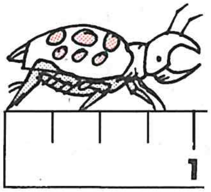




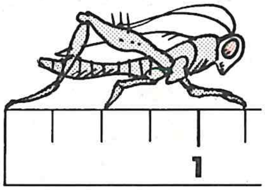
2. What fraction compares the bee with the inch?

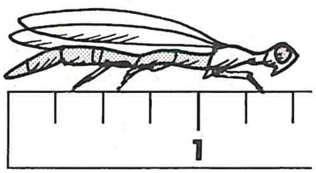


3. You can use the fraction $\frac{4}{4}$ to compare the beetle with the inch. What fraction would you use to compare the grasshopper with the inch?



4. What fraction compares the dragonfly with the inch?



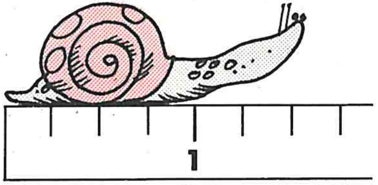


5. Which is larger, a worm that is 2 inches long, or one that is $\frac{7}{4}$ inches long?

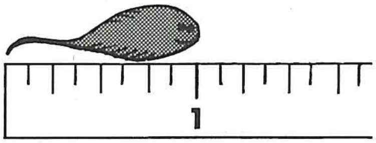
▶ A fraction with its numerator equal to or greater than its denominator is sometimes called an improper fraction.

EXERCISES

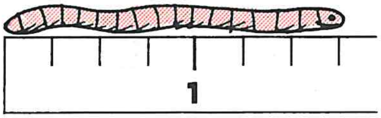
Give the improper fraction for each exercise.



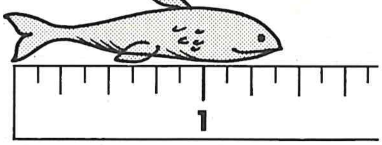
1. The fraction $\frac{\square}{4}$ compares the snail with the inch.



2. The fraction $\frac{\square}{8}$ compares the tadpole with the inch.



3. The fraction $\frac{\square}{4}$ compares the worm with the inch.



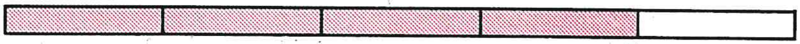
4. The fraction $\frac{\square}{8}$ compares the minnow with the inch.

Zero numerators

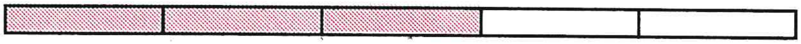
EXERCISES

1. Give a fraction to tell what part of each bar is red.

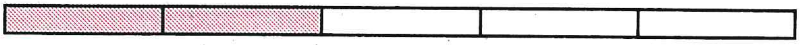
[A]



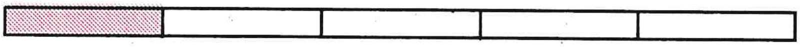
[B]



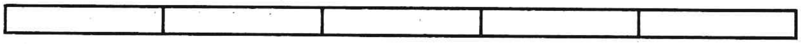
[C]



[D]



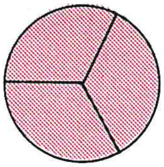
[E]



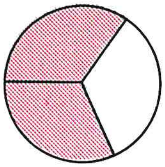
2. For each bar in exercise 1, give a fraction to tell what part is white.

3. Give a fraction to tell what part of each region is red.

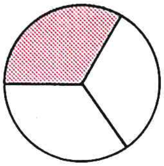
[A]



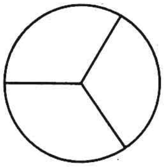
[B]



[C]



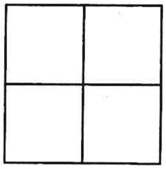
[D]



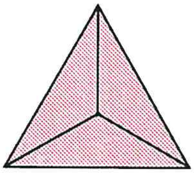
4. For each region in exercise 3, give a fraction to tell what part is white.

5. Give a fraction to tell what part of each region is red.

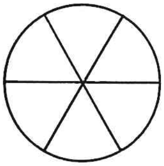
[A]



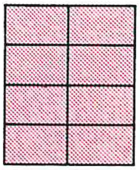
[B]



[C]



[D]



6. For each region in exercise 5, give a fraction to tell what part is white.

7. Give five more fractions for this set.

$$\left\{\frac{0}{1}, \frac{0}{2}, \frac{0}{3}, \frac{0}{4}, \frac{0}{5}, \frac{0}{6}, \frac{0}{7}, \cdot\ \cdot\ \cdot\right\}$$

Equivalent-fraction exercises

Copy the fractions on your paper.
Write three more fractions for each set.

1. [A] $\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\right\}$
   [B] $\left\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\right\}$
   [C] $\left\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \ldots\right\}$
   [D] $\left\{\frac{1}{5}, \frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \ldots\right\}$
   [E] $\left\{\frac{1}{6}, \frac{2}{12}, \frac{3}{18}, \ldots\right\}$
   [F] $\left\{\frac{1}{8}, \frac{2}{16}, \frac{3}{24}, \ldots\right\}$
   [G] $\left\{\frac{1}{10}, \frac{2}{20}, \ldots\right\}$
   [H] $\left\{\frac{1}{12}, \frac{2}{24}, \ldots\right\}$

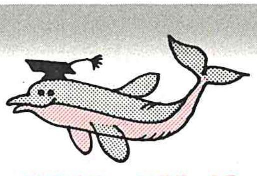
2. [A] $\left\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots\right\}$
   [B] $\left\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \ldots\right\}$
   [C] $\left\{\frac{2}{7}, \frac{4}{14}, \frac{6}{21}, \ldots\right\}$
   [D] $\left\{\frac{2}{9}, \frac{4}{18}, \frac{6}{27}, \ldots\right\}$
   [E] $\left\{\frac{0}{4}, \frac{0}{8}, \frac{0}{12}, \frac{0}{16}, \ldots\right\}$
   [F] $\left\{\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \ldots\right\}$
   [G] $\left\{\frac{3}{7}, \frac{6}{14}, \frac{9}{21}, \ldots\right\}$
   [H] $\left\{\frac{3}{10}, \frac{6}{20}, \frac{9}{30}, \ldots\right\}$

3. [A] $\left\{\frac{3}{2}, \frac{6}{4}, \frac{9}{6}, \ldots\right\}$
   [B] $\left\{\frac{4}{3}, \frac{8}{6}, \frac{12}{9}, \ldots\right\}$
   [C] $\left\{\frac{5}{2}, \frac{10}{4}, \frac{15}{6}, \ldots\right\}$
   [D] $\left\{\frac{5}{3}, \frac{10}{6}, \frac{15}{9}, \ldots\right\}$
   [E] $\left\{\frac{7}{4}, \frac{14}{8}, \frac{21}{12}, \ldots\right\}$
   [F] $\left\{\frac{9}{2}, \frac{18}{4}, \frac{27}{6}, \ldots\right\}$
   [G] $\left\{\frac{8}{3}, \frac{16}{6}, \frac{24}{9}, \ldots\right\}$
   [H] $\left\{\frac{6}{5}, \frac{12}{10}, \frac{18}{15}, \ldots\right\}$

4. [A] $\left\{\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \ldots\right\}$
   [B] $\left\{\frac{2}{1}, \frac{4}{2}, \frac{6}{3}, \ldots\right\}$
   [C] $\left\{\frac{3}{1}, \frac{6}{2}, \frac{9}{3}, \ldots\right\}$
   [D] $\left\{\frac{4}{1}, \frac{8}{2}, \frac{12}{3}, \ldots\right\}$
   [E] $\left\{\frac{10}{1}, \frac{20}{2}, \frac{30}{3}, \ldots\right\}$
   [F] $\left\{\frac{25}{1}, \frac{50}{2}, \frac{75}{3}, \ldots\right\}$
   [G] $\left\{\frac{50}{1}, \frac{100}{2}, \frac{150}{3}, \ldots\right\}$
   [H] $\left\{\frac{100}{1}, \frac{200}{2}, \frac{300}{3}, \ldots\right\}$

Think

Equivalent fractions you will see
When you view one half and me.
I'm thirteen above the line.
Find the name you think is mine.



WHO AM I?

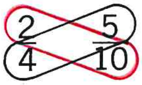
A check for equivalent fractions

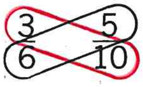
EXERCISES

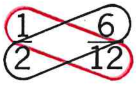
1. Each pair of fractions is from this set of equivalent fractions.

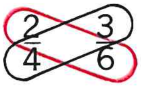
$$\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}\right\}$$

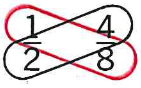
For each exercise, first find the product of the numbers in the red ring. Then find the product of the numbers in the black ring.

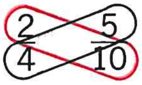
[A] 

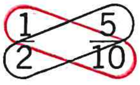
[B] 

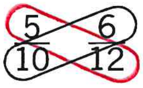
[C] 

[D] 

[E] 

[F] 

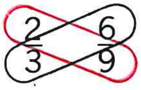
[G] 

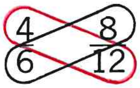
[H] 

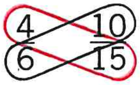
2. Each pair of fractions is from this set of equivalent fractions.

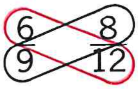
$$\left\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \frac{12}{18}\right\}$$

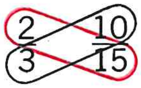
For each exercise, first find the product of the numbers in the red ring. Then find the product of the numbers in the black ring.

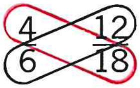
[A] 

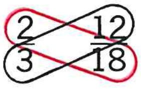
[B] 

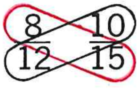
[C] 

[D] 

[E] 

[F] 

[G] 

[H] 

3. Here is another set of equivalent fractions.

$$\left\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}, \frac{6}{24}\right\}$$

Read the sentence. Then find the products.

[A] $\frac{2}{8}$ is equivalent to $\frac{3}{12}$.

$2 \times 12 = \boxed{n}$

$8 \times 3 = \boxed{n}$

[B] $\frac{3}{12}$ is equivalent to $\frac{5}{20}$.

$3 \times 20 = \boxed{n}$

$12 \times 5 = \boxed{n}$

On the opposite page, you explored a test for equivalent fractions. You can tell whether or not two fractions are equivalent by using this test. Study these examples.

$$\frac{3}{4} \; \frac{9}{12} \rightarrow \begin{array}{l} 4 \times 9 = 36 \\ 3 \times 12 = 36 \end{array} \rightarrow \frac{3}{4} \text{ is equivalent to } \frac{9}{12}.$$

$$\frac{2}{3} \; \frac{6}{8} \rightarrow \begin{array}{l} 3 \times 6 = 18 \\ 2 \times 8 = 16 \end{array} \rightarrow \frac{2}{3} \text{ is not equivalent to } \frac{6}{8}.$$

If the two products are the same, then the fractions are equivalent.

If the two products are not the same, then the fractions are not equivalent.

EXERCISES

Tell whether or not the two fractions are equivalent.

1. $\frac{1}{2}, \frac{7}{14}$
2. $\frac{3}{8}, \frac{9}{24}$
3. $\frac{6}{8}, \frac{7}{10}$
4. $\frac{6}{10}, \frac{9}{15}$
5. $\frac{12}{10}, \frac{6}{5}$
6. $\frac{3}{3}, \frac{8}{8}$
7. $\frac{6}{14}, \frac{2}{5}$
8. $\frac{0}{9}, \frac{0}{1}$
9. $\frac{9}{5}, \frac{18}{9}$
10. $\frac{15}{35}, \frac{3}{7}$

Think

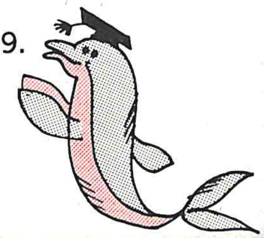
Find an improper fraction

$\frac{a}{b}$ so that

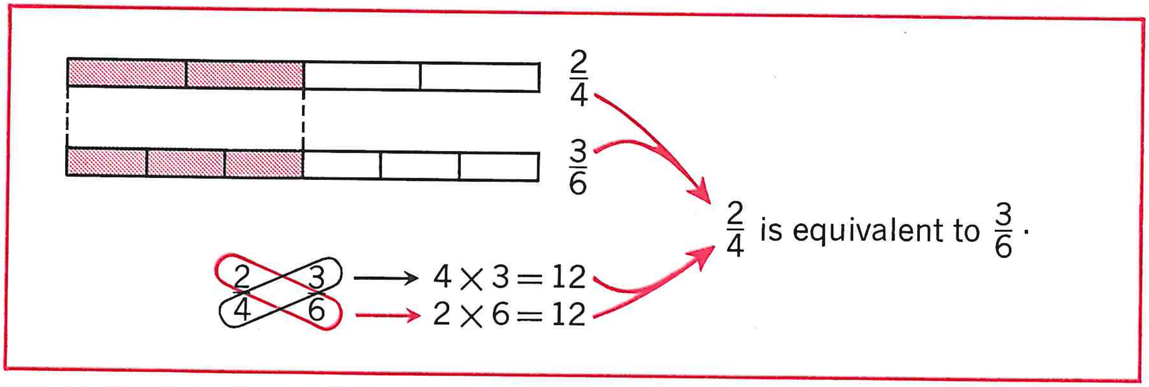
1. $a = 2 \times b$

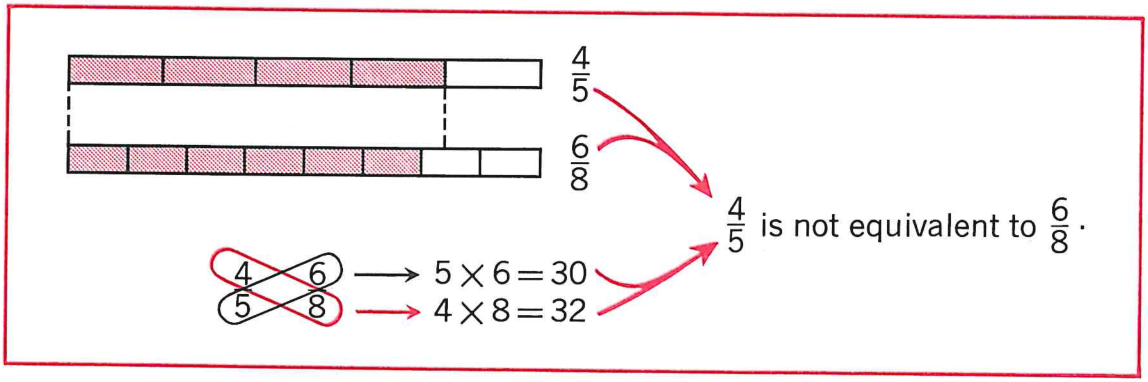
and

2. $a + b = 39$.



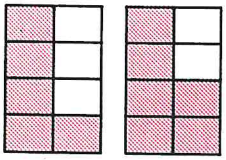
Equivalent fractions again

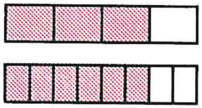

$\frac{2}{4}$
$\frac{3}{6}$
$\frac{2}{4}$ is equivalent to $\frac{3}{6}$.
$\frac{2}{4}$ $\frac{3}{6}$
$4 \times 3 = 12$
$2 \times 6 = 12$

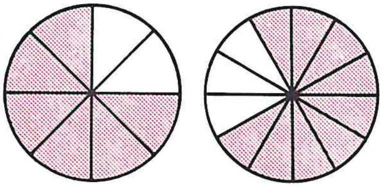

$\frac{4}{5}$
$\frac{6}{8}$
$\frac{4}{5}$ is not equivalent to $\frac{6}{8}$.
$\frac{4}{5}$ $\frac{6}{8}$
$5 \times 6 = 30$
$4 \times 8 = 32$

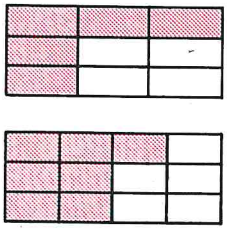
EXERCISES

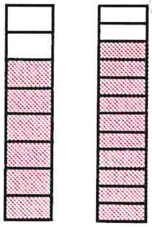
1. For each exercise, write the two fractions suggested by the shaded parts of the two regions given. Then tell whether or not the two fractions are equivalent.

[A]


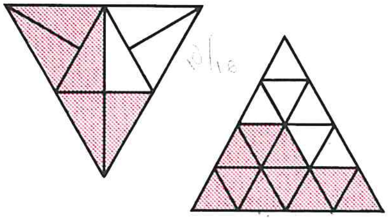
[B]


[C]


[D]


[E]


9/12

[F]


5/8

8/16

2. Use the product method to check your answers to exercise 1.

$\frac{15}{30}$ of the children are girls!

$\frac{2}{3}$ of the children in our club are boys!

EXERCISES

1. There are 30 children in Ann and Janet's class. 15 of the children are girls. Ann said, "$\frac{15}{30}$ of the children are girls." Janet said, "$\frac{3}{6}$ of the children are girls."

[**A**] Use the product method to show that the two fractions are equivalent.

[**B**] Are both girls right?

2. Mike and Tim are both in the school science club. Mike said, "$\frac{2}{3}$ of the children in our club are boys." Tim said, "$\frac{7}{10}$ of the children in our club are boys."

[**A**] Use the product method to show that these two fractions are not equivalent.

[**B**] Did one of the boys make a mistake?

3. There are 528 children in Mountain View School. $\frac{3}{4}$ of the children live in Mountain View. The other children live in different towns. Give five other fractions that tell what part of the children live in Mountain View.

$\frac{1}{2}$ of the boys in my Cub Pack are in my class!
Hmph!

★ **4.** Jim said, "$\frac{1}{2}$ of the boys in my Cub Pack are in my class at school. $\frac{3}{4}$ of the boys are in my school."

[**A**] What part of the Cub Pack is not in Jim's school?

[**B**] What part of the Cub Pack is not in Jim's class?

[**C**] What part of the Cub Pack is in Jim's school but not in Jim's class?

Lower and higher terms

Think of the set of all fractions equivalent to $\frac{1}{2}$.

$$\underbrace{\frac{1}{2}, \frac{2}{4}, \frac{3}{6},} \frac{4}{8}, \underbrace{\frac{5}{10}, \frac{6}{12}, \frac{7}{14}, \frac{8}{16}, \ldots}$$

These fractions are in lower terms than $\frac{4}{8}$.

These fractions are in higher terms than $\frac{4}{8}$.

$$\underbrace{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10},} \frac{6}{12}, \underbrace{\frac{7}{14}, \frac{8}{16}, \ldots}$$

These fractions are in lower terms than $\frac{6}{12}$.

These fractions are in higher terms than $\frac{6}{12}$.

Think of the set of all fractions equivalent to $\frac{3}{5}$.

$$\underbrace{\frac{3}{5}, \frac{6}{10},} \frac{9}{15}, \underbrace{\frac{12}{20}, \frac{15}{25}, \frac{18}{30}, \frac{21}{35}, \frac{24}{40}, \ldots}$$

These fractions are in lower terms than $\frac{9}{15}$.

These fractions are in higher terms than $\frac{9}{15}$.

$$\underbrace{\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \frac{12}{20},} \frac{15}{25}, \underbrace{\frac{18}{30}, \frac{21}{35}, \frac{24}{40}, \ldots}$$

These fractions are in lower terms than $\frac{15}{25}$.

These fractions are in higher terms than $\frac{15}{25}$.

▶ We use higher and lower terms only when the fractions are equivalent.

EXERCISES

1. [A] Give four fractions in lower terms than $\frac{5}{10}$.
[B] Give four fractions that are in higher terms than $\frac{5}{10}$.

2. [A] Give three fractions in lower terms than $\frac{12}{20}$.
[B] Give five fractions in higher terms than $\frac{12}{20}$.

More about lower and higher terms

On the opposite page, you learned about higher and lower terms for equivalent fractions. You should remember:

> We speak of higher and lower terms only when the fractions are equivalent.

For example:

Ⓐ $\frac{3}{4}$ is equivalent to $\frac{9}{12}$. We say that $\frac{3}{4}$ is in lower terms than $\frac{9}{12}$ and that $\frac{9}{12}$ is in higher terms than $\frac{3}{4}$.

Ⓑ $\frac{2}{3}$ is **not** equivalent to $\frac{6}{8}$. We do not say $\frac{2}{3}$ is in lower terms than $\frac{6}{8}$.

EXERCISES

1. $\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \frac{12}{18}, \frac{14}{21}, \ldots$ →

[A] What fractions are in lower terms than $\frac{6}{9}$?

[B] List two fractions that are in higher terms than $\frac{6}{9}$.

[C] List four fractions that are in lower terms than $\frac{12}{18}$.

[D] Give two fractions that are in higher terms than $\frac{12}{18}$.

[E] What fraction in the set is in lower terms than any other fraction in the set?

2. ▨, $\frac{2}{16}$, $\frac{3}{24}$, ▨, ▨, ▨, ▨, →

[A] Give two fractions in higher terms than $\frac{2}{16}$.

[B] Give two fractions in lower terms than $\frac{3}{24}$.

3. ▨, $\frac{10}{12}$, ▨, ▨, ▨, →

[A] Give a fraction in higher terms than $\frac{10}{12}$.

[B] Give a fraction in lower terms than $\frac{10}{12}$.

Lowest terms

In this lesson, you will learn about fractions that are in lowest terms.

Study the examples.

$$\left\{\underset{\uparrow}{\frac{3}{4}}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \frac{18}{24}, \frac{21}{28}, \ldots\right\}$$

- The fraction $\frac{3}{4}$ is in lower terms than any other fraction in the set.
- There is no fraction that is in lower terms than $\frac{3}{4}$.
- The fraction $\frac{3}{4}$ is in lowest terms.

$$\left\{\underset{\uparrow}{\frac{5}{8}}, \frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \frac{25}{40}, \frac{30}{48}, \frac{35}{56}, \ldots\right\}$$

- The fraction $\frac{5}{8}$ is in lower terms than any other fraction in the set.
- There is no fraction that is in lower terms than $\frac{5}{8}$.
- The fraction $\frac{5}{8}$ is in lowest terms.

EXERCISES

Each set is a set of equivalent fractions. Each set has one fraction in lowest terms. Find that fraction.

1. $\left\{\frac{3}{6}, \frac{2}{4}, \frac{5}{10}, \frac{1}{2}, \frac{10}{20}\right\}$

2. $\left\{\frac{8}{12}, \frac{20}{30}, \frac{14}{21}, \frac{6}{9}, \frac{2}{3}\right\}$

3. $\left\{\frac{7}{8}, \frac{70}{80}, \frac{14}{16}, \frac{35}{40}, \frac{21}{24}\right\}$

4. $\left\{\frac{8}{6}, \frac{16}{12}, \frac{40}{30}, \frac{4}{3}, \frac{32}{24}\right\}$

5. $\left\{\frac{30}{10}, \frac{15}{5}, \frac{3}{1}, \frac{6}{2}, \frac{9}{3}\right\}$

6. $\left\{\frac{15}{20}, \frac{3}{4}, \frac{9}{12}, \frac{21}{28}, \frac{12}{16}\right\}$

7. $\left\{\frac{10}{100}, \frac{2}{20}, \frac{1}{10}, \frac{5}{50}, \frac{7}{70}\right\}$

8. $\left\{\frac{80}{100}, \frac{8}{10}, \frac{40}{50}, \frac{16}{20}, \frac{4}{5}\right\}$

9. $\left\{\frac{21}{14}, \frac{3}{2}, \frac{30}{20}, \frac{12}{8}, \frac{9}{6}\right\}$

10. $\left\{\frac{1}{4}, \frac{4}{16}, \frac{2}{8}, \frac{10}{40}, \frac{5}{20}\right\}$

More about lowest terms

Here is a special rule for telling whether or not a fraction is in lowest terms.

Study the examples.

$\frac{3}{4}$	3 and 4 have no common factors except 1.	$\longrightarrow$ $\frac{3}{4}$	is in lowest terms.
$\frac{6}{10}$	6 and 10 have 2 as a common factor.	$\longrightarrow$ $\frac{6}{10}$	is not in lowest terms.
$\frac{9}{8}$	9 and 8 have no common factors except 1.	$\longrightarrow$ $\frac{9}{8}$	is in lowest terms.
$\frac{9}{12}$	9 and 12 have 3 as a common factor.	$\longrightarrow$ $\frac{9}{12}$	is not in lowest terms.

If the numerator and denominator have no common factor except 1, then the fraction is in lowest terms.

If the numerator and denominator have a common prime factor, then the fraction is not in lowest terms.

EXERCISES

1. Tell whether or not the fraction is in lowest terms. The prime factors of the numerator and denominator are given in red.

[A] $\frac{10}{14}\quad\begin{matrix}2, 5\\ 2, 7\end{matrix}$ [B] $\frac{6}{35}\quad\begin{matrix}2, 3\\ 5, 7\end{matrix}$ [C] $\frac{12}{15}\quad\begin{matrix}2, 3\\ 3, 5\end{matrix}$

[D] $\frac{7}{30}\quad\begin{matrix}7\\ 2, 3, 5\end{matrix}$ [E] $\frac{16}{15}\quad\begin{matrix}2\\ 3, 5\end{matrix}$ [F] $\frac{27}{24}\quad\begin{matrix}3\\ 2, 3\end{matrix}$

2. Tell whether or not the fraction is in lowest terms.

[A] $\frac{3}{6}$ [B] $\frac{8}{12}$ [C] $\frac{9}{15}$ [D] $\frac{8}{9}$ [E] $\frac{9}{14}$ [F] $\frac{15}{25}$

Lowest-terms fractions

You have learned how to build a set of equivalent fractions from a lowest-terms fraction. Here is an example.

$$\frac{3}{4} \longrightarrow \frac{2\times3}{2\times4} \longrightarrow \frac{3\times3}{3\times4} \longrightarrow \frac{4\times3}{4\times4} \longrightarrow \frac{5\times3}{5\times4} \longrightarrow \cdots$$

This process gives the set:

$$\left\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \cdots\right\}$$

Choose two fractions from this set and show that they are equivalent.

EXERCISES

1. Build a set of equivalent fractions from each lowest-terms fraction. Find at least six fractions for each set.

[A] $\frac{1}{2}$ [B] $\frac{2}{3}$ [C] $\frac{1}{5}$ [D] $\frac{3}{10}$ [E] $\frac{5}{6}$ [F] $\frac{7}{4}$

2. In each exercise, find the fraction for the ▨.

[A] $\left\{\frac{1}{8}, \frac{2}{16},\right.$ ▨ $\left., \frac{4}{32}, \cdots\right\}$

[B] $\left\{\frac{5}{4}, \frac{10}{8}, \frac{15}{12},\right.$ ▨ $\left., \cdots\right\}$

[C] $\left\{\frac{1}{1},\right.$ ▨ $\left., \frac{3}{3}, \frac{4}{4}, \cdots\right\}$

[D] $\left\{\frac{0}{1}, \frac{0}{2}, \frac{0}{3},\right.$ ▨ $\left., \cdots\right\}$

[E] $\left\{\frac{2}{1}, \frac{4}{2},\right.$ ▨ $\left., \frac{8}{4}, \cdots\right\}$

[F] $\left\{\frac{5}{8},\right.$ ▨ $\left., \frac{15}{24}, \cdots\right\}$

[G] $\{$ ▨ $, \frac{2}{18}, \frac{3}{27}, \cdots\}$

[H] $\{$ ▨ $, \frac{4}{14}, \frac{6}{21}, \cdots\}$

[I] $\{$ ▨ $, \frac{6}{4}, \frac{9}{6}, \cdots\}$

[J] $\{$ ▨ $, \frac{12}{10}, \frac{18}{15}, \cdots\}$

3. Give the lowest-terms fraction for each part of exercise 2.

Think

My head and feet are just alike.
My terms, as low as they can be.
As a fraction, I'm not proper.
I am one you seldom see.

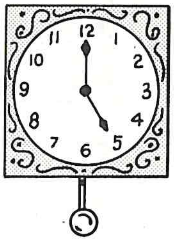
WHO AM I?

Fraction exercises

1. Write a fraction for each ▥.

(Example: A second is $\frac{1}{60}$ of a minute.)

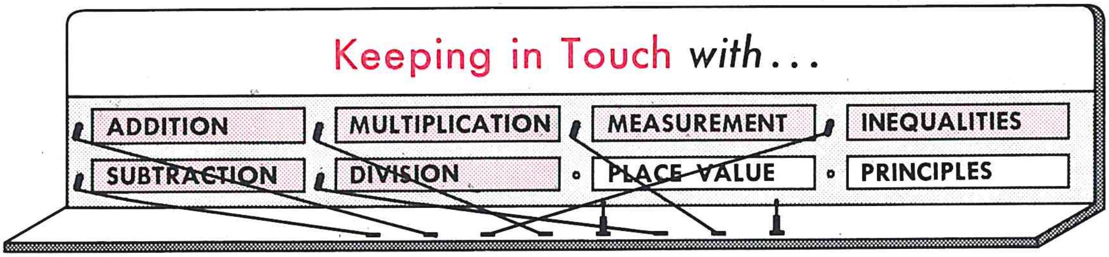
[A] A minute is ▥ of an hour.
[B] An hour is ▥ of a day.
[C] A day is ▥ of a week.
[D] A month is ▥ of a year.
[E] A year is a ▥ of a decade.
[F] A decade is ▥ of a century.
[G] An inch is ▥ of a foot.
[H] A foot is ▥ of a yard.
[I] A yard is ▥ of a mile.
[J] An ounce is ▥ of a pint.
[K] A pint is ▥ of a quart.
[L] A quart is ▥ of a gallon.
[M] An ounce is ▥ of a pound.
[N] A pound is ▥ of a ton.



★ **2.** Give the lowest-terms fraction for each ▥.

[A] 4 months is ▥ of a year.
[B] 40 minutes is ▥ of an hour.
[C] A 12-ounce steak weighs ▥ of a pound.
[D] 9 inches is ▥ of a foot.
[E] 9 inches is ▥ of a yard.
[F] 880 yards is ▥ of a mile.
[G] A quarter is ▥ of a dollar.
[H] A dime is ▥ of a dollar.
[I] A nickel is ▥ of a dollar.
[J] A half foot is ▥ of a yard.
[K] 12 seconds is ▥ of a minute.
[L] A half centimeter is about ▥ of an inch.

Keeping in Touch with . . .



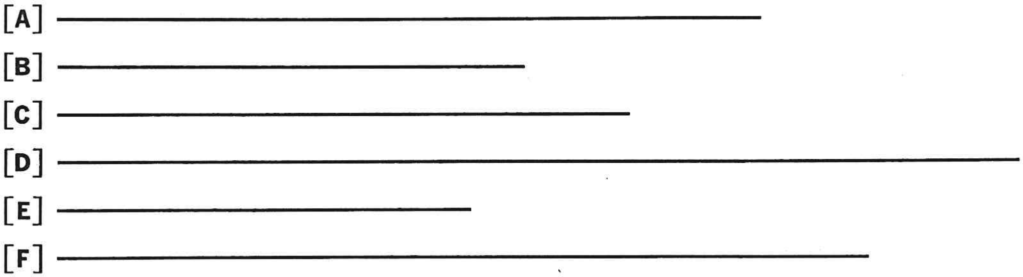
a/o **1.** Give the correct sign (+, −, ×, ÷) for each ●.

[A] 48 ● 48 > 100
[B] 127 ● 29 = 98
[C] 564 ● 1 > 564
[D] 329 ● 0 ≠ 329
[E] 0 ● 28 > 0

[F] 764 ● 764 < 1
[G] 168 ● 12 < 156
[H] 67 ● 35 = 102
[I] 369 ● 7 > 376
[J] 0 ● 1 ≠ 0

[K] 1 ● 1 < 1
[L] 4500 ● 12 < 4000
[M] 10 ● 100 = 1000
[N] 1000 ● 1 = 999
[O] 12 ● 12 > 140

a/o **2.** Find the length of each segment to the nearest half centimeter.



w/o **3.** Find the sums and differences.

[A] $\begin{array}{r} 347 \\ +685 \\ \hline \end{array}$ [B] $\begin{array}{r} 982 \\ -643 \\ \hline \end{array}$ [C] $\begin{array}{r} 803 \\ -265 \\ \hline \end{array}$ [D] $\begin{array}{r} 8654 \\ +7283 \\ \hline \end{array}$ [E] $\begin{array}{r} 9402 \\ -657 \\ \hline \end{array}$

[F] $\begin{array}{r} 842 \\ -764 \\ \hline \end{array}$ [G] $\begin{array}{r} 927 \\ +849 \\ \hline \end{array}$ [H] $\begin{array}{r} 600 \\ -475 \\ \hline \end{array}$ [I] $\begin{array}{r} 8020 \\ -6735 \\ \hline \end{array}$ [J] $\begin{array}{r} 9830 \\ +7692 \\ \hline \end{array}$

w/o **4.** Find the products and quotients.

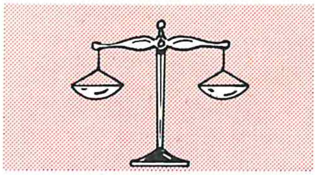
[A] 83 × 5
[B] 648 ÷ 4
[C] 76 × 13
[D] 108 ÷ 12
[E] 319 × 7
[F] 4285 × 3

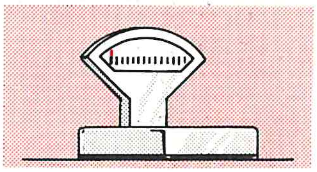
[G] 545 ÷ 5
[H] 207 ÷ 25
[I] 2816 × 27
[J] 264 ÷ 52
[K] 9024 × 35
[L] 391 ÷ 43

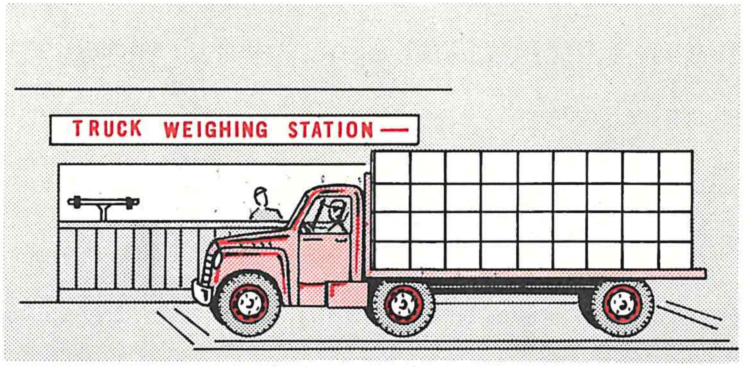
[M] 128 ÷ 34
[N] 3276 × 51
[O] 600 ÷ 70
[P] 4379 × 68
[Q] 700 ÷ 73
[R] 3269 × 48

[S] 527 ÷ 62
[T] 1346 × 37
[U] 435 × 6
[V] 252 ÷ 12
[W] 3412 × 12
[X] 4444 ÷ 4

WEIGHTS






TRUCK WEIGHING STATION —

1 pound (lb) is 16 ounces (oz). 1 ton is 2000 pounds.

1. [A] How many ounces are in 5 pounds?
[B] How many pounds are in 5 tons?
[C] How many ounces are in 78 pounds?
[D] How many pounds are in 17 tons?
[E] How many ounces are in 1 ton?

2. [A] How many ounces does a 7-pound beef roast weigh?
[B] How many ounces does a 13-pound ham weigh?
[C] If a turkey weighs 24 pounds and 7 ounces, how many ounces does it weigh?
[D] If a car weighs 1 ton and 1287 pounds, how many pounds does it weigh?

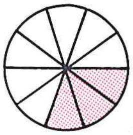
3. [A] 16 ounces is how many pounds?
[B] 32 ounces is how many pounds?
[C] 48 ounces is how many pounds?
[D] 320 ounces is how many pounds?
[E] 416 ounces is how many pounds?
[F] 720 ounces is how many pounds?

★**4.** Tell what should go in each ▥.
[A] 412 ounces is 25 pounds and ▥ ounces.
[B] 829 ounces is ▥ pounds and 13 ounces.
[C] 604 ounces is ▥ pounds and ▥ ounces.

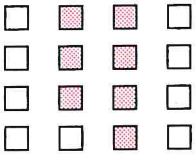
Chapter review

1. Give a fraction to tell what part of each picture is red.

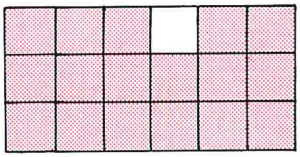
[A]



[B]

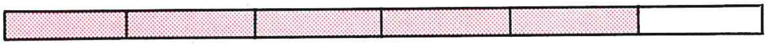


[C]

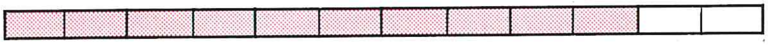


2. Give a fraction to tell what part of each bar is red.

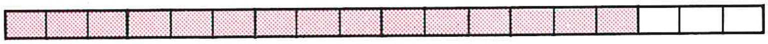
[A]



[B]



[C]



3. Use the product method to show that your fractions in exercise 2 are equivalent.

4. Give three more fractions for each set.

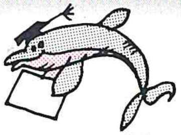
[A] $\left\{\frac{3}{7}, \frac{6}{14}, \frac{9}{21}, \ldots\right\}$

[B] $\left\{\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \ldots\right\}$

5. Tell whether or not the two fractions are equivalent.

[A] $\frac{2}{4}$ $\frac{8}{16}$ [B] $\frac{8}{3}$ $\frac{25}{9}$ [C] $\frac{10}{8}$ $\frac{15}{12}$ [D] $\frac{7}{8}$ $\frac{21}{16}$

Think

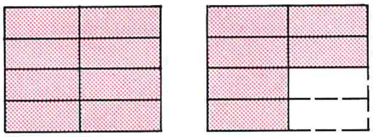


I like to think that I am big,
But no one will agree.
One in a million is your clue.
Use lowest terms for me.

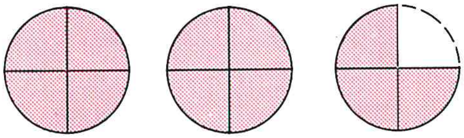
WHO AM I?

6. Write an improper fraction for each exercise.

[A]



[B]

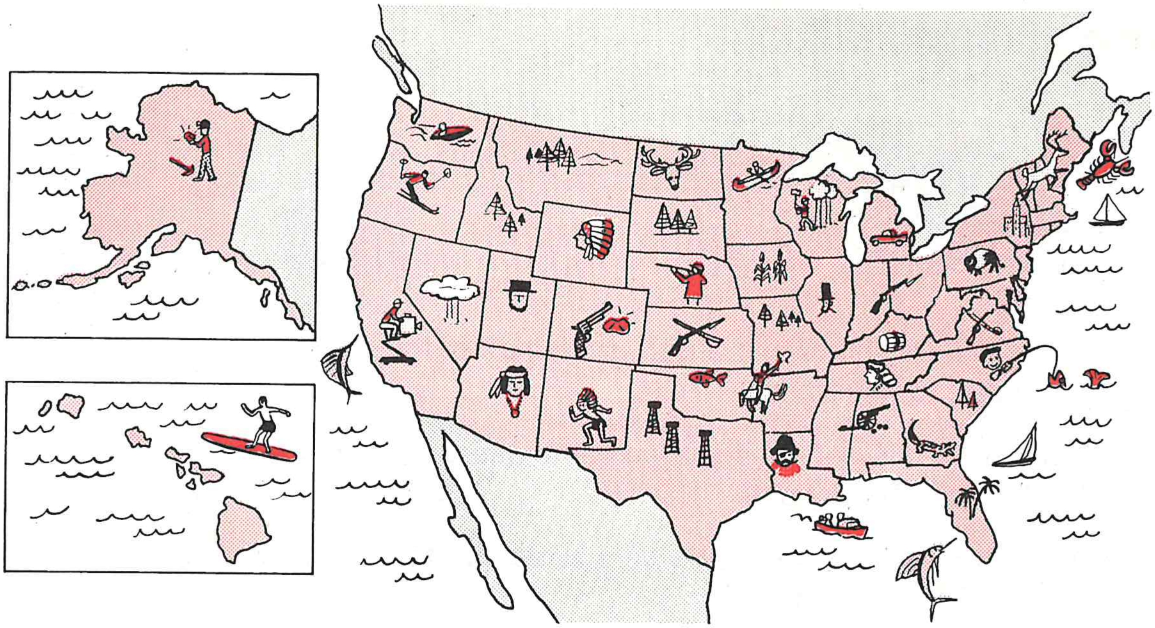


7. Write a set of six equivalent fractions for each lowest-terms fraction.

[A] $\frac{1}{2}$ [B] $\frac{1}{4}$ [C] $\frac{2}{3}$ [D] $\frac{3}{4}$ [E] $\frac{7}{8}$ [F] $\frac{3}{10}$

8. Why is $\frac{9}{21}$ not in lowest terms?

THE 50 STATES



1. The United States has 50 states. Two of them do not border on any other states. Give two fractions that tell what part of the states do not border on other states.

2. Michigan touches four of the five Great Lakes. What fraction of the lakes touch Michigan? What fraction of the lakes do not touch Michigan?

3. Eight of the 50 states touch at least one of the Great Lakes. Give two fractions to tell what part of the states this is.

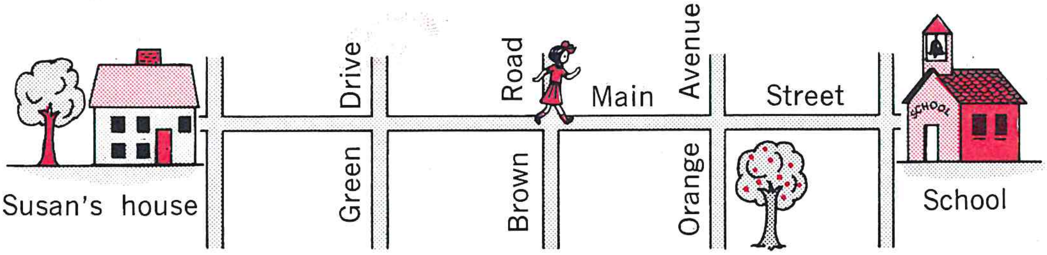
4. Ten of the states border Canada. Give three fractions to tell what part of the states border Canada.

5. Five of the states border the Gulf of Mexico. Give two fractions to tell what part of the states border the Gulf of Mexico.

★ 6. Four states border on Mexico, and five states border on the Pacific Ocean. California borders on both Mexico and the Pacific Ocean. What part of the 50 states border on Mexico or on the Pacific Ocean or both?

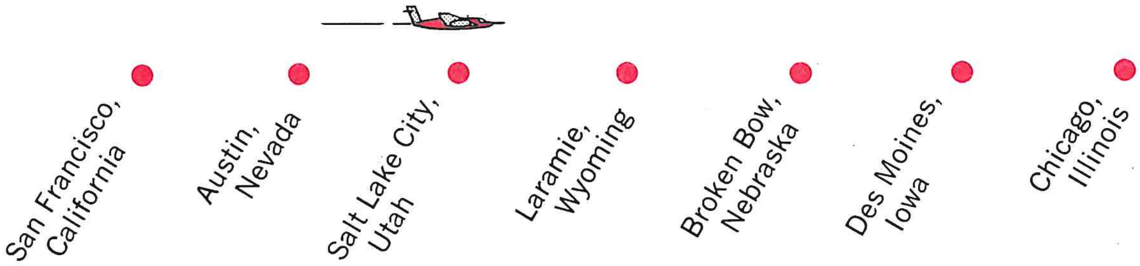
CHAPTER 11 Rational numbers

More about fractions



The picture shows that Susan is one half of the way from her house to the school.

1. Where would Susan be if she were one quarter of the way from her house to the school?
2. Where would Susan be if she were three quarters of the way from her house to the school?

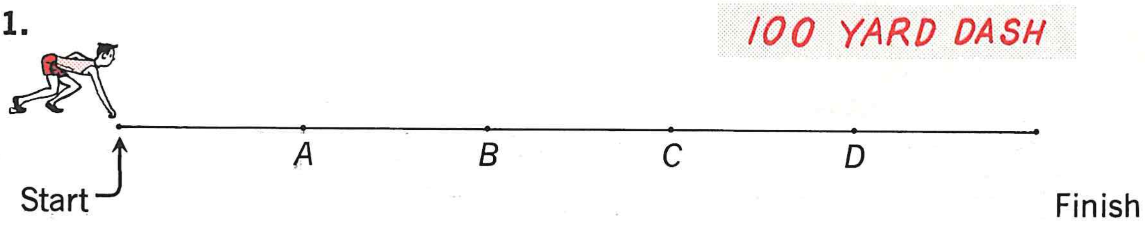


The picture shows the jet over Salt Lake City. It is about two sixths of the way from San Francisco to Chicago.

3. About where would the jet be if it were five sixths of the way from San Francisco to Chicago?
4. About where would the jet be if it were two thirds of the way from San Francisco to Chicago?
5. About what part of the distance would the jet have traveled if it were over Austin?
6. Give two fractions to tell about what part of the distance the jet has traveled when it is over Laramie.

EXERCISES

1.



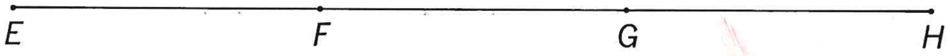
[A] When the runner is at point *B*, what fraction of the race has he run?

[B] When the runner has run three fifths of the race, where will he be?

[C] When the runner is at point *D*, what fraction of the race has he run?

★[D] About how many yards has the runner gone when he is at point *A*?

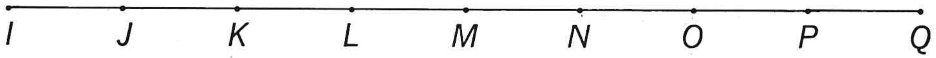
2.



[A] Which point is one third of the way from *E* to *H*?

[B] Point *G* above is what fraction of the way from *E* to *H*?

3.



[A] Which point is one eighth of the way from *I* to *Q*?

[B] Point *L* is what fraction of the way from *I* to *Q*?

[C] Which point is seven eighths of the way from *I* to *Q*?

[D] Which point is six eighths of the way from *I* to *Q*?

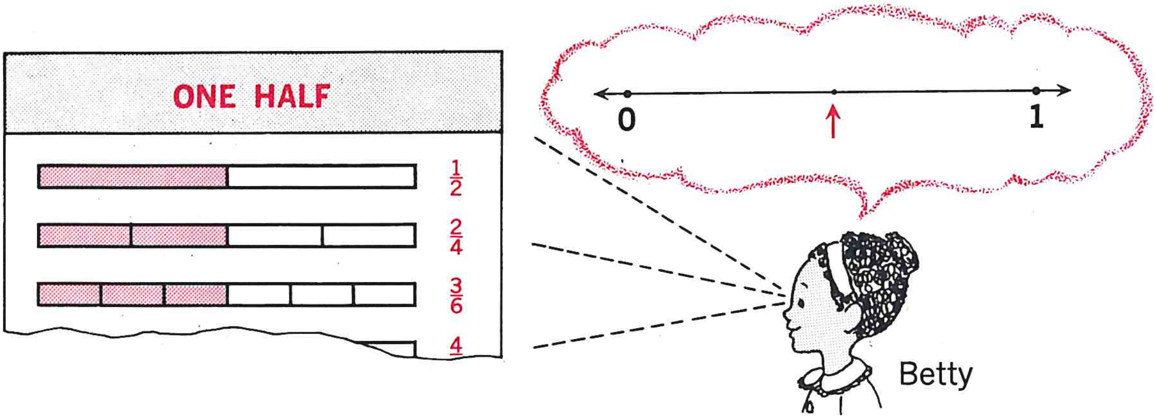
[E] Which point is three fourths of the way from *I* to *Q*?

★[F] Why did you get the same answer for exercises **D** and **E**?

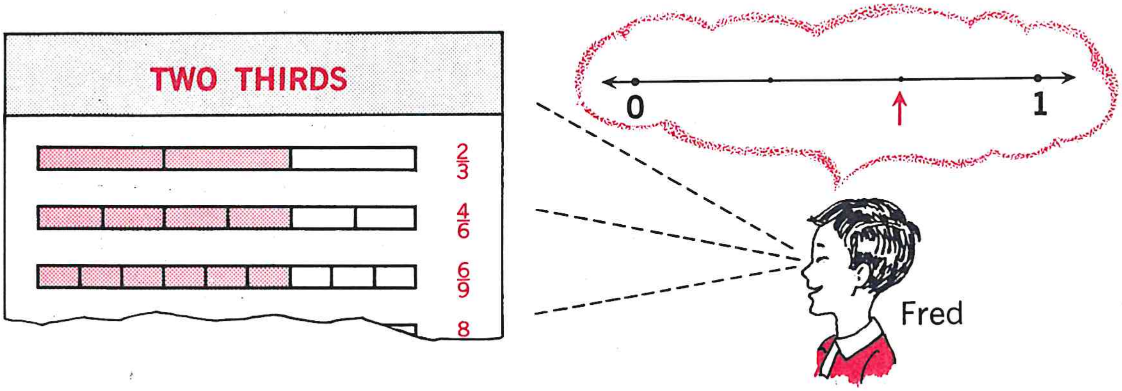
★[G] Give three fractions to tell what part of the way point *M* is from *I* to *Q*.

★[H] Give a fraction to tell what part of the way point *Q* is from *I* to *Q*.

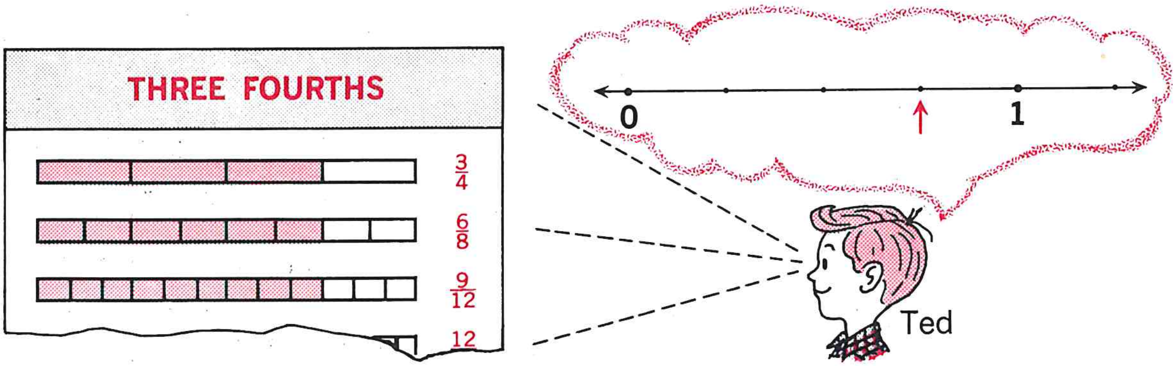
From fractions to numbers



1. Betty is thinking of a rational number. Explain how she can find the point on the number line for this number.



2. Fred is thinking of a rational number. Explain how to find the point on the number line for Fred's number.

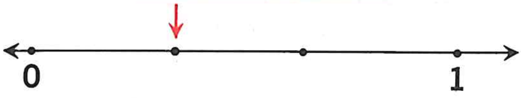


3. How do you think Ted found the point for his rational number?

For each set of equivalent fractions, we think of just one number.

We call this number a rational number.

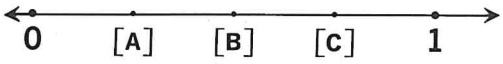
For this set of equivalent fractions, we think of one rational number and one point on the number line. → $\left\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\right\}$



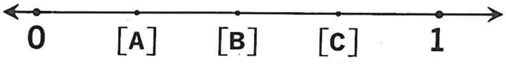
EXERCISES

Answer **A**, **B**, or **C** to give the point on the number line for each set of equivalent fractions.

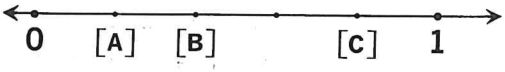
1. $\left\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \ldots\right\}$



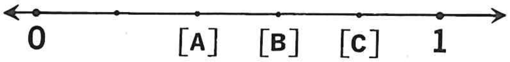
2. $\left\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \ldots\right\}$



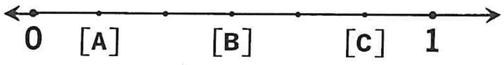
3. $\left\{\frac{1}{5}, \frac{2}{10}, \frac{3}{15}, \ldots\right\}$



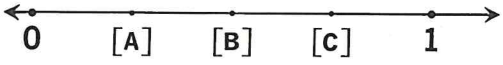
4. $\left\{\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \ldots\right\}$



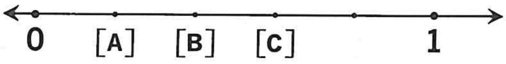
5. $\left\{\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \ldots\right\}$



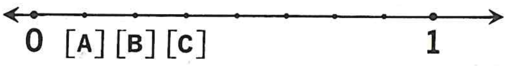
6. $\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \ldots\right\}$



7. $\left\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \ldots\right\}$

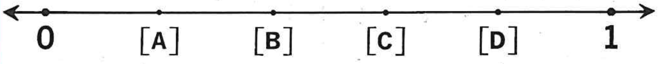
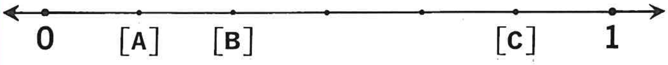
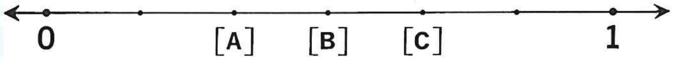
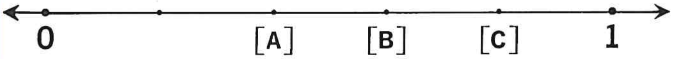
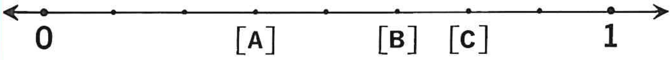
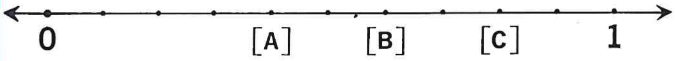
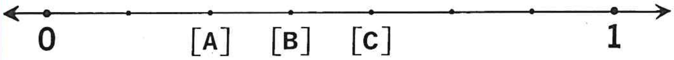
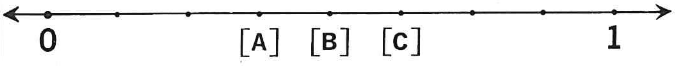
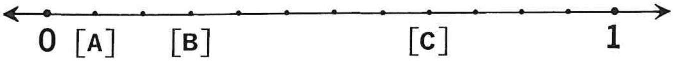
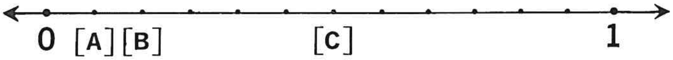
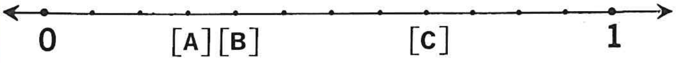


8. $\left\{\frac{1}{8}, \frac{2}{16}, \frac{3}{32}, \ldots\right\}$



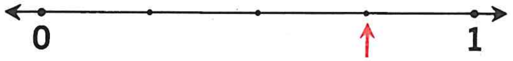
Rational-number exercises

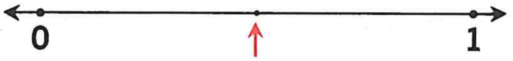
For each set of equivalent fractions in the table below there is a rational number. Give the point for the number on the number line.

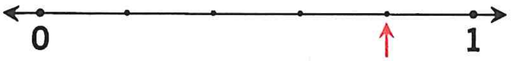
	Set of fractions for the number	Which point on the number line goes with the number?
1.	$\left\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \dots\right\}$	
2.	$\left\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \dots\right\}$	
3.	$\left\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \dots\right\}$	
4.	$\left\{\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \dots\right\}$	
5.	$\left\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \dots\right\}$	
6.	$\left\{\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \frac{16}{20}, \dots\right\}$	
7.	$\left\{\frac{3}{7}, \frac{6}{14}, \frac{9}{21}, \frac{12}{28}, \dots\right\}$	
8.	$\left\{\frac{5}{8}, \frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \dots\right\}$	
9.	$\left\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \dots\right\}$	
10.	$\left\{\frac{1}{6}, \frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \dots\right\}$	
11.	$\left\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \dots\right\}$	

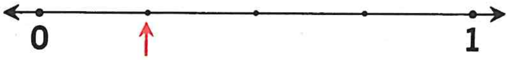
EXERCISES

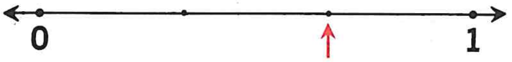
1. Match the sets of fractions with the number-line pictures.

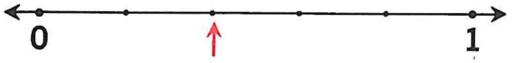
[A]


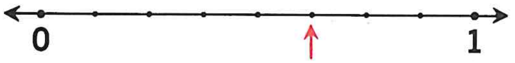
[B]


[C]


[D]


[E]


[F]


[G]


(1) $\left\{\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \ldots\right\}$

(2) $\left\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \ldots\right\}$

(3) $\left\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \ldots\right\}$

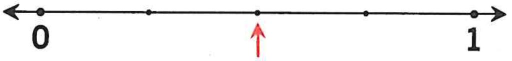
(4) $\left\{\frac{5}{8}, \frac{10}{16}, \frac{15}{24}, \ldots\right\}$

(5) $\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \ldots\right\}$

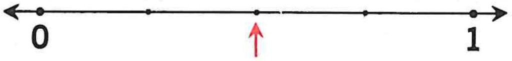
(6) $\left\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \ldots\right\}$

(7) $\left\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \ldots\right\}$

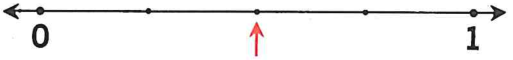
2. Choose the fraction for the point over the red arrow.

[A]


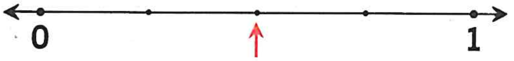
$\frac{6}{8}, \frac{2}{3}, \frac{1}{2}, \frac{1}{4}$

[B]


$\frac{1}{3}, \frac{2}{4}, \frac{5}{8}, \frac{7}{7}$

[C]


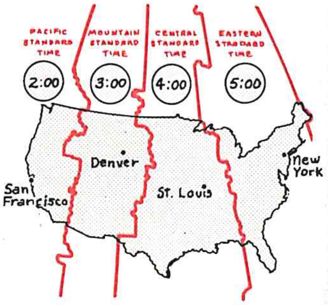
$\frac{5}{10}, \frac{3}{8}, \frac{1}{5}, \frac{6}{8}$

[D]


$\frac{4}{9}, \frac{3}{5}, \frac{8}{16}, \frac{7}{8}$

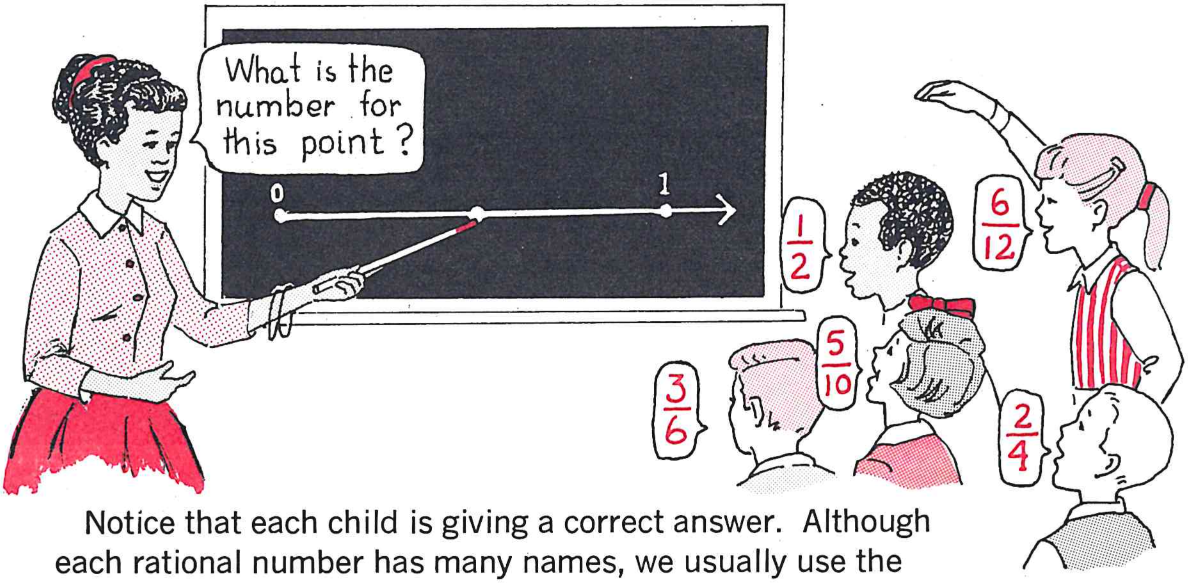
Think

1. From the map, you can see that it is 2:00 in San Francisco when it is 4:00 in St. Louis. What time is it in New York when it is 4:00 in St. Louis?
2. When it is 7:30 in Denver, what time is it in [A] San Francisco, [B] New York, [C] St. Louis?
3. What time zone are you in if you are eating lunch when the children in San Francisco are just starting school?



Names for rational numbers

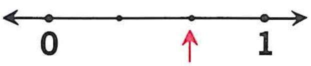
Any fraction from a set of equivalent fractions can be used to name the rational number for that set.

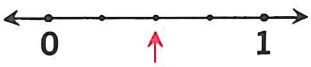

What is the number for this point?
0
1
1/2
6/12
5/10
3/6
2/4

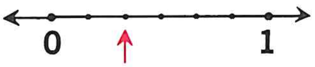
Notice that each child is giving a correct answer. Although each rational number has many names, we usually use the lowest-terms fraction to name the number.

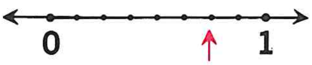
EXERCISES

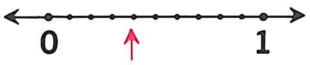
1. Give the lowest-terms fraction that names the rational number for the point over the red arrow.

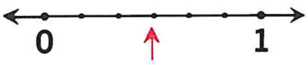
[A]


[B]


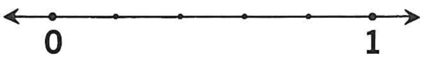
[C]


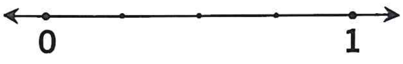
[D]


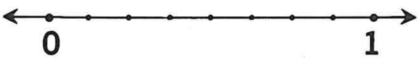
[E]


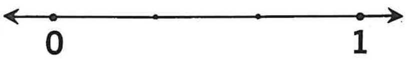
[F]


2. Trace the number line on your paper. Then mark with an arrow the point named by the fraction.

[A] $\frac{2}{5}$


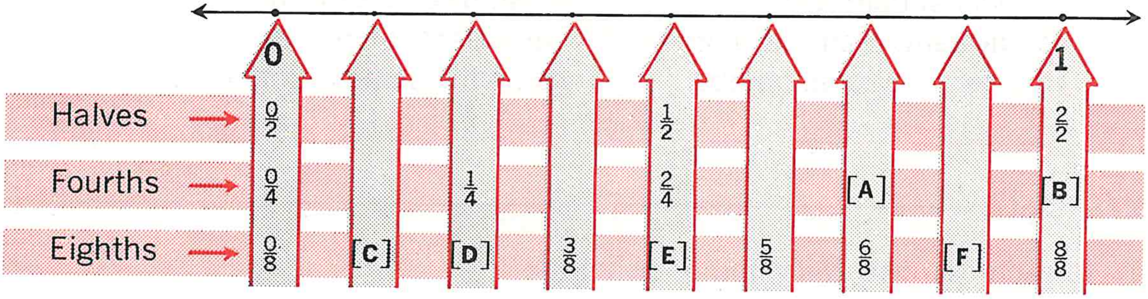
[C] $\frac{6}{8}$


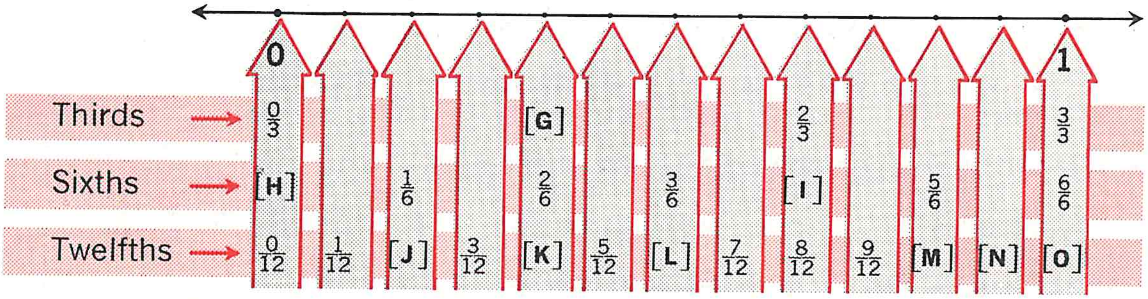
[B] $\frac{5}{8}$


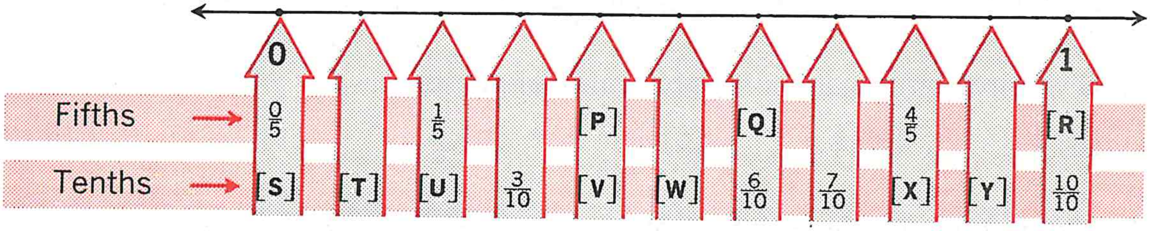
[D] $\frac{6}{9}$


EXERCISES

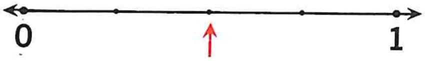
1. Just one rational number goes with each point. Some of the fractions that name the rational number are given. Give the missing fractions.

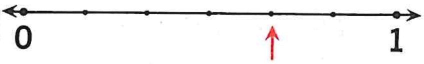


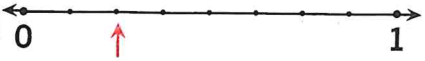


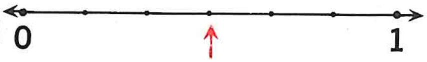


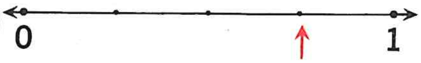
2. In each exercise, give two fractions to name the rational number for the point over the red arrow.

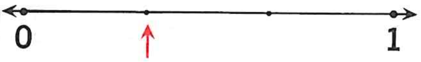
[A]


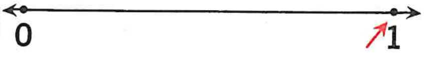
[B]


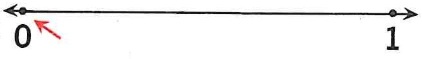
[C]


[D]


[E]


[F]


[G]


[H]


Equality of rational numbers

Study this definition.

If two fractions are equivalent, then they name the same rational number.	If two fractions are not equivalent, then they do not name the same rational number.

EXERCISES

1. In each exercise, tell whether or not the two fractions name the same rational number.

[A] $\frac{1}{2}$ $\frac{2}{4}$
[B] $\frac{2}{3}$ $\frac{8}{12}$
[C] $\frac{4}{16}$ $\frac{1}{3}$
[D] $\frac{5}{8}$ $\frac{15}{24}$
[E] $\frac{7}{10}$ $\frac{20}{30}$
[F] $\frac{9}{12}$ $\frac{12}{16}$
[G] $\frac{1}{8}$ $\frac{10}{80}$
[H] $\frac{0}{8}$ $\frac{0}{7}$
[I] $\frac{3}{10}$ $\frac{9}{20}$
[J] $\frac{4}{9}$ $\frac{12}{27}$

2. Tell which fraction does not belong in each set.

[A] $\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{5}, \frac{4}{8}, \ldots\right\}$
[B] $\left\{\frac{1}{3}, \frac{2}{4}, \frac{3}{9}, \frac{4}{12}, \ldots\right\}$
[C] $\left\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{5}{16}, \ldots\right\}$
[D] $\left\{\frac{2}{3}, \frac{4}{8}, \frac{6}{9}, \frac{8}{12}, \ldots\right\}$
[E] $\left\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{15}, \ldots\right\}$
[F] $\left\{\frac{5}{8}, \frac{10}{15}, \frac{15}{24}, \frac{20}{32}, \ldots\right\}$
[G] $\left\{\frac{2}{5}, \frac{4}{10}, \frac{6}{16}, \frac{8}{20}, \ldots\right\}$
[H] $\left\{\frac{0}{1}, \frac{1}{2}, \frac{0}{3}, \frac{0}{4}, \ldots\right\}$
[I] $\left\{\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \frac{3}{4}, \ldots\right\}$

3. Give three other names for each rational number.

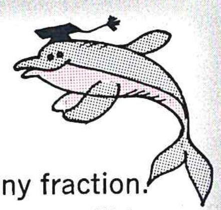
[A] $\frac{4}{8}$
[B] $\frac{5}{6}$
[C] $\frac{1}{4}$
[D] $\frac{3}{9}$
[E] $\frac{8}{12}$
[F] $\frac{3}{10}$

4. Match the fractions with the sets.

[A] $\frac{10}{20}$
[B] $\frac{100}{300}$
[C] $\frac{30}{40}$
[D] $\frac{300}{800}$
[E] $\frac{50}{60}$
[F] $\frac{50}{75}$
[G] $\frac{1000}{4000}$

(1) $\left\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\right\}$
(2) $\left\{\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \ldots\right\}$
(3) $\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\right\}$
(4) $\left\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots\right\}$
(5) $\left\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \ldots\right\}$
(6) $\left\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \ldots\right\}$
(7) $\left\{\frac{3}{8}, \frac{6}{16}, \frac{9}{24}, \frac{12}{32}, \ldots\right\}$

Think



Now look at any fraction.
As long as it's my name,
No matter which you choose,
The top number is the same.

WHO AM I?

When we write "=" between two fractions, we mean that the two fractions name the same rational number.

Study these examples.

$\frac{2}{4}$ names this rational number $\longrightarrow \{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\}$.

$\frac{3}{6}$ names this rational number $\longrightarrow \{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\}$.

We write: $\frac{2}{4} = \frac{3}{6}$.

$\frac{3}{9}$ names this rational number $\longrightarrow \{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\}$.

$\frac{1}{3}$ names this rational number $\longrightarrow \{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\}$.

We write: $\frac{3}{9} = \frac{1}{3}$.

$\frac{6}{8}$ names this rational number $\longrightarrow \{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \ldots\}$.

$\frac{12}{16}$ names this rational number $\longrightarrow \{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \ldots\}$.

We write: $\frac{6}{8} = \frac{12}{16}$.

DISCUSSION EXERCISES

1. Explain the two statements at the top of page 298.
2. Explain how you can check to see if $\frac{10}{12} = \frac{25}{30}$.

EXERCISES

Answer **T** (true) or **F** (false).

1. $\frac{1}{2}$ is equivalent to $\frac{2}{4}$.

2. $\frac{3}{8}$ is equivalent to $\frac{7}{16}$.

3. $\frac{8}{12}$ is equivalent to $\frac{2}{3}$.

4. $\frac{12}{30}$ is equivalent to $\frac{3}{10}$.

5. $\frac{2}{12}$ is equivalent to $\frac{5}{30}$.

6. $\frac{1}{2} = \frac{2}{4}$

7. $\frac{3}{8} = \frac{7}{16}$

8. $\frac{8}{12} = \frac{2}{3}$

9. $\frac{12}{30} = \frac{3}{10}$

10. $\frac{2}{12} = \frac{5}{30}$

11. $\frac{3}{4} = \frac{15}{20}$

12. $\frac{1}{8} = \frac{8}{64}$

13. $\frac{3}{5} = \frac{9}{10}$

14. $\frac{1}{2} = \frac{50}{100}$

15. $\frac{1}{4} = \frac{25}{100}$

More about equality

EXERCISES

1. Copy the sets of fractions on your paper and give five more fractions for each set.

[A] $\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\right\}$

[B] $\left\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\right\}$

[C] $\left\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \ldots\right\}$

[D] $\left\{\frac{1}{5}, \frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \ldots\right\}$

[E] $\left\{\frac{1}{6}, \frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \ldots\right\}$

[F] $\left\{\frac{1}{8}, \frac{2}{16}, \frac{3}{24}, \frac{4}{32}, \ldots\right\}$

[G] $\left\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots\right\}$

[H] $\left\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \ldots\right\}$

[I] $\left\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \ldots\right\}$

[J] $\left\{\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \ldots\right\}$

[K] $\left\{\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \frac{16}{20}, \ldots\right\}$

[L] $\left\{\frac{3}{8}, \frac{6}{16}, \frac{9}{24}, \frac{12}{32}, \ldots\right\}$

[M] $\left\{\frac{5}{8}, \frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \ldots\right\}$

[N] $\left\{\frac{7}{8}, \frac{14}{16}, \frac{21}{24}, \frac{28}{32}, \ldots\right\}$

[O] $\left\{\frac{1}{7}, \frac{2}{14}, \frac{3}{21}, \frac{4}{28}, \ldots\right\}$

2. Give the lowest-terms fraction for each set in exercise 1.

3. [A] One half is equal to how many eighths?

[B] One third is equal to how many sixths?

[C] One fourth is equal to how many twelfths?

[D] One sixth is equal to how many eighteenths?

[E] One eighth is equal to how many sixteenths?

[F] One fifth is equal to how many tenths?

[G] Four twelfths is equal to how many thirds?

[H] Three sixths is equal to how many halves?

[I] Two eighths is equal to how many fourths?

[J] Four sixteenths is equal to how many fourths?

4. Find the missing numerators and denominators.

[A] $\frac{2}{8} = \frac{\square}{4}$

[B] $\frac{4}{8} = \frac{1}{\square}$

[C] $\frac{6}{8} = \frac{\square}{4}$

[D] $\frac{2}{6} = \frac{1}{\square}$

[E] $\frac{3}{6} = \frac{1}{\square}$

[F] $\frac{4}{6} = \frac{\square}{3}$

[G] $\frac{2}{10} = \frac{\square}{5}$

[H] $\frac{4}{10} = \frac{2}{\square}$

[I] $\frac{5}{10} = \frac{\square}{2}$

[J] $\frac{6}{10} = \frac{\square}{5}$

[K] $\frac{8}{10} = \frac{\square}{5}$

[L] $\frac{2}{12} = \frac{1}{\square}$

[M] $\frac{3}{12} = \frac{\square}{4}$

[N] $\frac{4}{12} = \frac{\square}{3}$

[O] $\frac{6}{12} = \frac{\square}{2}$

[P] $\frac{8}{12} = \frac{2}{\square}$

[Q] $\frac{9}{12} = \frac{3}{\square}$

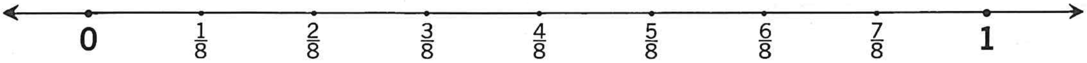
[R] $\frac{10}{12} = \frac{\square}{6}$

[S] $\frac{5}{15} = \frac{\square}{3}$

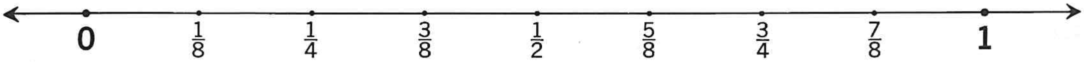
[T] $\frac{6}{18} = \frac{1}{\square}$

The lowest-terms fraction is the name often used for a rational number.

We could label points on the number line like this:

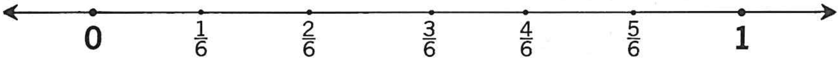


More often, we will label the points like this:



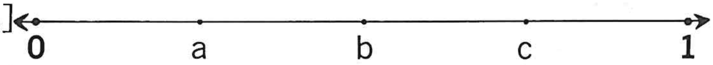
EXERCISES

1. Give the lowest-terms fraction for each point on this number line.

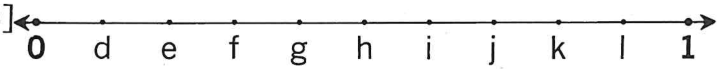


2. Copy the number lines on your paper. Give the lowest-terms fraction for the point above each letter.

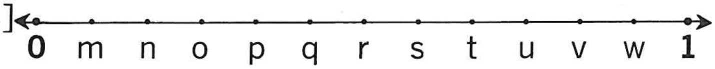
[A]



[B]



[C]



3. Answer **T** (true) or **F** (false).

[A] $\frac{5}{10} = \frac{1}{2}$

[B] $\frac{3}{8} = \frac{9}{24}$

[C] $\frac{5}{6} = \frac{5}{7}$

[D] $\frac{6}{10} = \frac{3}{5}$

[E] $\frac{4}{8} = \frac{20}{40}$

[F] $\frac{50}{100} = \frac{1}{2}$

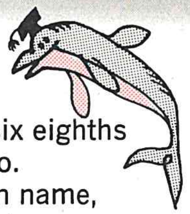
[G] $\frac{3}{12} = \frac{1}{3}$

[H] $\frac{1}{4} = \frac{10}{40}$

[I] $\frac{0}{8} = \frac{1}{10}$

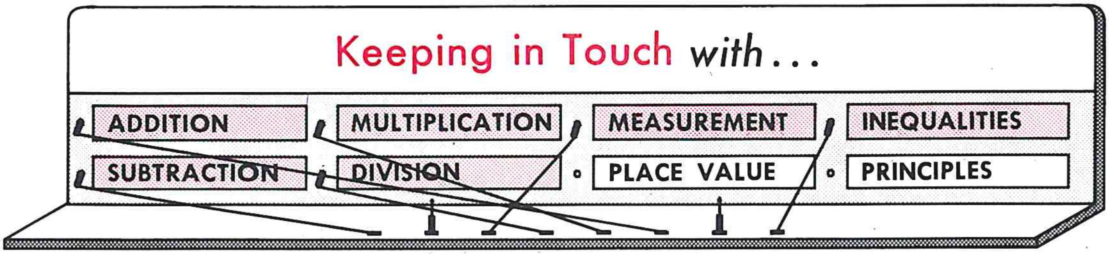
[J] $\frac{6}{12} = \frac{2}{4}$

Think



Sometimes I'm called six eighths
Nine twelfths is O.K. too.
Please give my common name,
Though any one would do.

WHO AM I?



1. Which number is larger?

[A] 362,807 or 359,968 [C] 78,076 or 78,100 [E] 640,000 or 98,000

[B] 4,027,340 or 3,928,643 [D] 9,284,316 or 9,283,978

2. Find the sums.

[A] 3287 + 642 + 93,216 + 49 [C] 37,064 + 85 + 79 + 3246

[B] 43,826 + 925 + 8301 + 9640 [D] 9324 + 657 + 8421 + 38

3. Find the differences.

[A] 9264 − 381 [C] 6403 − 856 [E] 6000 − 87

[B] 731 − 285 [D] 7218 − 3451 [F] 4006 − 198

4. Find the products.

[A] 37 × 26 [B] 58 × 79 [C] 283 × 46 [D] 349 × 38

5. Find the quotients and remainders.

[A] 348 ÷ 7 [B] 625 ÷ 20 ★[C] 482 ÷ 36 ★[D] 1927 ÷ 51

6. Measure each segment to the nearest half inch.

[A]

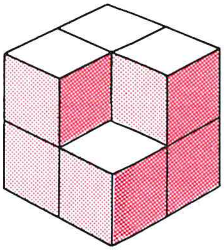
[B]

[C]

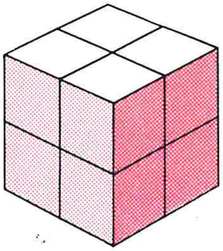
[D]

7. Find the volume for each figure.

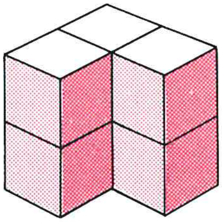
[A]



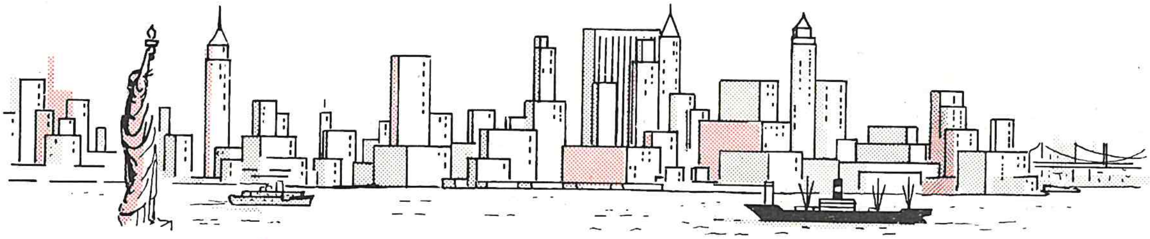
[B]



[C]



TALL BUILDINGS



Building	Location	Height	Stories
Empire State	New York City	1250 feet	102
John Hancock	Chicago, Illinois	1150 feet	100
Chrysler	New York City	1046 feet	77
Bank of Manhattan	New York City	900 feet	71
Rockefeller Center	New York City	850 feet	70
Terminal Tower	Cleveland, Ohio	708 feet	52
Humble Oil	Houston, Texas	606 feet	44
Board of Trade	Chicago, Illinois	605 feet	44

1. How much taller is the Empire State Building than the John Hancock Building?

2. The Board of Trade Building in Chicago has a 311-foot tower at the top. How high above the ground is the top of the tower?

3. About how tall is each story in these buildings?

   [A] Rockefeller Center
   [B] Board of Trade Building

4. [A] About how tall is a building if it has 58 stories and each story is about 12 feet tall?

   [B] If this building has a 175-foot television tower on top, about how far is the top of the tower from the ground?

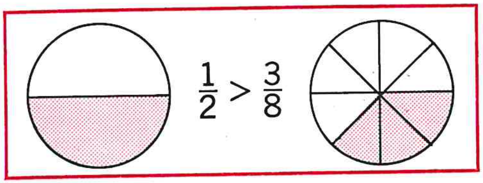
5. A television tower in Missouri is 1676 feet tall. If it were placed on top of the Chrysler Building, how far would the top of the tower be from the ground?

6. How much taller is Rockefeller Center than Terminal Tower?

Inequalities for rational numbers

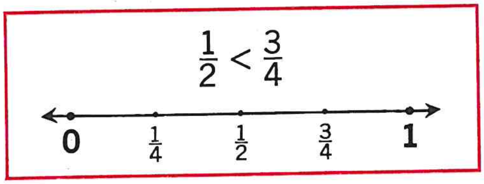
DISCUSSION EXERCISES

1. Explain how the shaded parts of the two circular regions help you see that $\frac{1}{2}$ is greater than $\frac{3}{8}$.



$\frac{1}{2} > \frac{3}{8}$

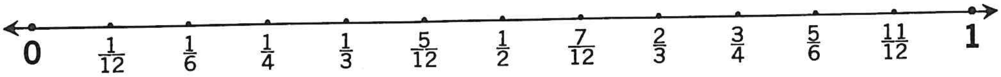
2. Explain how the number line picture helps you see that $\frac{1}{2}$ is less than $\frac{3}{4}$.



$\frac{1}{2} < \frac{3}{4}$

EXERCISES

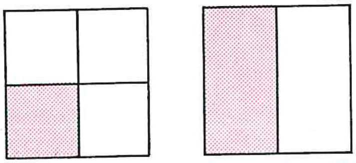
1. Study the number line. Then give the correct sign (<, =, or >) for each ●.



[A] $\frac{1}{4}$ ● $\frac{1}{3}$
[B] $\frac{1}{2}$ ● $\frac{5}{12}$
[C] $\frac{1}{12}$ ● $\frac{1}{4}$
[D] $\frac{3}{4}$ ● $\frac{5}{6}$
[E] $\frac{7}{12}$ ● $\frac{2}{3}$
[F] $\frac{1}{2}$ ● $\frac{6}{12}$
[G] $\frac{2}{3}$ ● $\frac{3}{4}$
[H] $\frac{5}{6}$ ● $\frac{11}{12}$
[I] $\frac{1}{12}$ ● 0
[J] $\frac{7}{12}$ ● $\frac{1}{2}$
[K] $\frac{5}{12}$ ● $\frac{1}{2}$
[L] $\frac{1}{2}$ ● $\frac{1}{3}$
[M] $\frac{2}{12}$ ● $\frac{1}{6}$
[N] $\frac{1}{4}$ ● $\frac{2}{12}$
[O] $\frac{1}{3}$ ● $\frac{3}{12}$

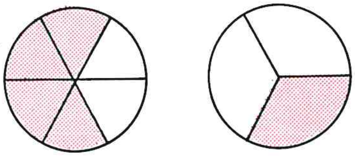
2. Write an inequality for each exercise.

[A]

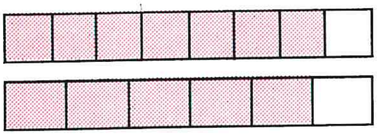


[Answer: $\frac{1}{4} < \frac{1}{2}$ (or $\frac{1}{2} > \frac{1}{4}$)]

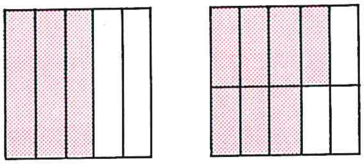
[B]



[C]

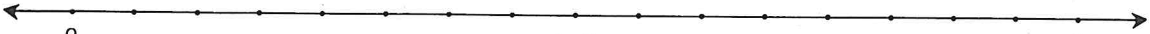


[D]



EXERCISES

1. Give the missing fractions.



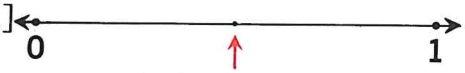
$\frac{0}{2}$ [A] $\frac{2}{2}$

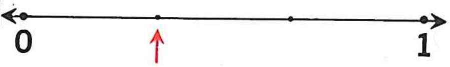
[B] $\frac{1}{4}$ $\frac{2}{4}$ [C] $\frac{4}{4}$

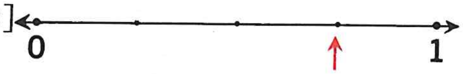
$\frac{0}{8}$ $\frac{1}{8}$ [D] $\frac{3}{8}$ [E] $\frac{5}{8}$ $\frac{6}{8}$ [F] [G]

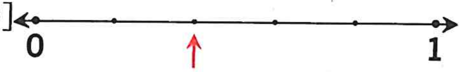
$\frac{0}{16}$ [H] $\frac{2}{16}$ $\frac{3}{16}$ $\frac{4}{16}$ $\frac{5}{16}$ [I] $\frac{7}{16}$ $\frac{8}{16}$ $\frac{9}{16}$ $\frac{10}{16}$ [J] [K] $\frac{13}{16}$ [L] $\frac{15}{16}$ $\frac{16}{16}$

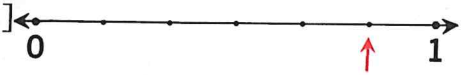
2. For each exercise, give a set of three equivalent fractions for the point over the red arrow.

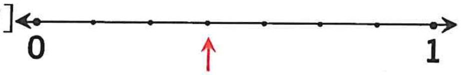
[A]


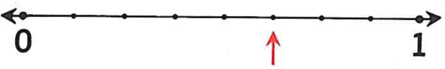
[B]


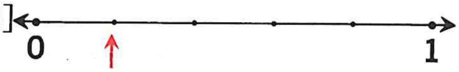
[C]


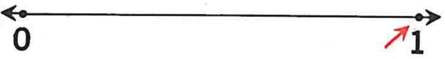
[D]


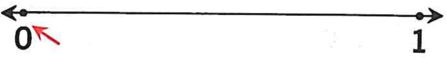
[E]


[F]


[G]


[H]


[I]


[J]


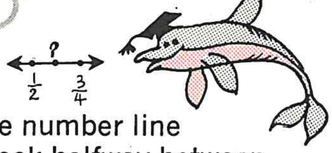
3. Answer **T** (true) or **F** (false).

[A] $\frac{1}{2} < \frac{5}{8}$
[B] $\frac{3}{8} > \frac{2}{2}$
[C] $\frac{1}{4} < \frac{1}{8}$
[D] $\frac{8}{8} = \frac{4}{4}$
[E] $\frac{8}{16} > \frac{4}{8}$
[F] $\frac{7}{8} < \frac{2}{2}$
[G] $\frac{0}{2} = \frac{0}{8}$
[H] $\frac{2}{3} = \frac{3}{4}$
[I] $\frac{4}{5} < \frac{5}{8}$
[J] $\frac{3}{4} < \frac{3}{8}$
★ [K] $\frac{4}{8} < \frac{4}{7}$
★ [L] $\frac{8}{14} < \frac{8}{15}$

4. Give the missing numerators and denominators.

[A] $\frac{1}{2} = \frac{\square}{4}$
[B] $\frac{1}{3} = \frac{2}{\square}$
[C] $\frac{3}{4} = \frac{\square}{8}$
[D] $\frac{3}{3} = \frac{\square}{9}$
[E] $\frac{1}{\square} = \frac{5}{10}$
[F] $\frac{\square}{3} = \frac{4}{6}$
[G] $\frac{1}{8} = \frac{3}{\square}$
[H] $\frac{4}{8} = \frac{\square}{4}$
[I] $\frac{\square}{4} = \frac{0}{2}$
[J] $\frac{4}{7} = \frac{8}{\square}$
[K] $\frac{2}{3} = \frac{\square}{15}$
[L] $\frac{2}{2} = \frac{8}{\square}$

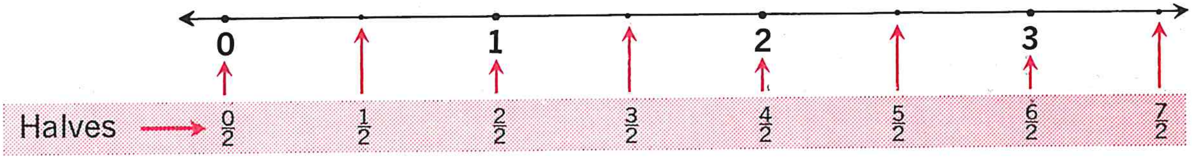
Think

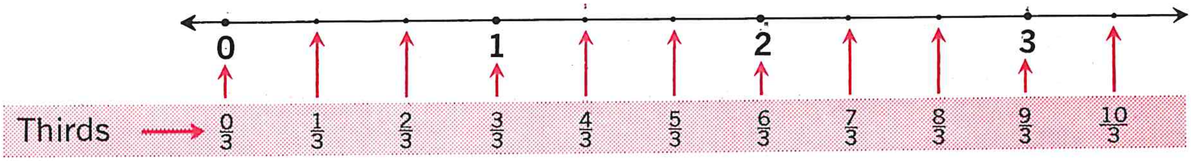


On the number line
Just look halfway between
Three-fourths and one-half.
That's where I'm often seen.

WHO AM I?

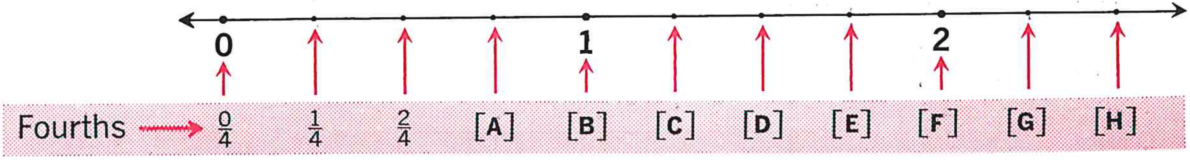
Rational numbers greater than 1

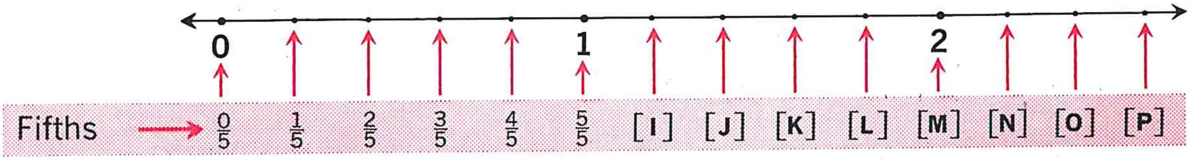


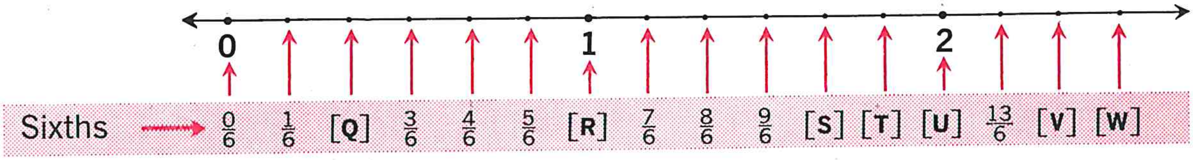


EXERCISES

1. Give the missing fractions.



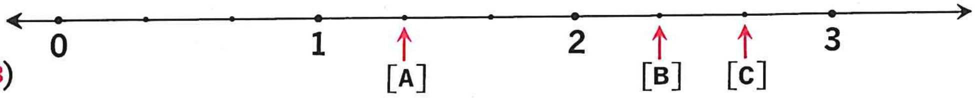




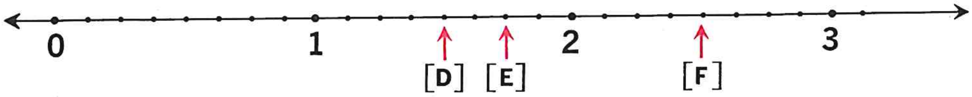
2. Choose the point for the number given.

[A] $\frac{7}{3}$

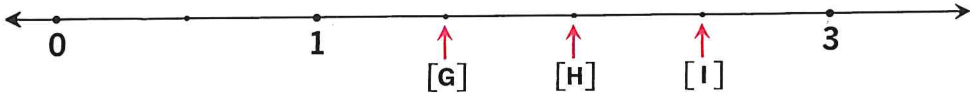
(Answer: B)



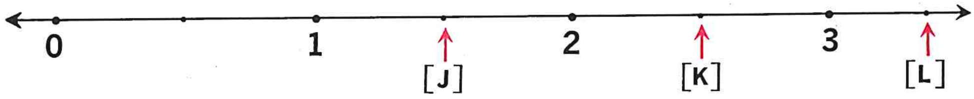
[B] $\frac{12}{8}$



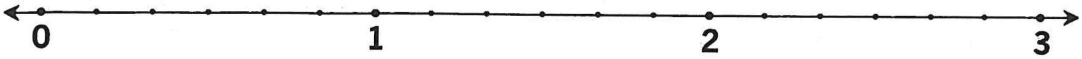
[C] $\frac{16}{8}$



[D] $\frac{10}{4}$



3. Give the missing fractions.



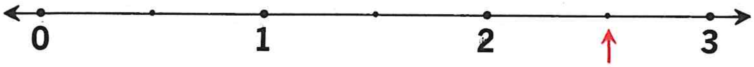
$\frac{0}{2}$ $\frac{1}{2}$ $\frac{2}{2}$ [A] [B] [C] $\frac{6}{2}$

[D] $\frac{1}{3}$ $\frac{2}{3}$ [E] $\frac{4}{3}$ [F] $\frac{6}{3}$ $\frac{7}{3}$ $\frac{8}{3}$ [G]

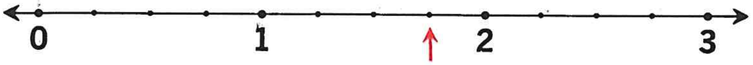
$\frac{0}{6}$ [H] $\frac{2}{6}$ $\frac{3}{6}$ $\frac{4}{6}$ $\frac{5}{6}$ $\frac{6}{6}$ $\frac{7}{6}$ [I] $\frac{9}{6}$ $\frac{10}{6}$ $\frac{11}{6}$ [J] $\frac{13}{6}$ $\frac{14}{6}$ [K] $\frac{16}{6}$ [L] $\frac{18}{6}$

4. Give a set of three equivalent fractions that name the rational number for the point above the red arrow.

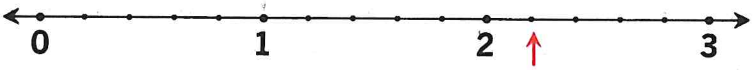
[A]



[B]



[C]



5. Answer **T** (true) or **F** (false).

[A] $\frac{2}{2} > \frac{1}{2}$

[B] $\frac{2}{2} < \frac{3}{2}$

[C] $\frac{3}{4} > \frac{3}{2}$

[D] $\frac{5}{2} > 2$

[E] $\frac{6}{8} = \frac{9}{12}$

[F] $\frac{5}{2} < 3$

[G] $\frac{6}{4} = \frac{3}{2}$

[H] $\frac{9}{4} < 2$

[I] $\frac{11}{4} > \frac{5}{2}$

[J] $\frac{12}{4} < 3$

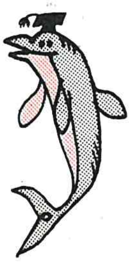
[K] $\frac{8}{8} > \frac{4}{4}$

[L] $\frac{6}{3} > \frac{5}{2}$

[M] $\frac{0}{4} < \frac{0}{8}$

[N] $\frac{0}{4} > \frac{0}{8}$

Think



Study the pattern.
Then solve the equations.

$37 \times (3 \times 1) = 111$

$37 \times (3 \times 2) = 222$

$37 \times (3 \times 3) = 333$

$37 \times (3 \times 4) = 444$

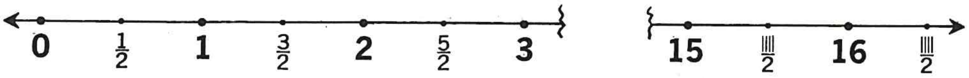
$37 \times (3 \times 5) = \boxed{n}$

$37 \times (3 \times 8) = \boxed{n}$

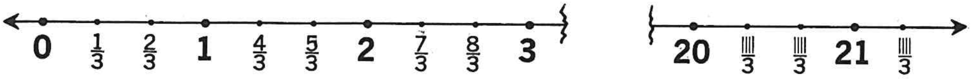
$37 \times (3 \times 9) = \boxed{n}$

★6. Find the missing numerators.

[A]



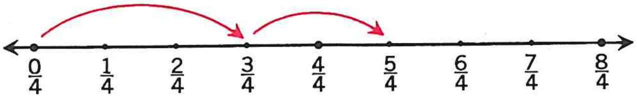
[B]



Addition of rational numbers

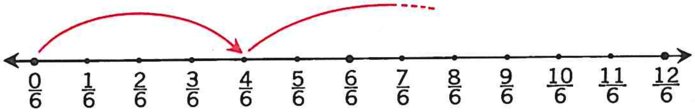
DISCUSSION EXERCISES

1. Explain how the number line picture helps you find the sum of $\frac{3}{4}$ and $\frac{2}{4}$.



$\frac{3}{4}+\frac{2}{4}=\boxed{n}$

2. Explain how you would decide on the second jump to find the sum of $\frac{4}{6}$ and $\frac{5}{6}$.

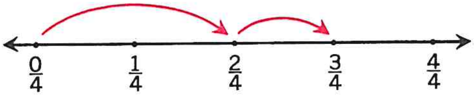


$\frac{4}{6}+\frac{5}{6}=\boxed{n}$

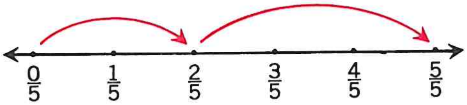
EXERCISES

1. Write an addition equation for each number-line picture.

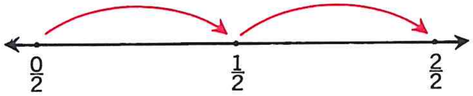
[A]



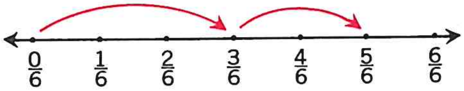
[B]



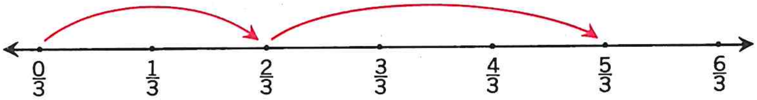
[C]



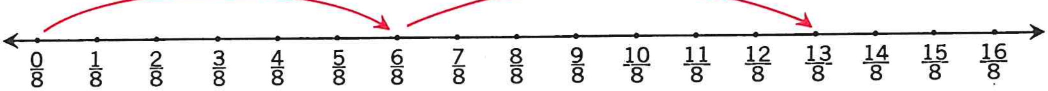
[D]



[E]



[F]



2. Find the sums.

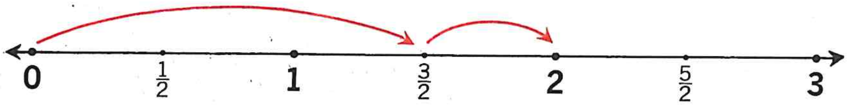
[A] $\frac{3}{8}+\frac{4}{8}$ **[C]** $\frac{6}{8}+\frac{1}{8}$ **[E]** $\frac{5}{7}+\frac{4}{7}$ **[G]** $\frac{4}{10}+\frac{5}{10}$ **[I]** $\frac{5}{10}+\frac{5}{10}$

[B] $\frac{2}{3}+\frac{4}{3}$ **[D]** $\frac{4}{6}+\frac{4}{6}$ **[F]** $\frac{5}{6}+\frac{1}{6}$ **[H]** $\frac{9}{10}+\frac{1}{10}$ **[J]** $\frac{9}{10}+\frac{11}{10}$

DISCUSSION EXERCISES

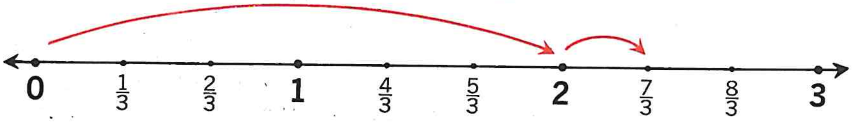
Explain each of the examples below.

1.



$\frac{3}{2} + \frac{1}{2} = 2$

2.

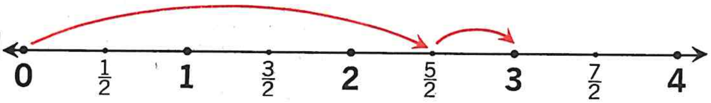


$2 + \frac{1}{3} = \frac{7}{3}$

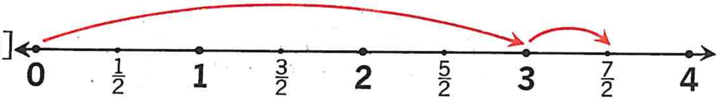
EXERCISES

1. Write an addition equation for each number-line picture.

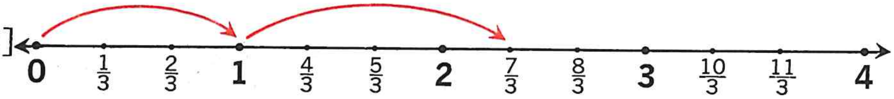
[A]



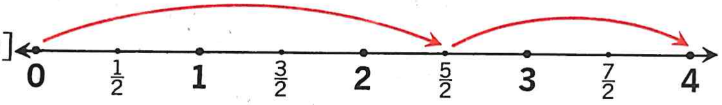
[B]



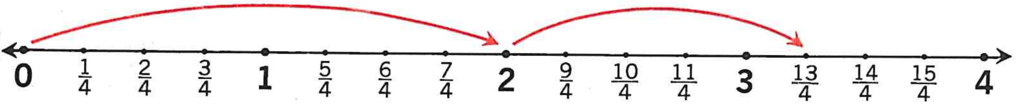
[C]



[D]



[E]



2. Find the sums.

[A] $1 + \frac{1}{2}$

[B] $2 + \frac{1}{2}$

[C] $1 + \frac{1}{3}$

[D] $2 + \frac{1}{3}$

[E] $1 + \frac{2}{3}$

[F] $1 + \frac{1}{4}$

[G] $2 + \frac{1}{4}$

[H] $1 + \frac{1}{5}$

[I] $2 + \frac{1}{5}$

★[J] $7 + \frac{1}{8}$

Whole numbers and rational numbers

EXERCISES

1. Give the missing fractions.

0	1	2	3	4	5	6	7	8
$\frac{0}{1}$	$\frac{1}{1}$	[A]	$\frac{3}{1}$	$\frac{4}{1}$	$\frac{5}{1}$	[B]	$\frac{7}{1}$	$\frac{8}{1}$
$\frac{0}{2}$	[C]	$\frac{4}{2}$	[D]	$\frac{8}{2}$	[E]	$\frac{12}{2}$	[F]	$\frac{16}{2}$
[G]	$\frac{3}{3}$	$\frac{6}{3}$	$\frac{9}{3}$	[H]	$\frac{15}{3}$	$\frac{18}{3}$	$\frac{21}{3}$	[I]
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

2. Match each whole number with a set of equivalent fractions.

[A] 3 (Answer: Set 4)	(1) $\left\{\frac{9}{1}, \frac{18}{2}, \frac{27}{3}, \frac{36}{4}, \ldots\right\}$
[B] 6	(2) $\left\{\frac{15}{1}, \frac{30}{2}, \frac{45}{3}, \frac{60}{4}, \ldots\right\}$
[C] 1	(3) $\left\{\frac{6}{1}, \frac{12}{2}, \frac{18}{3}, \frac{24}{4}, \ldots\right\}$
[D] 9	(4) $\left\{\frac{3}{1}, \frac{6}{2}, \frac{9}{3}, \frac{12}{4}, \ldots\right\}$
[E] 0	(5) $\left\{\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \ldots\right\}$
[F] 15	(6) $\left\{\frac{0}{1}, \frac{0}{2}, \frac{0}{3}, \frac{0}{4}, \ldots\right\}$

3. Give the missing numerators and denominators.

[A] $3 = \frac{6}{\square}$
[B] $6 = \frac{6}{\square}$
[C] $1 = \frac{\square}{5}$
[D] $9 = \frac{\square}{1}$
[E] $0 = \frac{\square}{6}$
[F] $15 = \frac{30}{\square}$
[G] $2 = \frac{\square}{4}$
[H] $7 = \frac{\square}{2}$
[I] $5 = \frac{15}{\square}$
[J] $8 = \frac{\square}{3}$
[K] $4 = \frac{12}{\square}$
[L] $10 = \frac{\square}{2}$
[M] $12 = \frac{\square}{1}$
[N] $20 = \frac{40}{\square}$
[O] $11 = \frac{77}{\square}$
[P] $\frac{1}{2} = \frac{\square}{10}$
[Q] $\frac{1}{4} = \frac{3}{\square}$
[R] $\frac{1}{2} = \frac{\square}{12}$
[S] $\frac{2}{3} = \frac{8}{\square}$
[T] $\frac{3}{4} = \frac{9}{\square}$

4. Find the sums.

[A] $\frac{4}{2} + \frac{1}{2}$
[B] $\frac{6}{2} + \frac{1}{2}$
[C] $3 + \frac{1}{2}$
[D] $\frac{6}{3} + \frac{1}{3}$
[E] $2 + \frac{1}{3}$
[F] $\frac{9}{3} + \frac{1}{3}$
[G] $3 + \frac{1}{3}$
[H] $\frac{12}{3} + \frac{2}{3}$
[I] $4 + \frac{2}{3}$
[J] $\frac{12}{2} + \frac{3}{2}$
[K] $6 + \frac{3}{2}$
[L] $\frac{8}{4} + \frac{3}{4}$
[M] $2 + \frac{3}{4}$
[N] $\frac{8}{8} + \frac{5}{8}$
[O] $1 + \frac{5}{8}$

Mixed numerals

Symbols such as $2\frac{1}{2}$ and $3\frac{1}{4}$ are called mixed numerals. The two examples on the right will help you understand mixed numerals.

$2\frac{1}{2}$ means $2 + \frac{1}{2}$

$3\frac{1}{4}$ means $3 + \frac{1}{4}$

DISCUSSION EXERCISES

1. Give the meaning of each of these mixed numerals.

[A] $4\frac{1}{8}$ [B] $1\frac{5}{6}$ [C] $7\frac{1}{2}$ [D] $8\frac{2}{3}$ [E] $9\frac{1}{4}$ [F] $6\frac{7}{8}$ [G] $4\frac{7}{9}$

2. Solve and explain each equation.

[A] $3 + \frac{1}{8} = n$ [B] $7 + \frac{1}{5} = n$ [C] $9 + \frac{1}{4} = n$ [D] $8 + \frac{2}{5} = n$

3. Explain each example below and give the mixed numeral for n.

Ⓐ $\frac{3}{2} = \frac{2}{2} + \frac{1}{2} = 1 + \frac{1}{2} = n$	Ⓑ $\frac{10}{4} = \frac{8}{4} + \frac{2}{4} = 2 + \frac{2}{4} = n$
Ⓒ $\frac{7}{3} = \frac{6}{3} + \frac{1}{3} = 2 + \frac{1}{3} = n$	Ⓓ $\frac{15}{14} = \frac{12}{4} + \frac{3}{4} = 3 + \frac{3}{4} = n$

EXERCISES

1. Give the correct mixed numeral for each sum.

[A] $2 + \frac{1}{4}$
(Answer: $2\frac{1}{4}$)

[B] $\frac{8}{4} + \frac{1}{4}$
(Answer: $2\frac{1}{4}$)

[C] $3 + \frac{1}{5}$

[D] $\frac{15}{5} + \frac{1}{5}$

[E] $1 + \frac{1}{8}$

[F] $\frac{8}{8} + \frac{1}{8}$

[G] $5 + \frac{1}{2}$

[H] $\frac{10}{2} + \frac{1}{2}$

[I] $1 + \frac{5}{6}$

[J] $\frac{6}{6} + \frac{5}{6}$

[K] $1 + \frac{3}{5}$

[L] $\frac{5}{5} + \frac{3}{5}$

[M] $8 + \frac{1}{4}$

[N] $\frac{32}{4} + \frac{1}{4}$

[O] $\frac{4}{9} + \frac{6}{9}$

[P] $\frac{4}{9} + \frac{15}{9}$

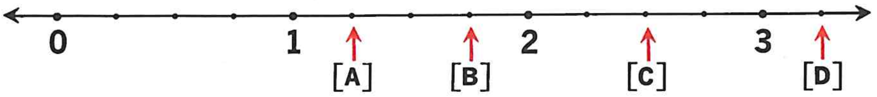
[Q] $\frac{6}{10} + \frac{7}{10}$

[R] $\frac{10}{10} + \frac{9}{10}$

2. Give a mixed numeral for each fraction. Use exercise 1.

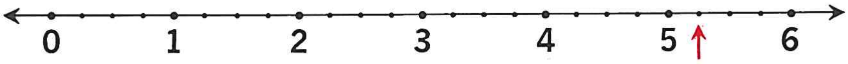
[A] $\frac{9}{4}$ [B] $\frac{16}{5}$ [C] $\frac{11}{6}$ [D] $\frac{33}{4}$ [E] $\frac{8}{5}$ [F] $\frac{9}{8}$ [G] $\frac{11}{2}$

3. Give a mixed numeral for each point A through D.



Numbers and points

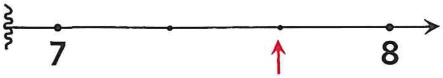
Often it is easier to think of a number for a point by thinking of a mixed numeral rather than an improper fraction.

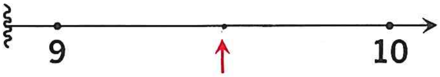


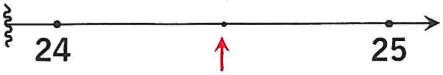
To find the number for the point over the red arrow, you could count by fourths and find 21 fourths. It may be easier just to think $5\frac{1}{4}$.

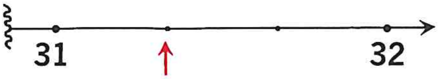
EXERCISES

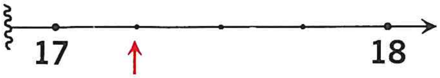
1. Give a mixed numeral for each exercise.

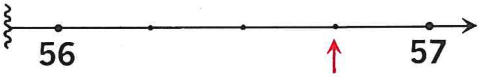
[A]


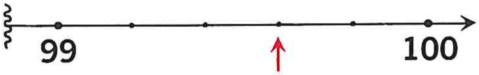
[B]


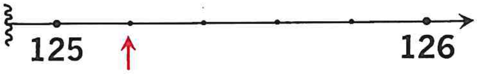
[C]


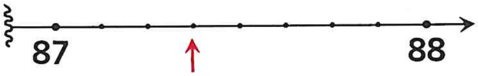
[D]


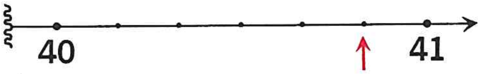
[E]


[F]


[G]


[H]


[I]


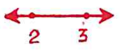
[J]


2. Give the correct sign (< or >) for each ▨.

[A] $2\frac{1}{2}$ ▨ 2
[B] $2\frac{1}{2}$ ▨ 3
[C] $5\frac{3}{4}$ ▨ 6
[D] $5\frac{3}{4}$ ▨ 5
[E] $7\frac{1}{8}$ ▨ $7\frac{3}{8}$
[F] $6\frac{3}{4}$ ▨ $7\frac{1}{4}$
[G] $2\frac{2}{3}$ ▨ $2\frac{1}{2}$
[H] $\frac{7}{8}$ ▨ 1

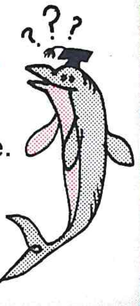
[I] 6 ▨ $5\frac{7}{8}$
[J] $9\frac{1}{2}$ ▨ $9\frac{3}{8}$
[K] 10 ▨ $9\frac{4}{5}$
[L] 13 ▨ $13\frac{1}{6}$
[M] $6\frac{7}{10}$ ▨ $6\frac{6}{10}$
[N] $3\frac{1}{10}$ ▨ $2\frac{9}{10}$
[O] $7\frac{9}{10}$ ▨ 8
[P] $8\frac{1}{10}$ ▨ 8

Think



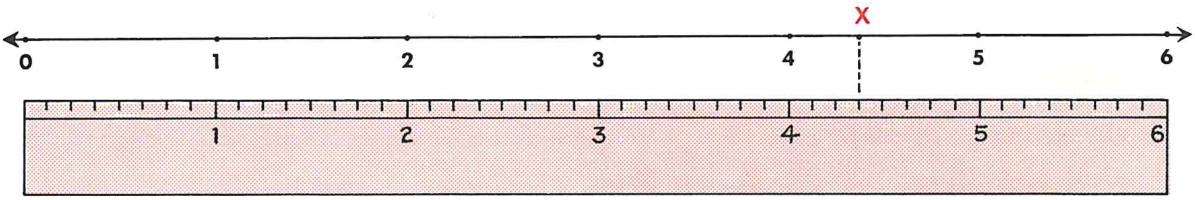
I'm more than two
And less than three.
Halfway between
You will find me.

WHO AM I?



Measurement

Using mixed numerals is very helpful in giving lengths. You can think of your ruler in the same way you think of a number line.

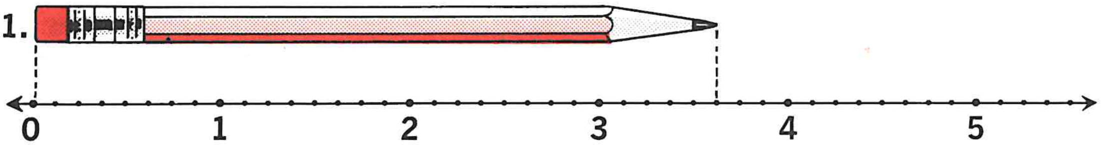


On most rulers the units are divided into eighths or sixteenths. Check to see how the units on the ruler above are divided. Now check your own ruler to see how it is divided. Notice in the picture that point X is $4\frac{3}{8}$ inches from the 0 point.

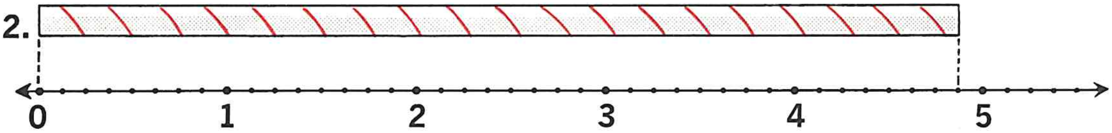
EXERCISES

The units on each number line are divided into eighths. Use mixed numerals to give the length of each object.

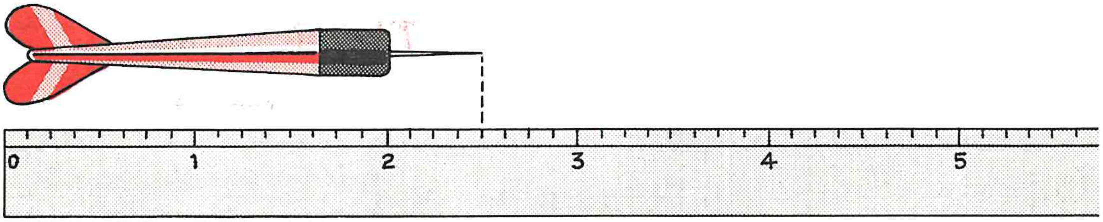
1.



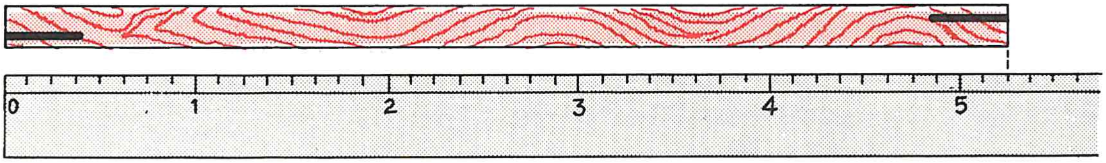
2.



3.

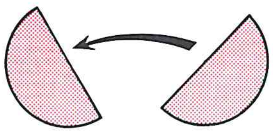


4.

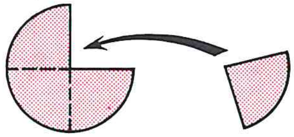


More about addition

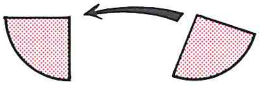
In your first work with addition of rational numbers, you used the number line. You can also use parts of objects to help you understand addition of rational numbers. Study the pictures below and try to give the sum as a whole number or as a lowest-terms fraction.



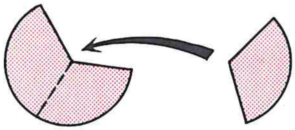
$\frac{1}{2}+\frac{1}{2}=\boxed{n}$



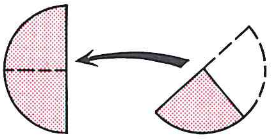
$\frac{3}{4}+\frac{1}{4}=\boxed{n}$



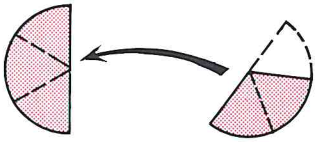
$\frac{1}{4}+\frac{1}{4}=\boxed{n}$



$\frac{2}{3}+\frac{1}{3}=\boxed{n}$



$\frac{1}{2}+\frac{1}{4}=\boxed{n}$



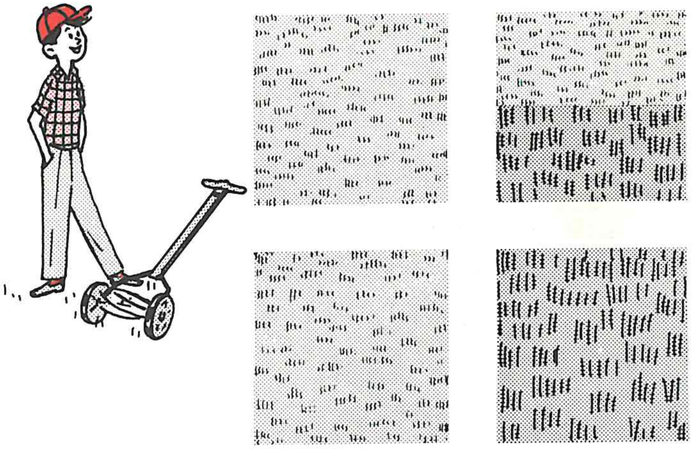
$\frac{1}{2}+\frac{1}{3}=\boxed{n}$

EXERCISES

Find the sums.

1. Since $\frac{1}{2}+\frac{1}{2}=1$, we know that $1\frac{1}{2}+\frac{1}{2}=\boxed{n}$.
2. Since $\frac{3}{4}+\frac{1}{4}=1$, we know that $2\frac{3}{4}+\frac{1}{4}=\boxed{n}$.
3. Since $\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$, we know that $3\frac{1}{4}+3\frac{1}{4}=\boxed{n}$.
4. Since $\frac{2}{3}+\frac{1}{3}=1$, we know that $2\frac{2}{3}+3\frac{1}{3}=\boxed{n}$.
5. Since $\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$, we know that $4\frac{1}{2}+1\frac{1}{4}=\boxed{n}$.
6. Since $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$, we know that $5\frac{1}{2}+2\frac{1}{3}=\boxed{n}$.

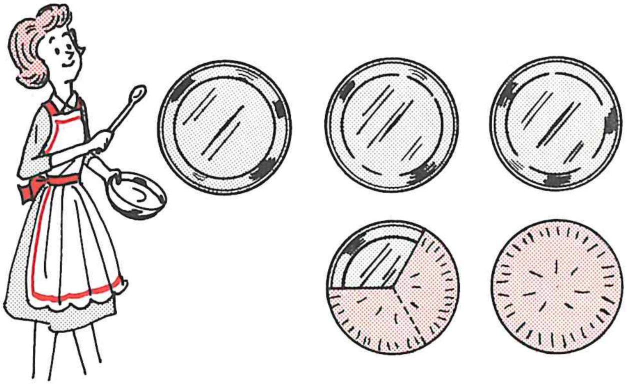
Rational-number problems



1. One Saturday, Jim had 4 lawns to mow. He mowed $2\frac{1}{2}$ lawns before lunch. How many lawns did he have left to mow in the afternoon? Solve these equations:

   [A] $2\frac{1}{2} + \boxed{n} = 4$

   [B] $4 - 2\frac{1}{2} = \boxed{n}$



2. Mrs. Brown baked 5 pies for a party. Only $3\frac{1}{3}$ pies were eaten. How many pies were left? Solve these equations:

   [A] $3\frac{1}{3} + \boxed{n} = 5$

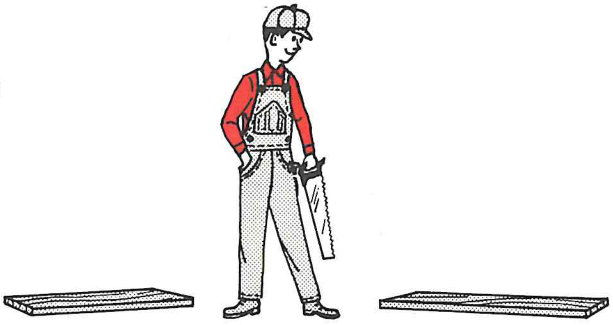
   [B] $5 - 3\frac{1}{3} = \boxed{n}$

3. Jane lived $3\frac{1}{2}$ blocks from Sally, and Sally lived $2\frac{1}{4}$ blocks from school. If Jane walked to Sally's house and then to school, how far did she walk? Solve this equation: ⟶ $3\frac{1}{2} + 2\frac{1}{4} = \boxed{n}$

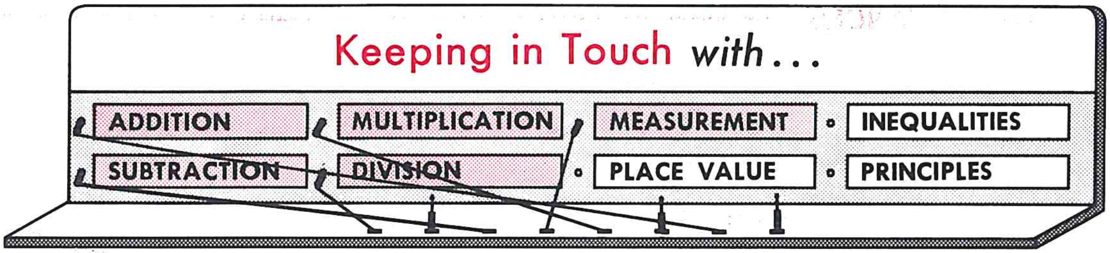
4. Mrs. White bought a beef roast that weighed $5\frac{1}{4}$ pounds and a steak that weighed $2\frac{1}{4}$ pounds. How many pounds of meat did she buy? Solve this equation: ⟶ $5\frac{1}{4} + 2\frac{1}{4} = \boxed{n}$

5. A carpenter cut a board into two pieces. One piece was $5\frac{1}{2}$ feet long, and the other was $4\frac{1}{3}$ feet long. How long was the board before he cut it? Solve this equation:

   $5\frac{1}{2} + 4\frac{1}{3} = \boxed{n}$



Keeping in Touch *with . . .*



1. Find the sums and differences.

[A] 27 + 58 [C] 728 + 694 [E] 9803 – 264 [G] 5043 – 3653
[B] 126 – 59 [D] 3427 + 96 [F] 5043 – 217 [H] 528 + 367 + 493

2. Find the products.

[A] 38 × 6 [C] 54 × 12 [E] 327 × 6 [G] 621 × 35 [I] 702 × 55
[B] 57 × 8 [D] 68 × 25 [F] 438 × 5 [H] 754 × 76 [J] 860 × 33

3. Find the quotients and remainders.

[A] 36 ÷ 4 [C] 68 ÷ 4 [E] 728 ÷ 5 [G] 52 ÷ 13 [I] 726 ÷ 91
[B] 75 ÷ 5 [D] 93 ÷ 6 [F] 436 ÷ 9 [H] 148 ÷ 23 [J] 343 ÷ 38

4. Find the total amounts.

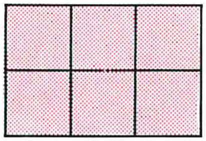
[A]	[B]	[C]	[D]	[E]
$6.23	$13.24	$33.02	$ 31.26	$ 76.38
5.37	.67	67.00	5.48	142.50
6.43	50.83	45.28	126.50	5.78

5. Find the difference of the amounts.

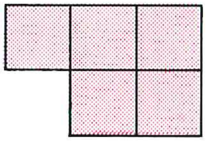
[A]	[B]	[C]	[D]	[E]
$9.28	$62.80	$37.86	$128.16	$675.24
5.67	5.43	29.59	54.37	99.98

6. Give the area and perimeter for each figure.

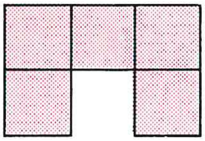
[A]



[B]



[C]

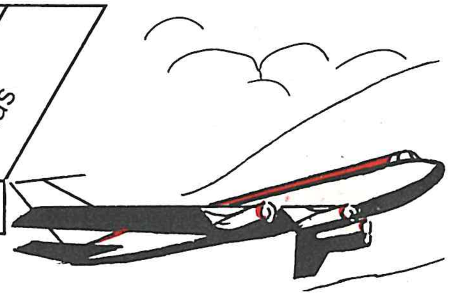


7. Which is more?

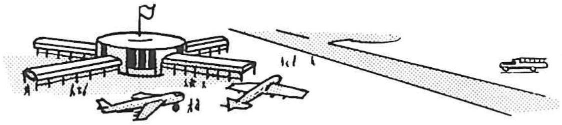
[A] 3 cups or 1 pint [C] 7 pints or 1 gallon [E] 3 gallons or 10 quarts
[B] 3 pints or 1 quart [D] 1 pint or 15 ounces [F] 50 cups or 3 gallons

AIRLINE DISTANCES

	New York	Miami	Los Angeles	Denver	Dallas
Chicago	713	1188	1745	920	803
Dallas	1374	1111	1240	663	
Denver	1631	1726	831		
Los Angeles	2451	2339			
Miami	1092				



The airline distance between Denver and Miami is 1726 miles.



EXERCISES

1. Use the chart to give the following airline distances.

[A] between Chicago and Denver
[B] between New York and Dallas
[C] between Miami and Dallas
[D] between New York and Miami
[E] between Miami and Los Angeles
[F] between Chicago and Los Angeles
[G] between Denver and New York
[H] between Los Angeles and New York

2. How much farther is it from Chicago to Los Angeles than it is from Chicago to Denver?

3. A salesman flew from New York to Miami, then from Miami to Denver and from Denver back to New York. How far did he travel?

4. If a plane takes 3 hours to fly from Chicago to Miami, what is its average speed in miles per hour?

5. An airliner made 7 round trips between Chicago and New York. How far did it fly?

★6. How much farther is it to go from Denver to Chicago to New York than it is to go from Denver to New York?

Chapter review

1. Give the missing words.

[**A**] For each set of equivalent fractions, there is one __?__ on the number line.

[**B**] For each rational number, there is one set of __?__ fractions.

[**C**] For each set of equivalent fractions, we think of just one __?__ number.

[**D**] To name a rational number, we can choose any __?__ from the set of equivalent fractions.

[**E**] The __?__ __?__ fraction is often used to name a rational number.

[**F**] If two fractions are __?__, then they name the same rational number.

[**G**] Each whole number is also a __?__ number.

[**H**] If two fractions name the same rational number, then they are __?__.

2. Match each set of fractions with a number-line picture.

[**A**] $\left\{\frac{1}{8}, \frac{2}{16}, \frac{3}{24}, \frac{4}{32}, \ldots\right\}$

[**B**] $\left\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\right\}$

[**C**] $\left\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{15}, \ldots\right\}$

[**D**] $\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\right\}$

[**E**] $\left\{\frac{5}{8}, \frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \ldots\right\}$

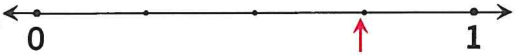
[**F**] $\left\{\frac{7}{10}, \frac{14}{20}, \frac{21}{30}, \frac{28}{40}, \ldots\right\}$

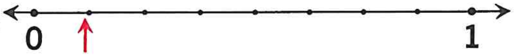
[**G**] $\left\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \ldots\right\}$

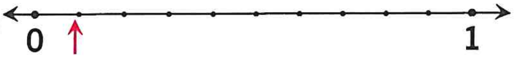
[**H**] $\left\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots\right\}$

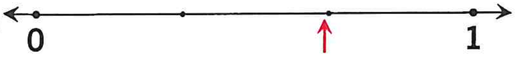
[**I**] $\left\{\frac{1}{6}, \frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \ldots\right\}$

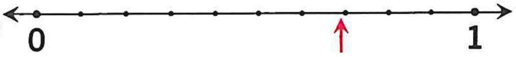
[**J**] $\left\{\frac{1}{10}, \frac{2}{20}, \frac{3}{30}, \frac{4}{40}, \ldots\right\}$

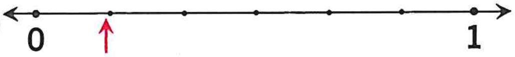
(1)


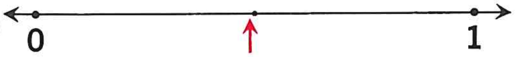
(2)


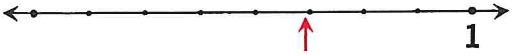
(3)


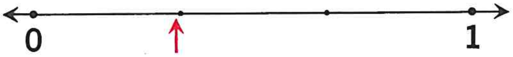
(4)


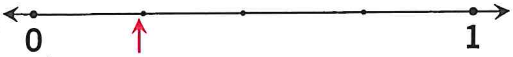
(5)


(6)


(7)


(8)


(9)


(10)


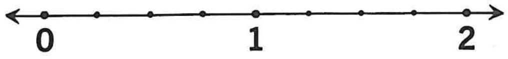
3. Give the missing numerators and denominators.

[A] $\frac{1}{2} = \frac{5}{\text{▥}}$ [D] $\frac{\text{▥}}{15} = \frac{1}{5}$ [G] $\frac{3}{4} = \frac{30}{\text{▥}}$ [J] $\frac{\text{▥}}{100} = \frac{3}{10}$ [M] $\frac{6}{7} = \frac{\text{▥}}{14}$

[B] $\frac{4}{\text{▥}} = \frac{1}{3}$ [E] $\frac{1}{8} = \frac{\text{▥}}{40}$ [H] $\frac{\text{▥}}{50} = \frac{2}{5}$ [K] $\frac{2}{9} = \frac{\text{▥}}{36}$ [N] $\frac{5}{\text{▥}} = \frac{50}{80}$

[C] $\frac{1}{4} = \frac{\text{▥}}{40}$ [F] $\frac{\text{▥}}{6} = \frac{2}{3}$ [I] $\frac{3}{8} = \frac{\text{▥}}{24}$ [L] $\frac{40}{\text{▥}} = \frac{4}{5}$ [O] $\frac{3}{10} = \frac{\text{▥}}{50}$

4. Copy the number line on your paper and give the missing fractions. Use only lowest-terms fractions.



5. Answer **T** (true) or **F** (false).

[A] $\frac{1}{2} > \frac{1}{4}$ [D] $\frac{5}{8} > \frac{1}{2}$ [G] $\frac{1}{4} < \frac{1}{3}$

[B] $\frac{1}{3} < \frac{1}{4}$ [E] $\frac{2}{3} < \frac{1}{2}$ [H] $\frac{4}{6} > \frac{1}{3}$

[C] $\frac{1}{8} < \frac{1}{4}$ [F] $\frac{3}{4} > \frac{1}{2}$ [I] $\frac{1}{6} < \frac{1}{3}$

6. Give a mixed numeral for each sum.

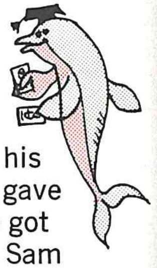
[A] $5 + \frac{1}{3}$ [B] $\frac{1}{4} + 2$ [C] $6 + \frac{1}{7}$

7. Give a lowest-terms fraction or a whole number for each sum.

[A] $\frac{1}{2} + \frac{1}{2}$ [C] $\frac{1}{4} + \frac{1}{4}$ [E] $\frac{1}{4} + \frac{1}{2}$

[B] $\frac{1}{4} + \frac{3}{4}$ [D] $\frac{1}{3} + \frac{2}{3}$ [F] $\frac{1}{3} + \frac{1}{2}$

Think



Fred gave Tom $\frac{1}{2}$ of his baseball cards. Tom gave Ned $\frac{1}{2}$ of the cards he got from Fred. Ned gave Sam $\frac{1}{2}$ the cards he got from Tom. Sam got only 6 cards. How many did Fred start with?

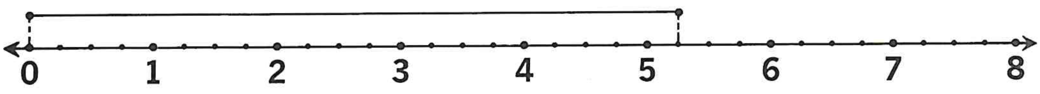
8. Give the correct sign (>, =, or <) for each ▥.

[A] 3 ▥ $3\frac{1}{4}$ [C] $\frac{5}{4}$ ▥ $\frac{6}{4}$ [E] $\frac{10}{4}$ ▥ $\frac{5}{2}$ [G] $2\frac{1}{3}$ ▥ $\frac{7}{3}$

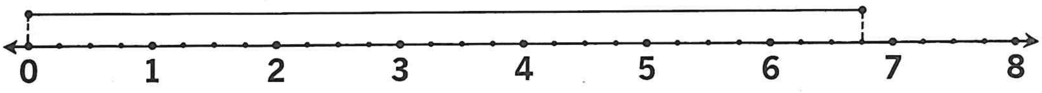
[B] $2\frac{7}{8}$ ▥ 3 [D] $\frac{5}{4}$ ▥ $\frac{5}{3}$ [F] $7\frac{1}{8}$ ▥ $6\frac{7}{8}$ [H] $\frac{10}{4}$ ▥ $2\frac{1}{2}$

9. Give the length of each segment.

[A]



[B]



Supplementary Exercises

Set 1 *For use after page 3*

Ⓐ

	a	b	c	d
1.	$36 = 30 + \boxed{n}$	$93 = \boxed{n} + 3$	$72 = 70 + \boxed{n}$	$37 = \boxed{n} + 7$
2.	$76 = \boxed{n} + 6$	$85 = \boxed{n} + 80$	$47 = 7 + \boxed{n}$	$99 = \boxed{n} + 9$

Ⓑ

1.	$67 = 60 + \boxed{n}$	$29 = \boxed{n} + 9$	$83 = \boxed{n} + 3$	$54 = 50 + \boxed{n}$
2.	$85 = \boxed{n} + 80$	$32 = 2 + \boxed{n}$	$89 = \boxed{n} + 9$	$19 = 9 + \boxed{n}$

CODED ANSWERS, SET 1, PART A: 1. (a) G, (b) IB, (c) C, (d) DB; **2.** (a) HB, (b) E, (c) FB, (d) IB

Set 2 *For use after page 5*

Ⓐ

	a	b	c
1.	$563 = \boxed{n} + 60 + 3$	$879 = 800 + \boxed{n} + 9$	$675 = 600 + 70 + \boxed{n}$
2.	$416 = 400 + \boxed{n} + 10$	$539 = \boxed{n} + 500 + 9$	$407 = 7 + 400 + \boxed{n}$

Ⓑ

1.	$327 = 300 + 20 + \boxed{n}$	$866 = 800 + \boxed{n} + 6$	$374 = \boxed{n} + 70 + 4$
2.	$521 = 1 + 20 + \boxed{n}$	$602 = \boxed{n} + 2 + 600$	$999 = 900 + \boxed{n} + 9$

CODED ANSWERS, SET 2, PART A: 1. (a) JDD, (b) GD, (c) J; **2.** (a) I, (b) HD, (c) D

Set 3 *For use after page 7*

Ⓐ

	a	b
1.	$3476 = 3000 + 400 + 70 + \boxed{n}$	$8572 = 8000 + \boxed{n} + 70 + 2$
2.	$4396 = \boxed{n} + 300 + 90 + 6$	$6713 = 6000 + 700 + \boxed{n} + 3$
3.	$5070 = 5000 + \boxed{n} + 70$	$3987 = \boxed{n} + 80 + 7 + 900$

Ⓑ

1.	$5347 = \boxed{n} + 300 + 40 + 7$	$6328 = 6000 + \boxed{n} + 20 + 8$
2.	$4165 = 4000 + 100 + \boxed{n} + 5$	$3219 = 3000 + 200 + 10 + \boxed{n}$
3.	$5989 = \boxed{n} + 5000 + 9 + 80$	$4307 = 4000 + 300 + \boxed{n} + 7$

CODED ANSWERS, SET 3, PART A: 1. (a) B, (b) JGG; **2.** (a) HGGG, (b) AG; **3.** (a) G, (b) CGGG

Set 4 *For use after page 23*

Ⓐ

	a	b	c	d
1.	$5+9=\boxed{n}$	$6+\boxed{n}=9$	$\boxed{n}+8=11$	$8+6=\boxed{n}$
2.	$\boxed{n}+7=15$	$9+6=\boxed{n}$	$7+\boxed{n}=12$	$\boxed{n}+8=17$
3.	$7+6=\boxed{n}$	$\boxed{n}+7=16$	$9+\boxed{n}=18$	$9+\boxed{n}=14$
4.	$13=5+\boxed{n}$	$\boxed{n}=4+9$	$11=\boxed{n}+3$	$\boxed{n}+8=16$
5.	$7+8=\boxed{n}$	$3+\boxed{n}=12$	$11=6+\boxed{n}$	$6+9=\boxed{n}$
6.	$9+\boxed{n}=17$	$13=8+\boxed{n}$	$6+\boxed{n}=14$	$\boxed{n}=7+7$

Ⓑ

1.	$8+6=\boxed{n}$	$9+\boxed{n}=16$	$\boxed{n}+4=11$	$8+7=\boxed{n}$
2.	$9+\boxed{n}=12$	$2+9=\boxed{n}$	$7+9=\boxed{n}$	$\boxed{n}+6=13$
3.	$8+5=\boxed{n}$	$4+\boxed{n}=12$	$5+8=\boxed{n}$	$\boxed{n}+7=14$
4.	$\boxed{n}=8+6$	$17=8+\boxed{n}$	$16=\boxed{n}+7$	$7+\boxed{n}=13$
5.	$\boxed{n}+4=13$	$11=\boxed{n}+5$	$\boxed{n}+7=10$	$3+\boxed{n}=11$
6.	$\boxed{n}+6=6$	$9+9=\boxed{n}$	$7+\boxed{n}=15$	$\boxed{n}+6=14$

CODED ANSWERS, SET 4, PART A: **1.** (a) IF, (b) B, (c) B, (d) IF; **2.** (a) A, (b) IC, (c) C, (d) J; **3.** (a) IB, (b) J, (c) J, (d) C; **4.** (a) A, (b) IB, (c) A, (d) A; **5.** (a) IC, (b) J, (c) C, (d) IC; **6.** (a) A, (b) C, (c) A, (d) IF

Set 5 *For use after page 25*

Ⓐ

	a	b	c	d
1.	$5+\boxed{n}=13$	$13-5=\boxed{n}$	$\boxed{n}+5=14$	$14-5=\boxed{n}$
2.	$13-6=\boxed{n}$	$\boxed{n}+9=14$	$14-9=\boxed{n}$	$18-9=\boxed{n}$
3.	$10-\boxed{n}=6$	$14-8=\boxed{n}$	$\boxed{n}+8=14$	$\boxed{n}-7=5$
4.	$15-9=\boxed{n}$	$16-\boxed{n}=8$	$13-7=\boxed{n}$	$14-\boxed{n}=5$
5.	$\boxed{n}-9=6$	$6+9=\boxed{n}$	$\boxed{n}-5=9$	$15-8=\boxed{n}$
6.	$16-\boxed{n}=9$	$17-9=\boxed{n}$	$\boxed{n}=15-6$	$8=14-\boxed{n}$

Ⓑ

1.	$8+\boxed{n}=13$	$13-8=\boxed{n}$	$\boxed{n}+7=15$	$15-7=\boxed{n}$
2.	$15-6=\boxed{n}$	$\boxed{n}+6=14$	$14-6=\boxed{n}$	$17-9=\boxed{n}$
3.	$13-6=\boxed{n}$	$11-\boxed{n}=7$	$\boxed{n}-7=9$	$\boxed{n}+7=16$
4.	$\boxed{n}-6=6$	$16-\boxed{n}=9$	$14-\boxed{n}=7$	$16-9=\boxed{n}$
5.	$15-\boxed{n}=15$	$\boxed{n}-9=2$	$11-\boxed{n}=4$	$16-8=\boxed{n}$
6.	$\boxed{n}-8=9$	$14-\boxed{n}=9$	$6=\boxed{n}-7$	$8=13-\boxed{n}$

CODED ANSWERS, SET 5, PART A: **1.** (a) E, (b) E, (c) G, (d) G; **2.** (a) C, (b) J, (c) J, (d) G; **3.** (a) B, (b) H, (c) H, (d) DF; **4.** (a) H, (b) E, (c) H, (d) G; **5.** (a) DJ, (b) DJ, (c) DB, (d) C; **6.** (a) C, (b) E, (c) G, (d) H

Set 6 *For use after page 34*

A	a	b	c	d	e	f	g	h	i	j
	5	8	8	9	9	8	7	8	9	6
	7	6	7	3	7	5	8	8	6	9
	+9	+9	+7	+8	+9	+7	+9	+8	+7	+8

B										
	8	9	8	5	8	8	9	5	8	9
	4	5	6	9	3	7	9	9	8	6
	+9	+7	+7	+8	+7	+8	+9	+9	+9	+9

CODED ANSWERS, SET 6, PART A: (a) AG, (b) AH, (c) AA, (d) AE, (e) AI, (f) AE, (g) AC, (h) AC, (i) AA, (j) AH

Set 7 *For use after page 35*

A	a	b	c	d	e	f	g	h	i	j
1.	5	4	9	9	9	7	6	9	9	7
	7	9	5	8	5	3	9	4	8	6
	6	2	8	7	7	9	7	6	3	0
	+9	+5	+5	+6	+6	+8	+8	+9	+8	+6
2.	4	6	7	9	8	4	6	5	7	9
	8	9	7	8	6	5	4	3	2	8
	3	1	2	9	9	8	6	7	5	4
	+2	+3	+8	+6	+5	+7	+9	+8	+9	+6

B	a	b	c	d	e	f	g	h	i	j	k
1.	4	9	5	3	2	8	4	9	9	6	8
	3	4	8	6	9	7	5	4	8	7	6
	7	8	9	6	8	4	3	9	8	9	2
	+9	+2	+1	+7	+5	+6	+8	+4	+3	+0	+7

2.	6	4	8	6	9	9	8	7	8	9
	5	8	7	6	5	9	8	7	8	9
	7	9	4	5	6	7	8	9	8	9
	+8	+3	+7	+6	+9	+5	+4	+2	+8	+9

CODED ANSWERS, SET 7, PART A: 1. (a) EA, (b) EF, (c) EA, (d) GF, (e) EA, (f) EA, (g) GF, (h) EC, (i) EC, (j) DJ; **2.** (a) DA, (b) DJ, (c) EB, (d) GE, (e) EC, (f) EB, (g) EH, (h) EG, (i) EG, (j) EA

Set 8 *For use after page 35*

Ⓐ	a	b	c
1.	$6+9+8+8=\boxed{n}$	$5+7+7+9=\boxed{n}$	$7+8+9+8=\boxed{n}$
2.	$80+70+60=\boxed{n}$	$90+90+60=\boxed{n}$	$70+60+80=\boxed{n}$
3.	$56+9+8=\boxed{n}$	$67+6+4=\boxed{n}$	$88+7+6=\boxed{n}$
4.	$86+9+7=\boxed{n}$	$158+6+9=\boxed{n}$	$137+9+5=\boxed{n}$

Ⓑ

1.	$8+9+5=\boxed{n}$	$9+9+5=\boxed{n}$	$8+9+6=\boxed{n}$
2.	$40+70+90=\boxed{n}$	$50+60+80=\boxed{n}$	$90+90+60=\boxed{n}$
3.	$46+9+7=\boxed{n}$	$64+9+5=\boxed{n}$	$86+8+9=\boxed{n}$
4.	$138+5+7=\boxed{n}$	$253+8+4=\boxed{n}$	$417+8+7=\boxed{n}$

CODED ANSWERS, SET 8, PART A: **1.** (a) CE, (b) JG, (c) CJ; **2.** (a) JED, (b) JFD, (c) JED; **3.** (a) IC, (b) II, (c) EDE; **4.** (a) EDJ, (b) EIC, (c) EHE

Set 9 *For use after page 37*

Ⓐ	a	b	c	d	e	f	g	h	i
1.	36 +45	53 +49	67 +56	84 +59	65 +86	78 +57	97 +48	89 +66	96 +38
2.	35 64 +23	33 57 +78	45 96 +67	64 70 +89	46 36 +97	49 53 +64	89 31 +79	54 56 +98	99 85 +47

Ⓑ

1.	43 +28	66 +48	59 +84	68 +75	57 +76	86 +48	99 +37	28 +98	86 +79
2.	48 39 +55	36 77 +65	27 47 +53	78 33 +59	47 64 +88	34 76 +95	25 86 +45	97 78 +56	89 67 +48
3.	73 67 +96	43 58 +84	99 84 +93	78 68 +87	87 46 +97	54 69 +75	73 80 +98	99 99 +99	89 76 +85

CODED ANSWERS, SET 9, PART A: **1.** (a) HG, (b) GIE, (c) GEA, (d) GCA, (e) GBG, (f) GAB, (g) GCB, (h) GBB, (i) GAC; **2.** (a) GEE, (b) GDH, (c) EIH, (d) EEA, (e) GFJ, (f) GDD, (g) GJJ, (h) EIH, (i) EAG

Set 10

For use after page 39

	a	b	c	d	e	f	g	h
A	426 +564	378 +483	507 +563	614 +496	834 +768	976 +457	638 +927	858 +937
B	573 +327	538 +423	675 +398	417 +895	629 +586	879 +654	469 +936	807 +996

CODED ANSWERS, SET 10, PART A: (a) IIB, (b) JGA, (c) ABHB, (d) AAAB, (e) AGBC, (f) AFDD, (g) AEGE, (h) AHIE

Set 11

For use after page 39

	a	b	c	d	e	f	g
A	4375 +2765	4832 +6549	5804 +3679	3768 +5954	6548 +7693	8743 +5697	9989 +8898
B	3624 +4567	4655 +5847	6038 +5967	7486 +5938	6785 +4389	8387 +9989	7648 +8395

CODED ANSWERS, SET 11, PART A: (a) GBAD, (b) BBHEB, (c) CAEH, (d) CGFF, (e) BAFAB, (f) BAAAD, (g) BEEEG

Set 12

For use after page 39

	a	b	c	d	e	f
A	34,785 +56,427	53,949 +49,038	64,838 +58,147	85,729 +67,256	76,617 +76,365	97,508 +85,479
B	43,356 +42,866	62,473 +58,778	51,584 +69,689	78,679 +77,594	97,487 +86,479	86,598 +95,386

CODED ANSWERS, SET 12, PART A: (a) IAFAF, (b) AGFIDE, (c) AFFIDJ, (d) AJFIDJ, (e) AJFIDF, (f) ADFIDE

Set 13

For use after page 39

	a	b	c	d
A	$563 + 79 = \boxed{n}$	$638 + 57 = \boxed{n}$	$5487 + 369 = \boxed{n}$	$8764 + 487 = \boxed{n}$
B	$648 + 35 = \boxed{n}$	$523 + 99 = \boxed{n}$	$8756 + 596 = \boxed{n}$	$6899 + 376 = \boxed{n}$

CODED ANSWERS, SET 13, PART A: (a) GFH, (b) GJC, (c) CACG, (d) JHCI

Set 14 *For use after page 40*

A

	a	b	c	d	e	f	g
1.	437 46 +5	639 417 +63	587 46 +394	786 497 +385	628 479 +853	5783 4969 +3807	8796 4357 +9438
2.	563 49 278 +7	537 685 29 +347	364 876 957 +138	878 349 565 +476	939 874 348 +763	8406 9327 4584 +3965	7965 9838 3107 +4782

B

	a	b	c	d	e	f	g
1.	52 349 +76	437 58 +9	865 437 +942	397 654 +836	798 374 +287	3468 6859 +4307	8799 8365 +4879
2.	567 38 298 +324	583 385 964 +469	283 479 263 +576	817 394 487 +635	639 978 684 +376	6847 3509 8458 +6542	3847 4158 3687 +4959

CODED ANSWERS, SET 14, PART A: 1. (a) BEE, (b) DDDG, (c) DAFC, (d) DHHE, (e) DGHA, (f) DBJJG, (g) FFJGD; **2.** (a) EGC, (b) DJGE, (c) FIIJ, (d) FFHE, (e) FGFB, (f) FHFEF, (g) FJHGF

Set 15 *For use after page 40*

Copy the problems. Write the missing digits instead of the ▥.

	a	b	c	d	e	f	g	h
1.	6▥ +▥4 97	▥9 +62 121	17▥ +▥5 207	16▥ +▥9 194	▥42 +▥9 38▥	▥34 +8▥▥ 1396	8▥▥ +▥47 1197	▥46 +5▥▥ 873
2.	▥▥ +39 107	783 +▥▥ 840	8▥2 +9▥ ▥37	3▥7 +▥48 95▥	▥5▥ +4▥9 1322	937 +▥▥▥ 1477	63▥ +2▥8 ▥26	▥76 +9▥4 156▥
3.	32 +▥▥ 131	▥4▥2 +57▥ 7▥50	▥0▥7 +2▥6▥ 7406	▥▥ +89 146	843 +▥▥▥ 1000	96 +▥▥ 165	8,▥▥7 +3,65▥ ▥▥,699	9▥ +9▥ 198

Set 16 *For use after page 42*

Find the total amounts.

Ⓐ

	a	b	c	d	e	f	g
1.	$3.14 6.95	$4.36 5.42	$6.86 7.54	$8.53 3.47	$14.38 9.76	$15.32 6.29	$8.56 .87
2.	$5.43 .62	$16.74 8.93	$ 9.86 14.49	$25.76 9.35	$16.49 6.36	$7.54 9.39	$38.76 18.85

Ⓑ

	a	b	c	d	e	f	g
1.	$4.91 3.13	$ 5.07 14.98	$5.43 6.65	$17.08 4.39	$6.99 9.49	$8.63 15.54	$7.99 .99
2.	$5.08 .37	$33.68 9.49	$8.98 6.98	$ 7.49 18.49	$18.62 9.37	$25.48 6.88	$47.88 53.88

CODED ANSWERS, SET 16, PART A: 1. (a) GE.EF, (b) F.JD, (c) GC.CE ,(d) GA.EE, (e) AC.GC, (f) AG.BG, (g) F.CH; **2.** (a) B.EI, (b) AI.BJ, (c) AC.HI, (d) HI.GG, (e) AA.DI, (f) GB.FH, (g) IJ.BG

Set 17 *For use after page 48*

Ⓐ

	a	b	c	d
1.	$63 = 50 + \boxed{n}$	$58 = 40 + \boxed{n}$	$79 = 60 + \boxed{n}$	$83 = 70 + \boxed{n}$
2.	$49 = 30 + \boxed{n}$	$96 = 80 + \boxed{n}$	$38 = 20 + \boxed{n}$	$27 = 10 + \boxed{n}$
3.	$34 = \boxed{n} + 14$	$75 = \boxed{n} + 15$	$86 = \boxed{n} + 16$	$68 = \boxed{n} + 18$
4.	$427 = 410 + \boxed{n}$	$738 = 720 + \boxed{n}$	$476 = 460 + \boxed{n}$	$353 = 340 + \boxed{n}$
5.	$430 = 300 + \boxed{n}$	$560 = 400 + \boxed{n}$	$870 = 700 + \boxed{n}$	$930 = 800 + \boxed{n}$
6.	$570 = \boxed{n} + 170$	$630 = \boxed{n} + 130$	$840 = \boxed{n} + 140$	$790 = \boxed{n} + 190$

Ⓑ

	a	b	c	d
1.	$54 = 40 + \boxed{n}$	$65 = 50 + \boxed{n}$	$87 = 70 + \boxed{n}$	$43 = 30 + \boxed{n}$
2.	$92 = 80 + \boxed{n}$	$37 = 20 + \boxed{n}$	$59 = 40 + \boxed{n}$	$24 = 10 + \boxed{n}$
3.	$58 = \boxed{n} + 18$	$73 = \boxed{n} + 13$	$81 = \boxed{n} + 11$	$95 = \boxed{n} + 15$
4.	$364 = 350 + \boxed{n}$	$847 = 830 + \boxed{n}$	$962 = 950 + \boxed{n}$	$658 = 640 + \boxed{n}$
5.	$580 = 400 + \boxed{n}$	$840 = 700 + \boxed{n}$	$750 = 600 + \boxed{n}$	$960 = 800 + \boxed{n}$
6.	$880 = \boxed{n} + 180$	$270 = \boxed{n} + 100$	$530 = \boxed{n} + 130$	$640 = \boxed{n} + 140$

CODED ANSWERS, SET 17, PART A: 1. (a) DG, (b) DC, (c) DJ, (d) DG; **2.** (a) DJ, (b) DI, (c) DC, (d) DA; **3.** (a) EF, (b) IF, (c) AF, (d) HF; **4.** (a) DA, (b) DC, (c) DI, (d) DG; **5.** (a) DGF, (b) DIF, (c) DAF, (d) DGF; **6.** (a) BFF, (b) HFF, (c) AFF, (d) IFF

Set 18 *For use after page 49*

	a	b	c	d	e	f	g	h	i
Ⓐ	54 −36	73 −9	83 −35	65 −47	74 −19	66 −48	53 −37	40 −25	87 −49
Ⓑ	47 −18	86 −8	64 −38	32 −17	81 −49	70 −62	63 −47	54 −28	96 −69

CODED ANSWERS, SET 18, PART A: (a) EG, (b) AF, (c) FG, (d) EG, (e) HH, (f) EG, (g) EA, (h) EH, (i) CG

Set 19 *For use after page 52*

	a	b	c	d	e	f	g
Ⓐ 1.	543 −57	462 −187	671 −63	884 −546	763 −268	9438 −6189	67,542 −15,673
2.	846 −457	572 −208	416 −397	735 −708	643 −576	9563 −3894	84,286 −46,789
Ⓑ 1.	733 −353	454 −328	676 −87	365 −176	522 −436	8742 −967	95,768 −35,899
2.	611 −584	840 −323	727 −458	450 −151	364 −265	7436 −2879	64,153 −13,697

CODED ANSWERS, SET 19, PART A: **1.** (a) CHD, (b) EFB, (c) DIH, (d) AAH, (e) CJB, (f) AECJ, (g) BGHDJ; **2.** (a) AHJ, (b) ADC, (c) GJ, (d) EF, (e) DF, (f) BDDJ, (g) AFCJF

Set 20 *For use after page 52*

	a	b	c
Ⓐ 1.	$568 - 9 = \boxed{n}$	$435 + 654 + 78 = \boxed{n}$	$847 - 50 = \boxed{n}$
2.	$436 - 428 = \boxed{n}$	$365 + 47 + 9 = \boxed{n}$	$648 - 39 = \boxed{n}$
Ⓑ 1.	$836 - 8 = \boxed{n}$	$347 + 298 + 36 = \boxed{n}$	$764 - 58 = \boxed{n}$
2.	$553 - 548 = \boxed{n}$	$58 + 326 + 8 = \boxed{n}$	$632 - 26 = \boxed{n}$

CODED ANSWERS, SET 20, PART A: 1. (a) EEI, (b) AAGH, (c) HIH; **2.** (a) J, (b) FCA, (c) GBI

Set 21 *For use after page 52*

Find the difference in the amounts given. Check your work.

Ⓐ	a	b	c	d	e	f	g
1.	$6.97 .83	$8.65 3.86	$9.42 .98	$10.43 8.75	$15.27 6.38	$9.46 3.83	$34.57 7.63
2.	$8.65 2.39	$19.63 13.56	$38.21 29.39	$16.54 7.82	$7.86 3.96	$5.17 1.98	$15.67 6.88

Ⓑ							
1.	$7.76 .64	$9.43 .89	$6.76 3.48	$11.42 5.67	$18.31 5.49	$7.63 2.49	$23.21 7.54
2.	$6.76 .93	$14.86 .97	$36.48 .58	$49.67 3.85	$21.17 4.38	$8.79 3.97	$53.23 17.45

CODED ANSWERS, SET 21, PART A: 1. (a) I.BA, (b) A.GC, (c) E.AA, (d) B.IE, (e) E.EC, (f) J.IH, (g) FI.CA; **2.** (a) I.FI, (b) I.DG, (c) E.EF, (d) E.GF, (e) H.CD, (f) H.BC, (g) E.GC

Set 22 *For use after page 58*

Ⓐ	a	b	c	d	e	f
1.	400 −376	508 −276	1600 −463	1403 −1074	1005 −937	1701 −1355
2.	303 −165	4204 −2475	3002 −1999	6034 −2447	4000 −2333	3200 −1655

	a	b	c	d
3.	307 − 48 = n	400 − 156 = n	1504 − 98 = n	3007 − 58 = n

Ⓑ						
1.	306 −189	200 −58	1503 −848	1700 −1177	2003 −947	1604 −764
2.	800 −401	4300 −3400	6004 −3775	2503 −1658	4062 −2739	5000 −3642

3.	500 − 198 = n	603 − 98 = n	1307 − 49 = n	4060 − 86 = n

CODED ANSWERS, SET 22, PART A: 1. (a) FH, (b) FCF, (c) AACE, (d) CFI, (e) BD, (f) CHB; **2.** (a) ACD, (b) AEFI, (c) AGGC, (d) CJDE, (e) ABBE, (f) AJHJ; **3.** (a) FJI, (b) FHH, (c) AHGB, (d) FIHI

Set 23 *For use after page 86—Multiplication review*

	a	b	c	d	e	f	g
1.	2×4	1×6	3×9	9×2	6×7	5×1	6×8
2.	1×9	8×3	7×5	0×0	4×6	1×8	4×4
3.	3×3	7×1	8×6	0×8	2×5	2×6	5×4
4.	7×2	6×9	8×1	3×7	4×2	6×5	9×0
5.	5×5	2×2	5×9	1×7	8×4	3×1	6×4
6.	0×6	2×8	4×8	6×2	7×3	1×3	5×3
7.	4×5	4×3	2×1	7×0	9×5	6×1	5×7
8.	1×4	2×7	7×8	9×3	9×9	5×2	4×7
9.	9×7	3×6	8×9	4×1	2×3	8×8	9×6
10.	7×4	3×8	8×2	5×0	9×8	8×5	1×1
11.	6×3	9×1	3×5	8×7	4×0	9×4	0×1
12.	5×8	1×2	5×6	1×5	4×9	7×9	0×3
13.	7×7	2×9	7×6	3×2	6×6	3×4	6×0

Set 24 *For use after page 93—Division review*

	a	b	c	d	e	f
1.	5÷1	10÷5	30÷5	5÷5	18÷6	56÷8
2.	21÷7	28÷4	24÷3	49÷7	27÷9	36÷6
3.	2÷1	6÷3	8÷4	36÷9	0÷5	14÷7
4.	40÷8	16÷2	54÷6	56÷7	4÷1	12÷2
5.	18÷3	24÷6	6÷2	2÷2	7÷7	6÷1
6.	72÷8	35÷7	6÷6	32÷4	14÷2	18÷9
7.	42÷7	54÷9	3÷1	15÷3	25÷5	9÷3
8.	12÷4	30÷6	15÷5	64÷8	18÷2	63÷9
9.	9÷1	35÷5	24÷8	24÷4	10÷2	20÷4
10.	12÷3	4÷2	81÷9	72÷9	12÷6	36÷4
11.	8÷1	40÷5	4÷4	0÷9	28÷7	21÷3
12.	48÷8	0÷1	20÷5	1÷1	9÷9	27÷3
13.	45÷5	7÷1	16÷8	45÷9	63÷7	8÷8
14.	32÷8	48÷6	3÷3	8÷2	42÷6	16÷4

Set 25 *For use after page 93*

A

	a	b	c	d
1.	$5 \times \boxed{n} = 40$	$\boxed{n} \times 8 = 32$	$56 = \boxed{n} \times 8$	$72 = 8 \times \boxed{n}$
2.	$40 \div 5 = \boxed{n}$	$32 \div \boxed{n} = 8$	$\boxed{n} \div 8 = 7$	$72 \div \boxed{n} = 9$
3.	$\boxed{n} \times 9 = 54$	$54 \div 9 = \boxed{n}$	$54 \div \boxed{n} = 9$	$48 = \boxed{n} \times 6$
4.	$7 \times \boxed{n} = 42$	$42 \div \boxed{n} = 6$	$\boxed{n} = 63 \div 9$	$9 = 81 \div \boxed{n}$

B

1.	$9 \times \boxed{n} = 36$	$30 = 6 \times \boxed{n}$	$\boxed{n} \times 4 = 28$	$54 = 9 \times \boxed{n}$
2.	$\boxed{n} \div 9 = 4$	$\boxed{n} = 36 \div 4$	$54 \div \boxed{n} = 6$	$\boxed{n} \div 9 = 5$
3.	$21 = 7 \times \boxed{n}$	$21 \div \boxed{n} = 7$	$\boxed{n} \div 7 = 3$	$0 = 8 \times \boxed{n}$
4.	$9 \times \boxed{n} = 63$	$\boxed{n} \div 7 = 9$	$48 \div \boxed{n} = 6$	$6 = \boxed{n} \div 4$

CODED ANSWERS, SET 25, PART A: 1. (a) A, (b) F, (c) E, (d) J; **2.** (a) A, (b) F, (c) CG, (d) A; **3.** (a) G, (b) G, (c) G, (d) A; **4.** (a) G, (b) E, (c) E, (d) J

Set 26 *For use after page 135*

A

	a	b	c	d	e	f
1.	9 × 10	10 × 10	5 × 100	10 × 100	5 × 1000	10 × 1000
2.	53 × 10	30 × 10	71 × 10	192 × 10	51 × 100	63 × 1000
3.	47 × 10	10 × 47	136 × 10	10 × 84	100 × 36	41 × 1000
4.	0 × 100	1 × 1000	54 × 100	100 × 80	40 × 10	68 × 100
5.	100 × 39	10 × 43	561 × 10	61 × 10	100 × 42	841 × 1000
6.	50 × 10	100 × 60	10 × 90	36 × 10	100 × 17	960 × 10

B

1.	4 × 10	11 × 10	12 × 100	7 × 1000	100 × 10	6 × 1000
2.	42 × 10	20 × 10	37 × 10	135 × 10	63 × 100	67 × 1000
3.	54 × 10	10 × 54	10 × 157	100 × 9	93 × 100	1000 × 71
4.	436 × 10	29 × 100	100 × 20	0 × 10	1000 × 31	5671 × 10
5.	40 × 10	100 × 30	387 × 100	90 × 10	430 × 10	10 × 4760
6.	54 × 100	20 × 10	100 × 60	10 × 230	417 × 10	62 × 1000

CODED ANSWERS, SET 26, PART A: 1. (a) GA, (b) DAA, (c) JAA, (d) DAAA, (e) JAAA, (f) DAAAA; **2.** (a) JIA, (b) IAA, (c) CDA, (d) DGFA, (e) JDAA, (f) HIAAA; **3.** (a) BCA, (b) BCA, (c) DIHA, (d) EBA, (e) IHAA, (f) BDAAA; **4.** (a) A, (b) DAAA, (c) JBAA, (d) EAAA, (e) BAA, (f) HEAA; **5.** (a) IGAA, (b) BIA, (c) JHDA, (d) HDA, (e) BFAA, (f) EBDAAA; **6.** (a) JAA, (b) HAAA, (c) GAA, (d) IHA, (e) DCAA, (f) GHAA

Set 27 *For use after page 136*

A

	a	b	c	d	e	f
1.	5 × 40	8 × 300	9 × 60	3 × 700	6 × 70	4 × 300
2.	8 × 60	50 × 9	800 × 7	6 × 900	70 × 9	800 × 8

B

1.	4 × 70	6 × 800	7 × 50	5 × 900	70 × 8	400 × 4
2.	8 × 90	700 × 7	600 × 7	9 × 90	600 × 6	6 × 500

CODED ANSWERS, SET 27, PART A: 1. (a) AEE, (b) ACEE, (c) ICE, (d) AGEE, (e) CAE, (f) GAEE; **2.** (a) CDE, (b) CIE, (c) IBEE, (d) ICEE, (e) BHE, (f) BCEE

Set 28 *For use after page 144*

A

	a	b	c	d	e
1.	40 × 50	60 × 20	40 × 80	80 × 40	50 × 90
2.	50 × 50	30 × 70	60 × 90	70 × 20	0 × 80
3.	80 × 60	30 × 30	40 × 60	60 × 80	60 × 70
4.	900 × 20	30 × 200	200 × 40	70 × 90	40 × 90

B

1.	50 × 60	80 × 70	30 × 50	90 × 10	90 × 80
2.	30 × 80	70 × 40	90 × 60	80 × 80	90 × 70
3.	40 × 70	50 × 80	50 × 20	90 × 90	90 × 30
4.	20 × 700	90 × 50	900 × 40	700 × 70	80 × 90

CODED ANSWERS, SET 28, PART A: 1. (a) EFFF, (b) DEFF, (c) GEFF, (d) GEFF, (e) BHFF; **2.** (a) EHFF, (b) EDFF, (c) HBFF, (d) DBFF, (e) F; **3.** (a) BCFF, (b) JFF, (c) EBFF, (d) BCFF, (e) BEFF; **4.** (a) DCFFF, (b) IFFF, (c) CFFF, (d) IGFF, (e) GIFF

Set 29 *For use after page 171*

	a	b	c	d	e	f	g	h	i
A	$\begin{array}{r}43\\ \times 2\\ \hline\end{array}$	$\begin{array}{r}57\\ \times 4\\ \hline\end{array}$	$\begin{array}{r}36\\ \times 3\\ \hline\end{array}$	$\begin{array}{r}58\\ \times 5\\ \hline\end{array}$	$\begin{array}{r}49\\ \times 4\\ \hline\end{array}$	$\begin{array}{r}27\\ \times 6\\ \hline\end{array}$	$\begin{array}{r}34\\ \times 8\\ \hline\end{array}$	$\begin{array}{r}83\\ \times 7\\ \hline\end{array}$	$\begin{array}{r}76\\ \times 9\\ \hline\end{array}$
B	$\begin{array}{r}32\\ \times 3\\ \hline\end{array}$	$\begin{array}{r}74\\ \times 2\\ \hline\end{array}$	$\begin{array}{r}56\\ \times 5\\ \hline\end{array}$	$\begin{array}{r}77\\ \times 7\\ \hline\end{array}$	$\begin{array}{r}84\\ \times 6\\ \hline\end{array}$	$\begin{array}{r}39\\ \times 8\\ \hline\end{array}$	$\begin{array}{r}27\\ \times 9\\ \hline\end{array}$	$\begin{array}{r}35\\ \times 4\\ \hline\end{array}$	$\begin{array}{r}56\\ \times 7\\ \hline\end{array}$

CODED ANSWERS, SET 29, PART A: (a) GA, (b) JJG, (c) EDG, (d) JBD, (e) EBA, (f) EAJ, (g) JIJ, (h) HGE, (i) AGF

Set 30 *For use after page 173*

Ⓐ

	a	b	c	d	e	f	g	h
1.	325 ×3	146 ×5	749 ×2	358 ×4	2407 ×9	3581 ×7	7325 ×8	6043 ×6
2.	734 ×5	5036 ×4	416 ×3	2105 ×7	657 ×8	3456 ×9	255 ×6	4137 ×2
3.	8632 ×7	493 ×3	768 ×8	549 ×4	4768 ×9	3409 ×5	766 ×7	6548 ×6

Ⓑ

1.	435 ×2	276 ×4	4134 ×6	2483 ×8	787 ×3	3462 ×5	587 ×7	3429 ×9
2.	563 ×2	8765 ×9	406 ×3	3120 ×8	865 ×4	9673 ×7	472 ×5	6349 ×6
3.	408 ×9	967 ×8	3008 ×7	4675 ×6	9399 ×3	666 ×4	999 ×7	6379 ×8

CODED ANSWERS, SET 30, PART A: 1. (a) JFB, (b) FAI, (c) GCJH, (d) GCAE, (e) EGDDA, (f) EBIDF, (g) BHDII, (h) ADEBH; **2.** (a) ADFI, (b) EIGCC, (c) GECH, (d) GCFAB, (e) BEBD, (f) AGGIC, (g) GBAI, (h) HEFC; **3.** (a) DICEC, (b) GCFJ, (c) DGCC, (d) EGJD; (e) CEJGE, (f) GFICB, (g) BADE, (h) AJEHH

Set 31 *For use after page 173*

Ⓐ

	a	b	c
1.	$5 \times 6 \times 7 = \boxed{n}$	$8 \times 3 \times 4 \times 2 = \boxed{n}$	$7 \times 4 \times 8 \times 3 = \boxed{n}$
2.	$5 \times 4 \times 9 = \boxed{n}$	$6 \times 9 \times 5 \times 3 = \boxed{n}$	$4 \times 7 \times 6 \times 5 = \boxed{n}$
3.	$34 \times 3 \times 7 = \boxed{n}$	$46 \times 9 \times 3 = \boxed{n}$	$86 \times 7 \times 3 = \boxed{n}$
4.	$321 \times 6 \times 5 = \boxed{n}$	$463 \times 8 \times 6 = \boxed{n}$	$595 \times 9 \times 7 = \boxed{n}$

Ⓑ

1.	$6 \times 9 \times 8 = \boxed{n}$	$7 \times 4 \times 6 \times 8 = \boxed{n}$	$6 \times 7 \times 5 \times 4 = \boxed{n}$
2.	$5 \times 8 \times 7 = \boxed{n}$	$4 \times 3 \times 7 \times 6 = \boxed{n}$	$5 \times 4 \times 6 \times 9 = \boxed{n}$
3.	$46 \times 5 \times 8 = \boxed{n}$	$87 \times 6 \times 7 = \boxed{n}$	$94 \times 3 \times 9 = \boxed{n}$
4.	$341 \times 7 \times 4 = \boxed{n}$	$579 \times 5 \times 4 = \boxed{n}$	$378 \times 6 \times 6 = \boxed{n}$

CODED ANSWERS, SET 31, PART A: 1. (a) CAB, (b) AIC, (c) GHC; **2.** (a) AJB, (b) JAB, (c) JFB; **3.** (a) HAF, (b) ACFC, (c) AJBG; **4.** (a) IGDB, (b) CCCCF, (c) DHFJE

Set 32 *For use after page 178*

	a	b	c	d	e	f	g	h	i
Ⓐ	23 ×34	57 ×25	19 ×27	18 ×71	39 ×62	56 ×43	75 ×54	64 ×85	43 ×96
Ⓑ	36 ×45	45 ×98	15 ×66	77 ×59	68 ×18	59 ×77	41 ×26	35 ×32	83 ×44

CODED ANSWERS, SET 32, PART A: (a) EDF, (b) AHFJ, (c) JAC, (d) AFED, (e) FHAD, (f) FHGD, (g) HGJG, (h) JHHG, (i) HAFD

Set 33 *For use after page 182*

	a	b	c	d	e	f	g	h
Ⓐ	425 ×32	173 ×46	218 ×57	423 ×38	561 ×94	789 ×62	343 ×71	629 ×84
Ⓑ	785 ×75	936 ×68	421 ×56	507 ×49	683 ×34	590 ×28	789 ×87	378 ×95

CODED ANSWERS, SET 33, PART A: (a) IBGDD, (b) EJCA, (c) IHFHG, (d) IGDEF, (e) CHEBF, (f) FAJIA, (g) HFBCB, (h) CHABG

Set 34 *For use after page 184*

Ⓐ	a	b	c	d	e	f	g	h
1.	146 ×242	257 ×234	375 ×155	807 ×263	466 ×376	394 ×485	581 ×297	925 ×634
2.	648 ×725	647 ×192	852 ×380	739 ×478	139 ×637	820 ×445	324 ×352	911 ×169
Ⓑ	345 ×612	224 ×319	715 ×348	756 ×806	469 ×852	379 ×345	891 ×607	491 ×223

CODED ANSWERS, SET 34, PART A: **1.** (a) IJIIF, (b) HADIE, (c) JEDFJ, (d) FDFFBD, (e) DCJFDH, (f) DGDAGA, (g) DCFJJC, (h) JEHBJA; **2.** (a) BHGEAA, (b) DFBFFB, (c) IFICHA, (d) IJIFBF, (e) EEJBI, (f) IHBGAA, (g) DDBABE, (h) DJIGJG

Set 35 *For use after page 210*

Find the quotients and remainders.

Ⓐ	a	b	c	d	e	f
1.	$3\overline{)94}$	$4\overline{)137}$	$5\overline{)325}$	$9\overline{)456}$	$8\overline{)536}$	$7\overline{)463}$
2.	$6\overline{)378}$	$4\overline{)326}$	$3\overline{)243}$	$2\overline{)109}$	$8\overline{)436}$	$9\overline{)643}$
Ⓑ						
1.	$6\overline{)206}$	$5\overline{)359}$	$3\overline{)147}$	$4\overline{)278}$	$8\overline{)483}$	$7\overline{)649}$
2.	$6\overline{)568}$	$4\overline{)167}$	$8\overline{)365}$	$5\overline{)342}$	$9\overline{)672}$	$8\overline{)747}$

CODED ANSWERS, SET 35, PART A: (*r* stands for remainder) **1.** (a) GD*r*D, (b) GB*r*D, (c) IH, (d) HF*r*I, (e) IA, (f) II*r*D; **2.** (a) IG, (b) CD*r*E, (c) CD, (d) HB*r*D, (e) HB*r*B, (f) AD*r*B

Set 36 *For use after page 213*

Find the average of the numbers in each set.

Ⓐ	a	b	c	d
1.	{5, 3, 10}	{8, 4, 3}	{11, 9, 7}	{54, 7, 2}
2.	{26, 35, 29}	{80, 73, 30}	{105, 102, 87}	{8, 7, 9, 4}
Ⓑ				
1.	{6, 4, 8}	{5, 9, 7}	{12, 15, 9}	{97, 8, 33}
2.	{47, 56, 23}	{94, 88, 37}	{125, 78, 46}	{7, 9, 9, 15}

CODED ANSWERS, SET 36, PART A: 1. (a) A, (b) H, (c) B, (d) JE; **2.** (a) CD, (b) AE, (c) BG, (d) I

Set 37 *For use after page 216*

Find the quotients and remainders.

Ⓐ	a	b	c	d	e	f
1.	$2\overline{)672}$	$6\overline{)2438}$	$8\overline{)5307}$	$7\overline{)2359}$	$5\overline{)4865}$	$4\overline{)3346}$
2.	$9\overline{)5837}$	$8\overline{)5946}$	$9\overline{)6308}$	$6\overline{)4524}$	$7\overline{)5048}$	$9\overline{)3929}$
Ⓑ						
1.	$3\overline{)978}$	$4\overline{)1586}$	$5\overline{)5003}$	$6\overline{)4836}$	$7\overline{)3851}$	$8\overline{)7534}$
2.	$9\overline{)8463}$	$8\overline{)4008}$	$7\overline{)4376}$	$6\overline{)3542}$	$5\overline{)3964}$	$4\overline{)2746}$

CODED ANSWERS, SET 37, PART A: (*r* stands for remainder) **1.** (a) AAD, (b) CID*r*E, (c) DDA*r*A, (d) AAF, (e) JFA, (f) HAD*r*E; **2.** (a) DCH*r*B, (b) FCA*r*E, (c) FII*r*H, (d) FBC, (e) FEG*r*G, (f) CAD*r*B

Set 38 *For use after page 224*

Give the largest number that will make each sentence true.

Ⓐ

	a	b	c	d
1.	$\boxed{n} \times 31 < 206$	$\boxed{n} \times 29 < 193$	$\boxed{n} \times 52 < 413$	$\boxed{n} \times 48 < 463$
2.	$\boxed{n} \times 27 < 223$	$\boxed{n} \times 63 < 520$	$\boxed{n} \times 77 < 410$	$\boxed{n} \times 54 < 317$

Ⓑ

1.	$\boxed{n} \times 22 < 178$	$\boxed{n} \times 49 < 365$	$\boxed{n} \times 33 < 337$	$\boxed{n} \times 68 < 568$
2.	$\boxed{n} \times 19 < 138$	$\boxed{n} \times 21 < 87$	$\boxed{n} \times 79 < 693$	$\boxed{n} \times 82 < 571$

CODED ANSWERS, SET 38, PART A: 1. (a) G, (b) G, (c) H, (d) I; **2.** (a) J, (b) J, (c) E, (d) E

Set 39 *For use after page 225*

Find the quotients and remainders.

Ⓐ

	a	b	c	d	e	f
1.	$31\overline{)206}$	$29\overline{)193}$	$52\overline{)413}$	$48\overline{)463}$	$27\overline{)223}$	$63\overline{)520}$
2.	$77\overline{)410}$	$54\overline{)317}$	$61\overline{)271}$	$89\overline{)675}$	$43\overline{)294}$	$58\overline{)456}$

Ⓑ

1.	$22\overline{)178}$	$49\overline{)365}$	$33\overline{)237}$	$68\overline{)568}$	$19\overline{)138}$	$21\overline{)87}$
2.	$79\overline{)693}$	$82\overline{)571}$	$52\overline{)372}$	$36\overline{)200}$	$78\overline{)643}$	$81\overline{)707}$

CODED ANSWERS, SET 39, PART A: (*r* stands for remainder) **1.** (a) I*r*FD, (b) I*r*BC, (c) G*r*AC, (d) C*r*HB, (e) E*r*G, (f) E*r*BI; **2.** (a) J*r*FJ, (b) J*r*AG, (c) A*r*FG, (d) G*r*JF, (e) I*r*HI, (f) G*r*JD

Set 40 *For use after page 225*

Ⓐ

	a	b	c
1.	$846 \times 5 = \boxed{n}$	$756 \div 9 = \boxed{n}$	$635 - 97 = \boxed{n}$
2.	$3856 + 497 = \boxed{n}$	$\boxed{n} = 301 \div 43$	$537 \times 23 = \boxed{n}$

Ⓑ

1.	$854 - 57 = \boxed{n}$	$347 \times 6 = \boxed{n}$	$429 + 657 = \boxed{n}$
2.	$168 \div 6 = \boxed{n}$	$507 - 268 = \boxed{n}$	$34 \times 76 = \boxed{n}$
3.	$837 \div 31 = \boxed{n}$	$365 \times 23 = \boxed{n}$	$3056 - 1987 = \boxed{n}$

CODED ANSWERS, SET 40, PART A: 1. (a) HFCG, (b) DH, (c) JCD; **2.** (a) HCJC, (b) E, (c) AFCJA

Set 41 *For use after page 225*

Copy the problems. Write the missing digits instead of the ▨.

	a	b	c	d	e	f
1.	34▨ +5▨6 ▨81	5▨4 ×3 1▨7▨	▨3 −2▨ 26	▨ 41)24▨ ▨▨▨ 0	3▨ ×6 1▨0	5 39)▨▨▨ ▨▨▨ 0
2.	▨65 +9▨▨ 1382	6▨ ×7 ▨▨5	▨▨7 −128 27▨	▨ 52)36▨ ▨▨▨ 0	3▨2 ×8 ▨416	4 7▨)312 ▨▨▨ 0

Set 42 *For use after page 305*

Write the symbol (>, <, =) for each ●.

	a	b	c	d	e
1.	$\frac{1}{2}$ ● $\frac{2}{4}$	$\frac{1}{4}$ ● $\frac{1}{2}$	$\frac{1}{3}$ ● $\frac{1}{6}$	$\frac{1}{3}$ ● $\frac{2}{6}$	$\frac{1}{4}$ ● $\frac{1}{3}$
2.	$\frac{5}{8}$ ● $\frac{3}{8}$	$\frac{4}{12}$ ● $\frac{3}{9}$	$\frac{2}{3}$ ● $\frac{5}{6}$	$\frac{4}{8}$ ● $\frac{8}{16}$	$\frac{1}{12}$ ● $\frac{1}{2}$
3.	$\frac{1}{3}$ ● $\frac{4}{6}$	$\frac{1}{5}$ ● $\frac{1}{2}$	$\frac{1}{4}$ ● $\frac{1}{8}$	$\frac{1}{4}$ ● $\frac{2}{8}$	$\frac{5}{6}$ ● $\frac{1}{1}$
4.	$\frac{3}{4}$ ● $\frac{3}{8}$	$\frac{1}{10}$ ● $\frac{1}{5}$	$\frac{3}{4}$ ● $\frac{15}{20}$	$\frac{4}{4}$ ● $\frac{5}{5}$	$\frac{7}{8}$ ● $\frac{15}{16}$

Set 43 *For use after page 311*

Ⓐ	a	b	c	d
1.	$\frac{1}{2}+\frac{1}{2}=\boxed{n}$	$\frac{2}{5}+\frac{1}{5}=\boxed{n}$	$\frac{5}{2}+\frac{4}{2}=\boxed{n}$	$3+\frac{1}{3}=\boxed{n}$
2.	$2\frac{1}{5}+1\frac{2}{5}=\boxed{n}$	$5\frac{2}{6}+3\frac{1}{6}=\boxed{n}$	$4\frac{2}{3}+6\frac{1}{3}=\boxed{n}$	$3\frac{1}{12}+5\frac{3}{12}=\boxed{n}$

Ⓑ				
1.	$\frac{2}{4}+\frac{2}{4}=\boxed{n}$	$\frac{3}{7}+\frac{2}{7}=\boxed{n}$	$\frac{6}{8}+\frac{5}{8}=\boxed{n}$	$\frac{1}{5}+5\frac{1}{5}=\boxed{n}$
2.	$3\frac{3}{4}+2\frac{2}{4}=\boxed{n}$	$7\frac{1}{8}+6\frac{3}{8}=\boxed{n}$	$4\frac{1}{2}+7\frac{3}{2}=\boxed{n}$	$\frac{3}{8}+4\frac{5}{8}=\boxed{n}$

CODED ANSWERS, SET 43, PART A: 1. (a) I, (b) B/C, (c) J/H, (d) ID/B; **2.** (a) B B/C, (b) A B/G, (c) II, (d) A F/IH

Tables of Measures

LENGTH	
English System	Metric System
12 inches (in.) = 1 foot (ft)	10 centimeters (cm) = 1 decimeter (dm)
3 feet = 1 yard (yd)	10 decimeters = 1 meter (m)
5280 feet = 1 mile (mi)	1000 meters = 1 kilometer (km)
1760 yards = 1 mile	

TIME	
60 seconds (sec) = 1 minute (min)	52 weeks = 1 year (yr)
60 minutes = 1 hour (hr)	12 months (mo) = 1 year
24 hours = 1 day	365 days = 1 year
7 days = 1 week (wk)	366 days = 1 leap year

LIQUID	DRY
8 fluid ounces (oz) = 1 cup	2 pints = 1 quart
2 cups = 1 pint (pt)	8 quarts = 1 peck (pk)
2 pints = 1 quart (qt)	4 pecks = 1 bushel (bu)
4 quarts = 1 gallon (gal.)	

WEIGHT	
English System	Metric System
16 ounces (oz) = 1 pound (lb)	1000 grams (g) = 1 kilogram (kg)
2000 pounds = 1 ton	1000 kilograms = 1 metric ton

BOOKS TO EXPLORE

Adler, Irving. *The Giant Golden Book of Mathematics.* New York, Golden Press, 1960.

Have you ever wondered how a tree grows or why a volcano is shaped as it is or what makes a card trick work? This colorful book answers these and many other questions, through exploring the world of mathematics. You'll find all kinds of exciting ideas about numbers and what they mean in our daily lives. Here are just a few of the interesting topics:

What an Egyptian carpenter used for a ruler	14
How to get a $5\frac{1}{2}$ ft airplane through a doorway 3 by 5 feet	33
How to locate north with a stick and a watch	60
The speed of a falling satellite	65
Your chance of getting 2 heads when you toss 2 coins	73

Bendick, Jeanne. *How Much, and How Many, the Story of Weights and Measures.* New York, Whittlesey House, 1960.

Many interesting facts on weights and measures and how they were developed are included in this book. You'll find the measurements used in rocketry, atomic science, photography, guns, and space science exciting reading. Some unusual units of measure used in the shipping, building, and printing trades may tickle your tongue. For example, did you know that a "kip," a "last," and a "quarter" are units of measure? See pages 35 and 36 to find out where these units are used.

The book answers these questions, among others:

Which came first, measuring or weighing?	14
Does the weight of an object change with altitude?	26
What is the difference between net weight and gross weight?	30
Did you know that some "tons" are measures and some are weights?	35
What does a "clove" measure?	51

Hogben, Lancelot. *The Wonderful World of Mathematics.* Garden City, New York, Garden City Books, 1955.

This book shows that the story of man becoming civilized is also the story of the growth of mathematics as a science. You will enjoy going back to the time of the cave man and finding out how man learned to measure and to count, to build and to navigate, to design and to calculate with computers.

Some topics covered in this book are:

The geometry of land measuring	17
How the Greeks measured the earth	36
Hindu numerals	45
Arabian lattice multiplication	50
Invention of the telescope	61

Jonas, Arthur. *New Ways in Math.* Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1962.

Cartoons tell the story of mathematics, including sets, probability, and algebra, and make this book fun to read. The chapter "Men in Math" on page 62 describes contemporary mathematicians, like Einstein and Von Neumann, as well as those from the past, like Pythagoras and Archimedes.

Other chapters you'll probably find exciting include:

The magic of two	24
When 1001 = 9	31
What is your hunch?	45
Doughnuts and pretzels	50
Math without numbers	57

Kenyon, Raymond G. *I Can Learn About Calculators and Computers.* New York, Harper & Row, 1961.

This book is a brief history of numbers, calculators and computers. You'll read about the operation of modern computers, and the language of computer science. You will even be able to build a simple calculator or computer yourself; the book gives instructions for several, and tells where to get the materials you need.

Be sure to look into the chapters on the following subjects:

John Napier's bones	53
'Stepped Wheel' calculator	67
Digital computer	74
Analogue computer	92

Other books to look for in your library include those about numbers, measuring, puzzles and time. Books you may enjoy are discussed below:

Adler, Irving and Ruth. *Numerals: New Dresses for Old Numbers.* New York, The John Day Company, Inc., 1964.

Several new ways of counting are clearly explained in this little book. The authors tell how to change numerals to other bases, then add and multiply; they also explain place value.

Andrews, Frank Emerson. *Numbers, Please.* Boston, Little, Brown and Company, 1961.

A book about numbers from base 2 to 12, which tells how counting began, how to use an abacus, and what decimals mean. You'll learn some useful short cuts in figuring, too.

Bendick, Jeanne. *The First Book of Time.* New York, Watts, 1963.

Excellent pictures help trace the history of time and how we measure it. All kinds of clocks are described—from sun dials and water clocks to the atomic clock and clocks in your body.

Cooke, David C. *How Money is Made.* New York, Dodd, Mead and Company, 1962.
How does an expert tell if a bill is counterfeit? This book has the answer. The author also tells how real money is designed and made, and what happens to money when it wears out.

Feravolo, Rocco. *Wonders of Mathematics.* New York, Dodd, Mead and Company, 1963.
This book uses many activities to develop several bases and number systems.

Kettlekamp, Larry. *Spirals.* Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1964.
An enjoyable look at spirals—some are in nature, some man-made, and they go in and out, up and down.

Leeming, Joseph. *Fun with Puzzles.* Philadelphia, J. B. Lippincott Company, 1946. Also available in paperback from Scholastic Book Service, 1966.
This book contains more than 200 match, coin, paper and pencil, cut-out, and word puzzles. The answers are all in the back of the book.

Massoglia, Elinor. *Fun-Time Paper Folding.* Chicago, Children's Press, Inc., 1959.
Many kinds of objects and shapes to make. No cutting and pasting as each shape can be folded from a single piece of paper.

Neely, Henry M. *Triangles: Getting Ready for Trigonometry.* New York, The Crowell-Collier Publishing Company, 1962.
Without trigonometry, engineers couldn't lay out highways, and navigators couldn't chart the seas. This interesting book tells how they use circles and triangles in their jobs; you'll also learn how trigonometry began.

Shaw, H. Alan and Fuge, Keri. *The Story of Mathematics.* New York, St. Martin's Press, Inc., 1963.
You'll see how mathematics developed from man's need to count, compute, build and orientate himself. Short biographies of mathematicians from the Greek Thales to the German Leibniz give a look into history. Clocks and the calendar through the ages are discussed, using many pictures.

Ravielli, Anthony. *The World Is Round.* New York, The Viking Press, Inc., 1963.
An interesting picture story about why the earth seems flat, and how man discovered it was really round.

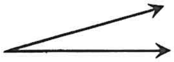
Simon, Leonard. *The Day the Numbers Disappeared.* New York, Whittlesey House, 1963.
The Egyptian, Greek and Roman methods for writing numbers are compared. You'll learn how to translate ∩∩||||| or κε or XXV into numbers you know.

GLOSSARY

addend Any one of a set of numbers to be added. In the equation $4 + 5 = 9$, the numbers 4 and 5 are addends.

addition An operation that combines a first number and a second number to give exactly one number. The two numbers are called addends, and the one number which is the result of combining the two numbers is called the sum of the addends.

angle Two rays from a single point.



approximation One number is an approximation of another number if the first number is suitably "close" (according to context) to the other number.

area The area of a closed figure or region is the measure of that region as compared to a given selected region called the unit, usually a square region in the case of area.

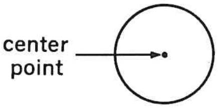
borrow A commonly used term for the regrouping process involved in certain types of subtraction.

Example:
$$\begin{array}{r} 4\;3 \\ -1\;7 \\ \hline 2\;6 \end{array} \rightarrow \begin{array}{r} 30 + 13 \\ -(10 + 7) \\ \hline 20 + 6 \end{array} = 26$$

carry A commonly used term for the regrouping that is involved in addition.

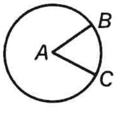
Example:
$$\begin{array}{r} 57 \\ +26 \\ \hline 83 \end{array} \rightarrow \begin{array}{r} 50 + 7 \\ 20 + 6 \\ \hline 70 + 13 \end{array} = 83$$

center point A given point in the interior of a circle, such that all the points on the circle are the same distance from this given point.

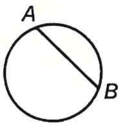


centimeter A unit of length. One centimeter is $\frac{1}{100}$ meter.

central angle In the figure below, angle BAC illustrates a central angle of a given circle with center A.

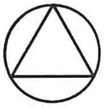


chord A line segment that has its end points on a given circle.



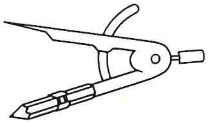
circle A set of points, all of which are a specified distance from a given point called the center or center point.

circumscribed circle A circle is circumscribed about a polygon when each vertex of the polygon is a point of the circle. In the figure, the circle is circumscribed about the triangle.



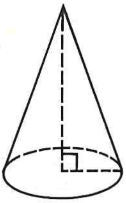
common factor When a number is a factor of two different numbers, it is said to be a common factor of the two numbers.

compass A device for drawing models of a circle.



composite number Any whole number greater than 1 that is not prime.

cone Generally thought of as a right circular cone, which is illustrated below.



coordinate Number pair used in graphing.

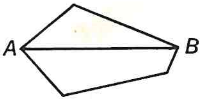
coordinate axes Two number lines intersecting at right angles at 0.

count To name numbers in regular succession.

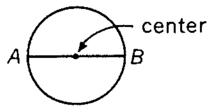
cube A rectangular prism (box) such that all faces are squares.

denominator The number indicated by the numeral below the line in a fraction symbol.

diagonal A segment joining two nonadjacent vertices of a polygon. In the figure, the diagonal is segment *AB*.



diameter A chord that passes through the center point of the circle.



difference The number resulting from the subtraction operation.

digits The basic Hindu-Arabic symbols used to write numerals. In the base-ten system, these are the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

dividend In the problem 33 ÷ 7, 33 is called the dividend.

$$\text{Example: } \begin{array}{r} 4 \\ 7\overline{)33} \leftarrow \text{dividend} \\ \underline{28} \\ 5 \end{array}$$

division An operation related to multiplication as illustrated:

$$\begin{tikzcd} & 12 \div 3 = 4 \\ 3 \times 4 = 12 \arrow[ur] \arrow[dr] & \\ & 12 \div 4 = 3 \end{tikzcd}$$

divisor In the problem 33 ÷ 7, 7 is called the divisor.

$$\text{Example: } \begin{array}{r} 4 \\ \text{divisor} \rightarrow 7\overline{)33} \\ \underline{28} \\ 5 \end{array}$$

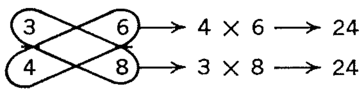
edge An edge of a space figure is one of the segments making up any one of the faces of the space figure.

empty set A set that has no objects in it.

equality (equals; or =) A mathematical relation of being exactly the same.

equation A mathematical sentence involving the use of the equality symbol. Examples: $5 + 4 = 9$; $7 + \square = 8$; $\mathbf{n} + 3 = 7$.

equivalent fractions Two fractions are equivalent when it can be shown that they each can be used to represent the same amount of a given object. Also, two fractions are equivalent if these two products are the same:



equivalent sets Two sets that may be placed in a one-to-one correspondence.

estimate To find an approximation for a given number. (Sometimes a sum, a product, etc.)

even numbers The whole-number multiples of 2 (0, 2, 4, 6, 8, 10, 12, · · ·).

face The face of a given space figure is any one of the plane geometric figures (regions) making up the space figure. For example, in a cube each of the square regions is a face of the cube.

factor *See* multiplication. The equation $6 \times 7 = 42$ illustrates that both 6 and 7 are factors of 42.

fraction A number pair or the symbol for a number pair, usually written $\frac{2}{3}, \frac{3}{4}, \frac{1}{2}$, and so on.

graph (1) A set of points associated with a given set of numbers or set of number pairs. (2) A picture used to illustrate a given collection of data. The data might be pictured in the form of a bar graph, a circle graph, a line graph, or a pictograph. (3) To draw the graph of.

greater than (>) One of the two basic inequality relations. Examples: $8 > 5$, $28 > 25$, $80 > 50$.

greatest common factor The largest, or greatest, number that is a factor of each of two numbers.

grouping principle (associative principle) When adding (or multiplying) three numbers, you can change the grouping and the sum (or product) is the same.

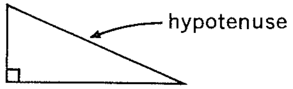
Example: $2 + (8 + 6) = (2 + 8) + 6$

hexagon A six-sided polygon.

higher terms A first fraction is in higher terms than a second fraction if the first fraction is equivalent to the second fraction and if the denominator of the first fraction is greater than the denominator of the second fraction.

Example: $\frac{9}{12}$ is in higher terms than $\frac{6}{8}$.

hypotenuse The side opposite the right angle in a right triangle.



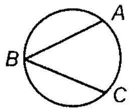
improper fraction A fraction in which the numerator is greater than or equal to the denominator.

Examples: $\frac{8}{5}, \frac{9}{6}, \frac{12}{3}, \frac{7}{7}$

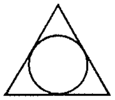
inch A unit of length. One inch is $\frac{1}{12}$ foot.

inequality (<, ≠, >) In arithmetic, a relation indicating that the two numbers are not the same.

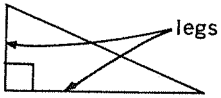
inscribed angle Angle *ABC* in the figure illustrates an angle inscribed in a given circle.



inscribed circle A circle is said to be inscribed in a polygon if the circle lies within the polygon and each side of the polygon is tangent to the circle.



legs of a right triangle The two sides of a right triangle other than the hypotenuse.



length (1) A number indicating the measure of one line segment with respect to another line segment, called the unit. (2) Sometimes used to denote one dimension (usually the greater) of a rectangle.

less than ($<$) One of the two basic inequality relations. Examples: $5 < 8$, $25 < 28$, $50 < 80$.

line segment See segment.

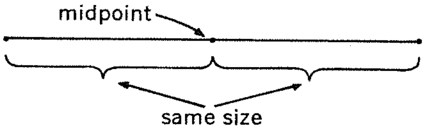
lower terms A first fraction is in lower terms than a second fraction if the first fraction is equivalent to the second fraction and if the denominator of the first fraction is less than the denominator of the second fraction.

Example: $\frac{6}{8}$ is in lower terms than $\frac{9}{12}$.

lowest terms A fraction is in lowest terms if the numerator and denominator of the fraction have no common factor greater than 1.

measure (1) A number indicating the relation between a given object and a suitable unit. (2) The process of finding the number described in (1).

midpoint A point that divides a line segment into two parts of the same size.



minus (—) Used to indicate the subtraction operation, as in $7 - 3 = 4$ (read, "7 minus 3 equals 4").

multiple A first number is a multiple of a second number if there is a whole number that multiplies by the second number to give the first number. Example: 24 is a multiple of 6 since $4 \times 6 = 24$.

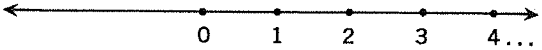
multiplication An operation that combines a first number and a second number to give exactly one number. The two numbers are called factors, and the one number which is a result of combining the two numbers is called the product of the two numbers.

multiplication-addition principle (distributive principle) This principle is sometimes described in terms of "breaking apart" a number before multiplying.
Example: $6 \times (20 + 4) = (6 \times 20) + (6 \times 4)$

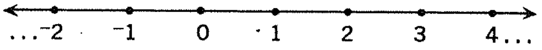
negative number If a number adds to a whole number to give 0, it is a negative number.

For example: $5 + {}^{-}5 = 0$
$19 + {}^{-}19 = 0$

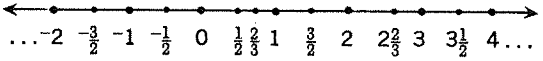
number line A line with a subset of its points matched with a subset of the real numbers. We say that the rational number line has "holes" in it because some points are not matched with rational numbers. The real number line is said to be "complete" because each point is matched with some real number.



The number line



The integer number line



The rational number line

number pair Any pair of numbers. In this book, usually a pair of whole numbers.

numeral A symbol for a number.

numerator The number indicated by the numeral above the line in a fraction symbol.

odd number Any whole number that is not even.

one-to-one correspondence A one-to-one correspondence exists between two sets when the elements of one can be matched with the elements of the other in such a way that each element of the first set is matched with exactly one element of the second set and each element of the second set is matched with exactly one element of the first set.

order principle (commutative principle) When adding (or multiplying) two numbers, the order of the addends (or factors) does not affect the sum (or product). Example: $4 + 5 = 5 + 4$

parallel lines Two lines which lie in the same plane and do not intersect.

parallelogram A quadrilateral with its opposite sides parallel.

parentheses (( )) Symbols used to indicate grouping or order of performing operations. Examples:

$(5 \times 4) - 2 = 18 \qquad 5 \times (4 - 2) = 10$

pentagon A five-sided polygon.

perfect number A number that is half the sum of its factors. Examples: 6, 28, and 496.

perimeter The sum of the lengths of the sides of a given polygon.

period In arithmetic, each set of three digits indicated by commas when writing a numeral is called a period. These periods are called (right to left) units period, thousands period, millions period, and so on.

Example:	**3 4 2 ,**	**6 7 4 ,**	**2 0 8**
	millions period	thousands period	units period

place value A system used for writing numerals for numbers, using only a definite number of symbols or digits. In the numeral 3257 the 5 stands for 50; in the numeral 36, 289 the 6 stands for 6000.

plus (+) Used to indicate the addition operation, as in $4 + 3 = 7$ (read, "4 plus 3 equals 7").

polygon A closed geometric figure made up of line segments.

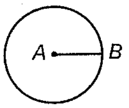
prime number A number greater than 1 whose only factors are itself and 1.

product The result of the multiplication operation. In 6 × 7 = 42, 42 is the product of 6 and 7.

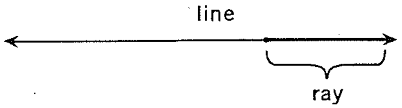
quadrilateral A four-sided polygon.

quotient The number (other than the remainder) that is the result of the division operation. It may be thought of as a factor in a multiplication equation.

radius (1) Any segment from the center point to a point on the circle. (2) The distance from the center point to any point on the circle.



ray The heavy part of the line shows a ray.



rectangle A quadrilateral that has four right angles.

regrouping A method of handling place value symbols in adding or subtracting numbers.

remainder

Example:
$$\begin{array}{r} 6 \\ 7\overline{)47} \\ 42 \\ \hline 5 \end{array} \longleftarrow \text{remainder}$$

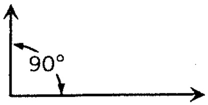
repeated addition Finding the sum of a set of numbers, each of which is the same.

Example: 5 + 5 + 5 + 5

repeated subtraction Starting with a number and repeatedly subtracting the same given number from each difference that is obtained.

rhombus A parallelogram with 4 sides of the same size.

right angle An angle that has the measure of 90 degrees.



A right angle

right triangle A triangle that has one right angle.

Roman numerals Numerals used by the Romans. Used primarily to record numbers rather than for computing. Examples: IV, IX, XIV.

segment Two points on a line and all the points on that line that are between the two points.

sequence A collection or set of numbers given in a specific order. Such numbers are commonly given according to some rule or pattern.

set A group or collection of objects.

skip count To count by multiples of a given number. Example: Counting by fives — 0, 5, 10, 15, 20, · · · .

solution The number or numbers which result from solving an equation or a given problem.

solve To find the number or numbers which, when substituted for the variable or placeholder, make the given equation true.

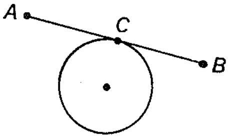
square A quadrilateral that has four right angles and four sides that are the same length.

subtraction An operation related to addition as illustrated:

$$\begin{tikzcd}[row sep=small] & 15 - 8 = 7 \\ 7 + 8 = 15 \arrow[ur] \arrow[dr] & \\ & 15 - 7 = 8 \end{tikzcd}$$

sum The result obtained by adding any set of numbers.

tangent A line is tangent to a circle if the two figures are in one plane and have exactly one point in common.

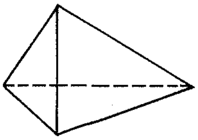


Line *AB* is tangent to the circle at point *C*.

times (×) Used to indicate the multiplication operation, as in 3 × 4 = 12 (read, "3 times 4 equals 12").

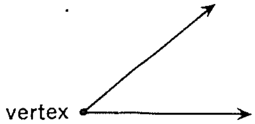
triangle A three-sided polygon.

triangular pyramid A 4-sided space figure that has triangular regions for all faces.



unit An amount or quantity adopted as a standard of measurement.

vertex The point that the two rays of an angle have in common.



volume The measure, obtained using an appropriate unit (usually a cube), of the interior region of a space figure.

whole number Any number in the set

{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, · · · }.

INDEX

Addends (missing), 24
Addition
  column, 34, 40
  estimation, 158
  grouping principle, 31
  inverse of, 24
  money problems, 42
  and multiplication, 74
  multiplication principle, 80, 168–169
  number line, 23
  order principle, 30
  rational numbers, 308
  reasoning, 33
  regrouping in, 36
  2-digit sums, 36
  3-digit sums, 38
Angles (central and inscribed), 150
Area
  finding, 118
  introduction to, 112
  of a rectangle, 120
Average, 212

Centimeter
  cubic, 122
  ruler, 115
  square, 118
Central angle, 150
Checking
  equivalent fractions, 276
  quotients, 210
Circles
  central and inscribed angles, 150
  circumscribed, 154
  inscribed, 152
  and pairs of points, 68
  and points, 66
  and tangents, 148
  and three points, 70
Circumscribed circles, 154
Common factors, 236; greatest common factor, 237
Column addition, 34, 40
Composite numbers, 241
Cone, 198
Coordinates, 244
Cube
  cubic centimeter and cubic inch, 122
  patterns for, 194
  volume, 112
Cup, 128
Cylinder, 198

Denominator of a fraction, 268
Differences (estimating), 158
Digits, 2
Division
  dividend, 202
  divisor, 73, 202
  2-digit divisors, 225
  estimating quotients, 163
  finding and checking quotients, 210
  finding quotients by subtraction, 204
  and multiplication, 76
  multiples of ten as divisors, 221
  quotients and missing factors, 92
  quotients and remainders, 202
  3-digit quotients, 215
  and sets, 73, 96
  and subtraction, 75
  zero and one, 94

Edge
  of a cube, 192
  of a pyramid, 196

Equivalent fractions
  checking, 276
  introduction, 262
  and rational numbers, 298
  sets of, 264, 266
Estimation
  of length, 116
  multiples of ten, 208
  multiples of one hundred, 214
  perimeter, 124
  products, 160
  of quotients, 163, 200
  sums and differences, 158
Even numbers, 232

Face
  of a cube, 192
  of a pyramid, 196
Factor(s)
  common, 236
  greatest common, 237
  finding, 235
  missing, 91–92, 138
  and products, 93, 234
  2-digit, 170, 176, 178
  3- and 4-digit, 172
  two 3-digit, 184
Foot, 129
Fractions
  checking equivalence, 276
  equivalent, 262
  higher and lower terms, 280
  improper, 272
  lowest terms, 282
  and number pairs, 252, 254
  numerator and denominator, 268
  and rational numbers, 292
  and sets, 258
  sets, equivalent, 264, 266
  zero numerators, 274
Function machine, 28, 100, 235, 240

Gallon, 128
Geometry
  central and inscribed angles, 150
  circles and pairs of points, 68
  circles and points, 66
  circles and tangents, 148
  circumscribed circles, 154
  cones, 198
  coordinates, 244
  cube, 192–195
  cylinders, 198
  graphing functions, 248
  graphing negative numbers, 250
  graphing number pairs, 246
  inscribed circles, 152
  simple closed curves, 64
  three points and a circle, 70
  triangular pyramid, 196
Graphing
  functions, 248
  negative numbers, 250
  number pairs, 246
Greatest common factor, 237
Grouping principle
  for addition, 31, 144
  for multiplication, 136, 144

Higher terms for fractions, 280

Improper fractions, 272
Inch(es)
  cubic, 122
  ruler, 115, 129
  square, 118
Inequalities
  place value, 8
  rational numbers, 304
Inscribed
  angle, 150
  circle, 152
Inverse
  addition and subtraction, 24
  multiplication and division, 76
Length
  estimating, 116
  estimating perimeter, 124
  finding, 112, 114
  inches, feet, yards, miles, 129
  measuring to find perimeter, 126
Liquid measure, 128
Lower terms for fractions, 280
Lowest terms for fractions, 282

Measurement
  area of a rectangle, 120
  converting units, 130
  estimating length, 116
  estimating perimeter, 124
  finding length, 114
  finding volume, 122
  inch and centimeter ruler, 115
  inches, feet, yards, miles, 129
  length, area, volume, 112
  liquid, 128
  to find perimeter, 126
  rational numbers, 313
Mile, 129
Million, 14
Mixed numerals, 311
Money, finding the total amount, 42
Multiples
  of one hundred in estimation, 214
  of ten as divisors, 221
  of ten in estimation, 208
  of ten, one hundred, one thousand, 134
Multiplication
  and addition, 74
  addition principle, 80, 168–169
  algorithm, 171
  common factors, 236
  and division, 76
  estimating products, 160
  and factors, 72
  finding factors, 235
  facts, 83–85
  greatest common factor, 237
  grouping principle, 136
  missing factors, 91, 138
  order principle, 79
  and pairing, 104, 107
  products and factors, 93, 234
  and sets, 72
  2-digit factors, 170, 176, 178
  3- and 4-digit factors, 172
  two 3-digit factors, 184
  zero and one, 82

Negative numbers, graphing, 250
Numbers
  even and odd, 232
  negative (graphing), 250
  prime, 239
  sets and numerals, 1
  subtraction (2-digit), 49
  subtraction (3-digit), 51
  sums (2-digit), 36
  sums (3-digit), 38
Number line
  addition, 23
  games, 88, 90
  rational numbers, 292, 312
  subtraction, 25
Number pairs
  and fractions, 252, 254
  graphing, 246
  multiplication and pairing, 107
Numerals
  2-digit, 2
  3-digit, 4
  4-digit, 6
  4-, 5-, and 6-digit, 12
  mixed, 311
  sets and numbers, 1
Numerator of a fraction, 268; zero, 274

Odd numbers, 232
One and zero
  in division, 94
  in multiplication, 82
Order principle
  for addition, 30
  for multiplication, 79, 144
Ounce (liquid), 128

Pairing and multiplication, 104, 107
Perimeter
  estimating, 124
  measuring, 126
Pint, 128
Place value
  comparison, 8
  grouping by tens, 2
  million, 14
  3-digit numerals, 4
  4-digit numerals, 6
  place-value machine, 16

  sets numbers, and numerals, 1
  thousands, 12
Points
  and circles, 66
  pairs of, and circles, 68
  three, and a circle, 70
Prime numbers, 239
Principle(s)
  grouping, for addition, 31, 144
  grouping, for multiplication, 136, 144
  multiplication-addition, 80, 168–169
  order, for addition, 30
  order, for multiplication, 79
Products
  estimating, 160
  and factors, 93, 234
  using principles to find, 136
Pyramid (triangular), 196

Quart (liquid), 128
Quotients
  estimating, 163
  finding and checking, 210
  and missing factors, 92
  and remainders, 202
  by subtraction, 204
  3-digit, 215

Rational numbers
  addition, 308
  equality, 298
  greater than one, 306, 311
  inequalities, 304
  introduction, 292
  measurement, 313
  names for, 296
  number line, 312
Reasoning, 33, 44, 61, 142
Rectangle, area of, 120
Regrouping, 48, 56
Ruler, inch and centimeter, 115

Segment, 112
Sequences, 45
Sets,
  addition and subtraction, 22
  and division, 73, 96
  of equivalent fractions, 264, 266
  and fractions, 258
  and multiplication, 72
  numbers and numerals, 1
Simple closed curves, 64
Square
  area, 112
  inch, centimeter, 118
Subtraction,
  and division, 75
  estimating sums and differences, 158
  finding quotients, 204
  missing addends, 24
  number line, 25
  of 2-digit numbers, 49
  of 3-digit numbers, 51
Sums
  estimating, 158
  of 2-digit numbers, 36
  of 3-digit numbers, 38

Tangent to a circle, 148
Terms for fractions
  higher and lower, 280
  lowest, 282
Thousands, 12

Units
  of area, 112, 118
  converting, 130
  inches, feet, yards, miles, 129
  of length, 112, 115
  liquid measure, 128
  of volume, 112, 122

Vertex
  of a cube, 192
  of a pyramid, 196
Volume
  finding, 122
  length and area, 112

Yard, 129

Zero
  in division, 94
  in multiplication, 82
  numerators, 274

EICHOLZ • O'DAFFER • BRUMFIEL • SHANKS

ESM

ADDISON-WESLEY PUBLISHING COMPANY